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Polyà distribution in a parton description of high-energy scattering

Lay-Nam Chang
Department of Physics, University of Pennsylvania, Philadelphia, Pennsylvania 19104

Ngee-Pong Chang*
Max-Planck-Institut für Physik und Astrophysik, München, Germany
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We derive a multiplicity distribution law based on parton model ideas and asymptotic Koba-Nielsen-Olesen scaling. Specifically, the Polyà distribution is found to result if one demands consistency among Feynman-Yang scaling, a linear fit to the Wroblewski plot, and independent emission of secondaries. A comparison with data is presented.

I. INTRODUCTION

One of the more easily accessible pieces of experimental data in high-energy scattering is the multiplicity distribution. Recently Koba, Nielsen, and Olesen[1] have shown that if one accepted the scaling ideas of Feynman and Yang,[2] then such distributions themselves may scale. More specifically, if $P(n,\bar{n})$ is the distribution function at an energy for which the average multiplicity is $\langle n\rangle$, then, as $s\rightarrow\infty$,

$$\langle n\rangle P(n,\langle n\rangle)=\psi(n/\langle n\rangle)\,. \tag{1.1}$$

The quantity $\langle n\rangle$ grows as the energy increases, and it is expected that the corrections to KNO scaling become small at asymptotically high energies.

Slattery[3] has fitted Eq. (1.1) to the available data, using for n the numbers of charged prongs, and concluded that at present energies, the corrections are very small. This result is not expected since the original derivation breaks down for nonasymptotic energies. Indeed, if one demands strict validity of Eq. (1.1), at finite energies, one runs into conflict with unitarity.[4]

Lacking a good understanding of the basic processes at work, the best way to examine such precocious scaling behavior perhaps will be within the context of models incorporating phenomenological features. The parton picture furnishes several models of this kind. The simplest realization of the short-range correlation feature basic to many such models is the multiperipheral model. The short-range correlation is explicit here; the secondaries are emitted independently. However, an examination of the resulting distribution shows *no* asymptotic KNO scaling.[5] This is of course not to be taken to mean that KNO scaling is incompatible with the parton picture. Rather, we may ask if such scaling may be used to constrain the picture.

In this paper, we indicate the form that such constraints may take. We shall abstract two simple properties of the parton picture: firstly, that scattering amplitudes exhibit Feynman-Yang scaling, and secondly, that secondaries are emitted independently. We shall demand compatibility with a linear fit to $\langle n\rangle$ of the dispersion $(\langle n^2\rangle-\langle n\rangle^2)^{1/2}$; that is, we shall demand a good fit to the Wroblewski plot.[6] We are then able to derive that the multiplicity follows a Polyà distribution[7] rather than the naively expected Poisson distribution. This distribution agrees well with the data, considering the crudity of our picture. The demand of consistency with the Wroblewski plot is made initially in our study on the empirical ground that it seems to fit the data very well; however, our results have led us to a possible picture of the production process, which we discuss in the conclusion.

In Sec. II we define our model and demonstrate the relevance of the Polyà distribution. We confine our attention to neutral particles here. Sec. III deals with the generalization to describe charged multiplicities. In line with the general spirit of the picture, we consider charge conservation only on the average. The consequent final state of the excited system, it turns out, is the coherent state of a ghost-free relativistic oscillator. A comparison with data is then carried out. We conclude with some remarks on possible implications in Sec. IV.

II. THE POLYÀ DISTRIBUTION

In this section we derive the basic law of statistics of multiparticle production in a parton picture from the following three assumptions:

(1) Feynman-Yang scaling, i.e., all inclusive cross sections are s-independent as $s\rightarrow\infty$, and are functions only of x and $\vec{p}_\perp$ for each observed particle;

(2) independent emission of particles; more pre-

cisely the multiparticle inclusive cross sections are factorizable in the form given by Eq. (2.6)[8];

(3) asymptotically, a linear fit to the Wroblewski plot holds.

The first assumption is very well justified experimentally, at least for one-particle inclusive cross sections, while the second assumption is on a less firm basis experimentally. The third assumption is totally motivated by the Wroblewski plot and the Slattery analysis of the KNO scaling.

In the parton picture, the infinite energy limit of the scattered state may be written as

$$\Phi = N^{-1}\left[|0\rangle + \lambda_1 \int \frac{dx}{x} d^2k\, \varphi_1(x,\vec{k})|x,\vec{k}\rangle + \frac{\lambda_2}{\sqrt{2!}}\int \frac{dx}{x} d^2k \frac{dx'}{x'} d^2k'\, \varphi_2(x,\vec{k},x',\vec{k}')|x,\vec{k},x',\vec{k}'\rangle + \cdots\right]. \tag{2.1}$$

In Eq. (2.1) we have suppressed the label for the leading particles so that $|0\rangle$ denotes the elastic state, while $|x,\vec{k}\rangle$ denotes the inelastic state with an additional "radiated" pion with $p_z = xW$, $\vec{p}_\perp = \vec{k}$. If x is positive, then, by momentum conservation the right-moving leading particle has its $p_z = (1-x)W$; $x<0$ implies that the pion is in the left-moving system.

In Eq. (2.1) we have built in the Feynman-Yang scaling limit by making the structure functions $\varphi_1(x,\vec{k})$, $\varphi_2(x,\vec{k},x',\vec{k}')$, ... independent of W. However, in principle the s dependence of exclusive cross sections is absorbed in the parameters λ_1, λ_2, ..., which may vanish with $s\to\infty$.

Now we make the ansatz for independent emission in the parton picture by assuming a factorizable φ (Ref. 8):

$$\varphi_n(x,\vec{k},x',\vec{k}',\ldots) = \varphi(x,\vec{k})\varphi(x',\vec{k}')\cdots . \tag{2.2}$$

It then follows that we can treat independently the right- and left-moving systems and derive relations among inclusive cross sections for the right and left fragments independently.

If we introduce the generating function

$$f(F) \equiv 1 + g_1F + g_2F^2 + \cdots , \tag{2.3}$$

$$F = \int_0^1 \frac{dx}{x} d^2k\,|\varphi(x,\vec{k})|^2 , \tag{2.4}$$

then

$$\langle A^\dagger(x,\vec{k})A(x,\vec{k})\rangle = |\varphi(x,\vec{k})|^2 \frac{d\ln f}{dF} , \tag{2.5}$$

$$\langle A^\dagger(x,\vec{k})A^\dagger(y,\vec{k}')A(y,\vec{k}')A(x,\vec{k})\rangle = |\varphi(x,\vec{k})|^2|\varphi(x',\vec{k}')|^2 \frac{1}{f}\frac{d^2f}{dF^2}, \tag{2.6}$$

etc. $A(x,\vec{k})$, $A^\dagger(x,\vec{k})$ are annihilation and creation operators for the pion. In Eq. (2.3) the constants $g_1, g_2, \ldots$ come from summing over the fragments of the left-moving system and are related to the parameters $\lambda_1, \lambda_2, \ldots$ in Eq. (2.1). [If in Eq. (2.1) we "forgot" about the left fragments, then the average expectation values in Eqs. (2.5) and (2.6) would have everywhere $|\lambda_1|^2$ instead of g_1 and $|\lambda_2|^2$ instead of g_2, etc.]

It should perhaps be emphasized again at this point that independent emission does not imply that the statistics are a Poisson distribution. However, by an additional assumption of "ignorance," one would be led to an f of the form $e^{\lambda F}$ and the correlation function $f_2 = \langle n^2\rangle - \langle n\rangle^2 - \langle n\rangle$ would vanish, in direct contradiction with experiment.[3]

Let us now insist on satisfying asymptotically the linear fit to the Wroblewski plot, i.e.,

$$D^2 \equiv \langle n^2\rangle - \langle n\rangle^2 = \frac{1}{\alpha}\langle n\rangle^2, \quad s\to\infty; \tag{2.7}$$

then it is easy to show that f must be given by

$$f = \left(1 - \frac{F}{c}\right)^{-\alpha}, \tag{2.8}$$

where

$$c = \langle n\rangle + \alpha . \tag{2.9}$$

Proof. By definition,

$$\langle n\rangle = F\frac{d\ln f}{dF} , \tag{2.10}$$

$$\langle n^2\rangle - \langle n\rangle = \frac{F^2}{f}\frac{d^2f}{dF^2} . \tag{2.11}$$

Wroblewski's linear fit translates into the differential equation (calling $d\ln f/dF \equiv y$)

$$\frac{dy}{dF} = \frac{1}{\alpha}y^2 , \tag{2.12}$$

whose solution is

$$\frac{1}{y} = -\frac{F}{\alpha} + \frac{c}{\alpha} \tag{2.13}$$

and Eq. (2.8) results upon performing one further integration.

Equation (2.8) is the generating function of the negative binomial statistics or the Polyà distribution. In the limit as $\alpha\to\infty$ and c/α is held fixed at $1/\lambda$, the Polyà statistics approach those of Poisson: $e^{\lambda F}$. An important feature to note here is the fact that for all finite, positive α, the Polyà statistics are broader than the Poisson statistics.

For convenience, we list here a few simple mathematical properties of the negative binomial or Polyâ statistics[7]:

$$P_n=(1-x)^\alpha x^n \frac{(n+\alpha-1)!}{n!(\alpha-1)!}\,, \tag{2.14}$$

$$\langle n\rangle=\frac{x}{1-x}\,\alpha\,, \tag{2.15}$$

$$\langle n^2\rangle=\frac{\alpha+1}{\alpha}\langle n\rangle^2+\langle n\rangle\,, \tag{2.16}$$

$$\langle n^3\rangle=\frac{(\alpha+1)(\alpha+2)}{\alpha^2}\langle n\rangle^3+\frac{3(\alpha+1)}{\alpha}\langle n\rangle^2+\langle n\rangle\,, \tag{2.17}$$

$$\langle n^4\rangle=\frac{(\alpha+1)(\alpha+2)(\alpha+3)}{\alpha^3}\langle n\rangle^4 +6\frac{(\alpha+1)(\alpha+2)}{\alpha^2}\langle n\rangle^3+7\frac{\alpha+1}{\alpha}\langle n\rangle^2+\langle n\rangle, \tag{2.18}$$

$$\langle n^5\rangle=\frac{(\alpha+4)!}{\alpha!\alpha^4}\langle n\rangle^5+10\frac{(\alpha+3)!}{\alpha!\alpha^3}\langle n\rangle^4 +25\frac{(\alpha+2)!}{\alpha!\alpha^2}\langle n\rangle^3+15\frac{\alpha+1}{\alpha}\langle n\rangle^2+\langle n\rangle. \tag{2.19}$$

In general, for q a positive integer

$$\langle n(n-1)\,\cdots\,(n-q+1)\rangle=\frac{(\alpha+q-1)!}{\alpha!\alpha^{q-1}}\langle n\rangle^q\,. \tag{2.20}$$

An important result we shall use for Sec. III is the composition law for two Polyâ statistics, which we shall state in the form of a theorem.[7]

Theorem. Let P_n, $P_{n'}$ be independent Polyâ distributions in n, n', respectively, with parameters α, x and α', x, respectively. Then the combined distribution law is again a Polyâ distribution parametrized by $\alpha+\alpha', x$.

Proof.

$$P_N\propto\sum_{n+n'=N} x^N\frac{(n+\alpha-1)!}{(\alpha-1)!n!}\,\frac{(n'+\alpha'-1)!}{(\alpha'-1)!n'!}=x^N\varphi_N\,,$$

where

$$\varphi_N=\frac{1}{2\pi i}\oint\frac{d\lambda}{\lambda^{N+1}}(1-\lambda)^{-\alpha}(1-\lambda)^{-\alpha'}$$

$$=\frac{1}{2\pi i}\oint\frac{d\lambda}{\lambda^{N+1}}(1-\lambda)^{-\alpha-\alpha'}$$

$$=\frac{(N+\alpha+\alpha'-1)!}{N!(\alpha+\alpha'-1)!}\,.$$

The properly normalized combined statistical distribution therefore is

$$P_N=(1-x)^{\alpha+\alpha'}x^N\frac{(N+\alpha+\alpha'-1)!}{N!(\alpha+\alpha'-1)!}\,.$$

Q.E.D.

Lastly, we remark that the Polyâ statistics can be simply expressed as a compound Poisson distribution,[7] viz.,

$$P_N(\alpha,x)=\int_0^\infty\frac{d\lambda}{\lambda}e^{-\lambda y}\frac{(\lambda y)^\alpha}{(\alpha-1)!}\left(e^{-\lambda}\frac{\lambda^n}{n!}\right), \qquad y\equiv\frac{1-x}{x},\quad 0<x<1\,. \tag{2.21}$$

III. CHARGED-PARTICLE DISTRIBUTION

In Sec. II we derived the law of multiparticle production statistics in a parton picture and relied heavily on the remarkable linear fit to the Wroblewski plot. Charge and all other quantum numbers were ignored in that derivation. For an actual comparison with experiment, of course, charge cannot be ignored, especially in view of the recently available data on charged and neutral multiplicity correlations. This problem of charge and isospin conservation has received considerable attention within the past year, resulting in several classes of models depending on whether pions are emitted independently (charge conservation on the average) or in charged and neutral pairs (σ, ρ, ω models).[9] From an experimental point of view, the problem of distinguishing between these different but simple models of partition of charge among the produced pions is complicated by the fact that observed multiplicity correlations depend also on the production probability of N $(=n_+ + n_- + n_0)$ pions.

There are two attitudes that one can take. One is to look directly at experimental data and apply the statistics test to n_-, say, and ask if the Polyâ distribution in n_- fits the available multiplicity data. The answer is yes, for lab energies greater than 60 GeV, with the fit getting better at higher energies. (This is in accord with the well-known observation that multiplicity distributions become broader than Poisson distributions as energy increases, while for lower energies they are narrower than those of Poisson.[3]) We shall return to this later.

The other attitude would be to make further assumptions about the production of mass clusters and their subsequent "decay" into pions and to then confront the resulting predictions with experiment. Let us therefore go back to our parton picture of the scattered state. In line with our remark about independent emission, we shall focus our attention on, say, the right-moving system and ask what the state vector looks like for an independent-emission Polyà distribution model. It is instructive to write down the corresponding state vector for a Poisson distribution model. It is simply the coherent state

$$\mathfrak{N}^{-1}\exp\left[\lambda\int_0^1 \frac{dx}{x}\, d^2k\, \varphi(x,\vec{k})a^\dagger(x,\vec{k})\right]|0\rangle\ . \tag{3.1}$$

[Energy-momentum conservation can be understood in the sense described in Eq. (2.1) of Sec. II.]

For a Polyà model, the new coherent state is

$$\mathfrak{N}^{-1}\exp\left[\lambda(\tau+\alpha-1)^{1/2}\int_0^1 \frac{dx}{x}\, d^2k\, \varphi(x,\vec{k})a^\dagger(x,\vec{k})\right]|0\rangle\ , \tag{3.2}$$

where

$$\tau = \int\frac{dx}{x}\, d^2k\, a^\dagger(x,\vec{k})a(x,\vec{k}) \tag{3.3}$$

is the number operator for the mesons. At this point it should be remarked that this coherent state has in fact already been discovered by Giovanini and Predazzi and by Fujiwara and Kitazoe,[10] starting from totally different physical grounds. It should also be remarked, and we admit that the connection is at best tenuous at present, that in a recent study of an infinite component field of a relativistic oscillator model with no ghosts[11] the resonance states in a unitary representation of SU(3, 1) were found to occur in a coherent state in the form given here, with the interpretation, however, that $\alpha-1$ is the mass squared of the Regge intercept, which for baryons would be equal to 1, i.e., $\alpha=2$ from the infinite-component-field point of view. This rather tenuous interpretation will be found to be in remarkable agreement with the ratio $D/\langle n\rangle=\frac{1}{2}$ in the Wroblewski plot.

But we now come back to the point about charge partition among the pions. Lacking detailed knowledge and on the grounds of simplicity we generalize Eq. (3.2) to the case of charged pions by simply writing

$$\mathfrak{N}^{-1}\exp\left[\lambda(\tau+\alpha-1)^{1/2}\int\frac{dx}{x}\, d^2k\, \varphi(x,\vec{k})[a_+^\dagger(x,\vec{k})+a_-^\dagger(x,\vec{k})+a_0^\dagger(x,\vec{k})]\right]|0\rangle\ , \tag{3.4}$$

where τ is the number operator for the charged and neutral mesons. Charge is conserved on the average in this model.

The multiplicity statistics can be obtained from Eq. (3.4) by noting that it is equivalent to the statistics of producing N pions (of whatever charge) being in a Polyà distribution

$$P_R(N)=(1-\lambda)^\alpha\lambda^N\frac{(N+\alpha-1)!}{N!(\alpha-1)!}\ , \tag{3.5}$$

and the partition of charge among these right-moving pions being given by

$$P_R(N|\,n_+,n_-,n_0)=\frac{N!}{n_+!\,n_-!\,n_0!}(\tfrac{1}{3})^N . \tag{3.6}$$

Therefore,

$$P_R(n_+,n_-,n_0)=(1-\lambda)^\alpha(\tfrac{1}{3}\lambda)^{n_++n_-+n_0}\times\frac{(n_++n_-+n_0+\alpha-1)!}{n_+!n_-!n_0!(\alpha-1)!}\ . \tag{3.7}$$

It is not *a priori* clear that this extremely crude statistical assumption about charge partition can be compatible with the recent CERN ISR data on charged neutral multiplicity correlation.[12] Indeed, if we used a Poisson distribution for $P(N)$ instead of Eq. (3.5), the resulting correlation (or rather the lack of it) would disagree with experiment.

To calculate the correlation, let us first sum over n_+ in Eq. (3.7):

$$P_R(n_-,n_0)=\left(\frac{1-\lambda}{1-\frac{1}{3}\lambda}\right)^\alpha\left(\frac{\lambda}{3-\lambda}\right)^{n_-+n_0}\times\frac{(n_-+n_0+\alpha-1)!}{n_-!\,n_0!\,(\alpha-1)!}\ . \tag{3.8}$$

Before we can confront these ideas with experiment, for pp scattering, we must recall that our discussion has been confined exclusively to the right-moving system. For an experiment, n_-, n_0 include those that come from both hemispheres. By symmetry in pp scattering, the left-moving system will have a Polyà distribution of the same parameters λ and α. By our theorem of Sec. II

TABLE I. Comparison of the KNO moment coefficients between Slattery's extrapolation from the data and those of the Polyà distribution. The value of 2α is chosen to be the expected asymptotic value of 4. The experimental values are taken from the curve and quoted numbers of Ref. 17.

q	c_q (Slattery)	c_q (Polyà)	c_q (expt.)
2	1.24	1.25	1.25 ± 0.01
3	1.81	1.80	1.82 ± 0.02
4	2.97	2.52	2.96 ± 0.05
5	5.36	4.00	5.27 ± 0.11
6	10.4	7.20	10.3 (from curve)
7	21.6	14.4	21 (from curve)

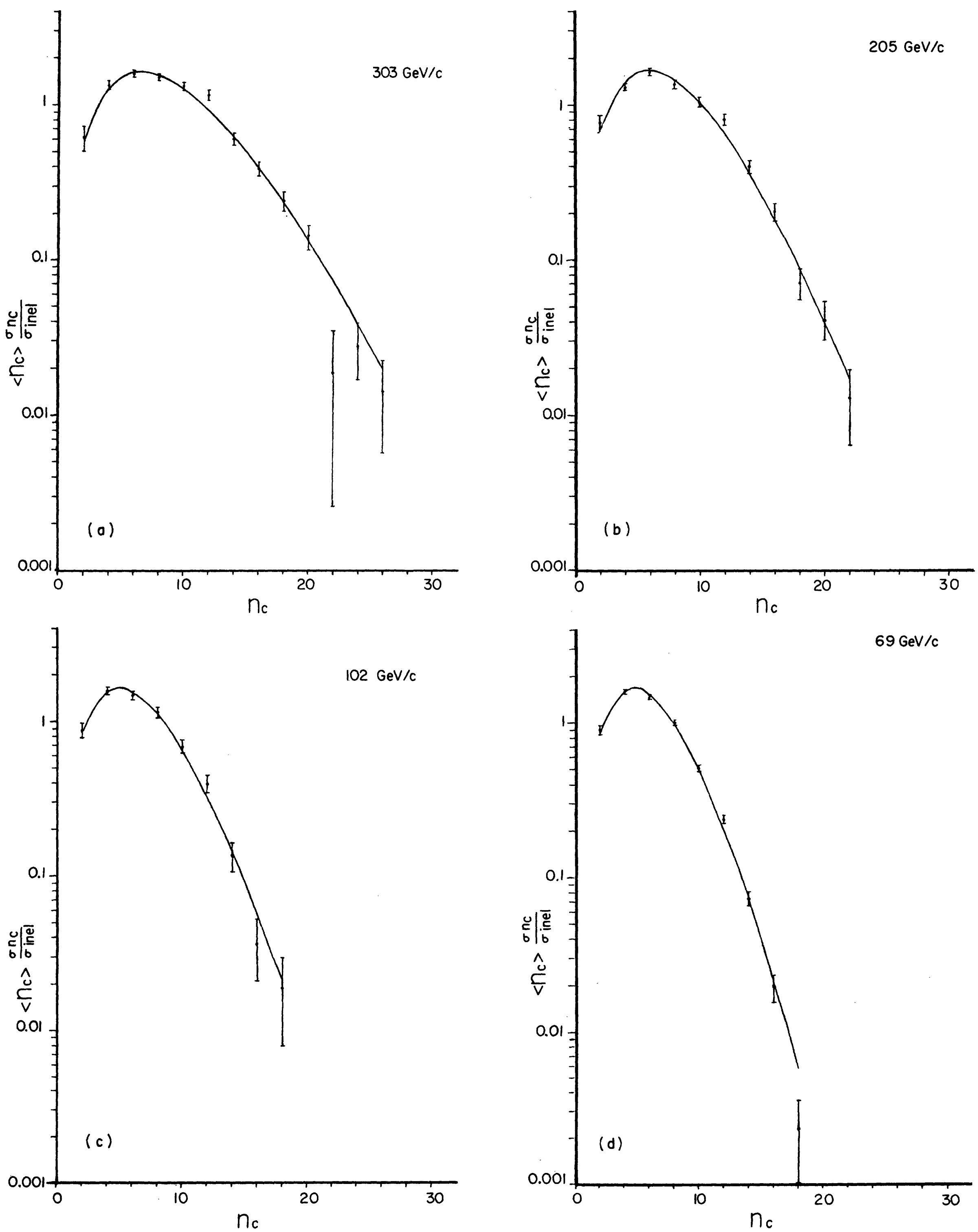

FIG. 1. (a)–(d) Comparison between data at 303, 205, 102, and 69 GeV/*c* and Polyâ distribution of the parameters given in Table II. The data points and errors are taken from the Slattery compilation, except for new data corrections given by F. T. Dao *et al.*, Phys. Lett. 45B, 399 (1973); G. Charlton *et al.*, paper submitted to the XVI International Conference on High Energy Physics, Chicago-Batavia, Ill., 1972 (unpublished); V. V. Ammosov *et al.*, Phys. Lett. 42B, 519 (1972). See also Ref. 13.

we find

$$P(n_-, n_0) = \left(\frac{1-\lambda}{1-\frac{1}{3}\lambda}\right)^{2\alpha}\left(\frac{\lambda}{3-\lambda}\right)^{n_-+n_0} \times \frac{(n_-+n_0+2\alpha-1)!}{n_-!\,n_0!(2\alpha-1)!}. \qquad (3.9)$$

Now, for fixed n_-, Eq. (3.9) is again a Polyâ distribution in n_0, with an effective "α" $= n_- + 2\alpha$, so from Sec. II, we know

$$\langle n_0\rangle_- = \frac{\lambda}{3-2\lambda}(n_- + 2\alpha), \qquad (3.10)$$

a linear rise, which is indicated by the ISR experiment.[12]

It is of course an obvious statement that our model makes this linear correlation for the multiplicities from each jet separately, so that in $\pi^- p$ collisions, the multiplicities among the fragments of p should obey this correlation, just as the multiplicities among the fragments of π should obey a similar charged-neutral correlation with, however, different slopes and intercepts.

Lastly, we sum over n_0 and find the multiplicity distribution for n_-, in pp experiments:

$$P(n_-) = (1-p)^{2\alpha} p^{n_-} \frac{(n_- + 2\alpha - 1)!}{n_-!(2\alpha-1)!}, \qquad (3.11)$$

$$p = \frac{\lambda}{3-2\lambda}, \quad 1 > p > 0. \qquad (3.12)$$

The parameters p and α are related to $\langle n_-\rangle$ and D_{n_-} in the following way:

$$\langle n_-\rangle = \frac{p}{1-p} 2\alpha, \qquad (3.13)$$

$$\langle n_-^2\rangle - \langle n_-\rangle^2 = \frac{1}{2\alpha}\langle n_-\rangle^2 + \langle n_-\rangle. \qquad (3.14)$$

As we remarked earlier, if α is related to the intercept of the linear baryon Regge trajectory in a way suggested by the ghostfree oscillator model, then $\alpha = 2$. Therefore, in the Wroblewski plot,

$$D = \frac{1}{(2\alpha)^{1/2}}\langle n_-\rangle, \quad s \to \infty \qquad (3.15)$$

$D/\langle n_-\rangle \to \frac{1}{2}$. However, until that connection is spelled out in more physical detail, it remains only a suggestion.

Koba, Nielsen, and Olesen[1] have suggested the scaling hypothesis for s large:

$$\langle n_-^q\rangle = c_q \langle n_-\rangle^q \qquad (3.16)$$

and derived from this, assuming *absence* of lower order terms in Eq. (3.16) the well-known universal function

$$\langle n_-\rangle P(n_-) = \psi(n_-/\langle n_-\rangle). \qquad (3.17)$$

TABLE II. Fitted values of 2α for a Polyâ distribution in n_-.

p_{lab} (GeV/c)	2α	χ^2
303	5.75	22
205	9.0	13.4
102	11.50	4.8
69	44	16.9

As has been shown by Chodos, Rubin, and Sugar,[4] Eq. (3.16) taken strictly violates the positivity of the probability distribution.

In our approach, there are correction terms to the asymptotic scaling which are not strictly zero, and even though we find asymptotic KNO scaling [see Eq. (2.20)] there is, strictly speaking, no energy-independent universal function, although it may well be true in some approximate sense.

Slattery[3] has analyzed the KNO scaling assumption and found precocious KNO scaling in n_{ch} experimentally for all lab energies above 59 GeV. In his analysis, he found asymptotic values of the coefficients c_q from data. It is amusing that the Polyâ distribution gives a prediction for those c_q which is not far from the quoted extrapolated values of Slattery. (See Table I.)

We have fitted the Polyâ distribution to available multiplicity data and found a reasonable fit of Eq. (3.11) to data (see Fig. 1). At 303 GeV, the best fit is for $2\alpha = 5.75$. Again, if $\alpha - 1$ is read as the mass squared of the lowest contributing resonance in an infinite component field, this value of α corresponds to the $N^*(1238)$ resonance. But this interpretation fails for lower energies since the best fitted value for α increases as energy decreases, in line with the fact that multiplicity distributions in n_- become narrower than Poisson at low energies.

The parameters α that correspond to best fit at each incident energy are given in Table II.[13] The general trend of larger values of α for lower energies is due to the well-known way in which the Polyâ distributions approach the Poisson distributions. Because of this fact, the precise value of α at lower energies is not to be taken seriously. We remark at this point that the extrapolation of the ideas of independent emission to lower energies involves the question of the choice of variable n_- or $n_{\text{ch}} - 2$, etc., which we have not fully investigated.

IV. CONCLUSION

Compared with other phenomenological fits to the data, such as the Slattery analysis of precocious KNO scaling in n_{ch} and other analyses involving truncated Gaussian distributions and compound Poisson distributions,[14] we cannot truly say our fit is better. We attribute this to the crudity of our model since, firstly, charge conservation has not been implemented except on the average and, secondly, the assumption of independent emission may not be justified until one gets to the ISR energies. However, at energies of the NAL range, the fit of our Polyâ distribution is as good as other reported fits to the multiplicity, indicating that the lack of strict charge conservation may not be an important factor. At lower energies, the question of a "right" choice of variables for the Polyâ distribution is an open one.

The independent-emission feature of our model is motivated from a different point of view than that in the multiperipheral model. In the simple parton picture we are using, both right- and left-moving systems, after initial collison, contain partons in various excited states. The number of levels in each system could be fairly large; indeed, we should allow the partons to interact after the collision according to their own interaction potential.[15] In terms of diagrams, we allow particles from each multiperipheral chain to further interact; we must, in the language of dualists, sum over "fishnet" diagrams.[16]

If the basic dynamical force holding the partons together is that of a ghost-free relativistic oscillator, then in the corresponding bremsstrahlung models the radiated partons will be in accordance with the Polyâ distribution. If a multiperipheral chain is to be thought of as a set of emission diagrams with no final-state interaction, then our model is to be thought of as emission with final-state (harmonic oscillator) interactions.

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*On sabbatical leave from The City College of the City University of New York, New York, N. Y. 10031.

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