

are described in terms of a statistical production process.

In this work it has been shown that the multiparticle variables may serve as tools for the study

of $\bar{p}p$ annihilation dynamics. It will thus be of interest to further explore the usefulness of these variables by applying them to a large variety of reactions at different energies and multiplicities.

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Description of diffractively produced multipion enhancements

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A model of the diffractive production of multipion states on hydrogen and heavy nuclei is presented. The model is statistical in nature and incorporates the effects due to the competition between many open channels at high mass. In this picture, multipion states are produced in a two-stage process: An excited meson state (fireball) is produced peripherally, through Pomeranchukon exchange, which then proceeds to decay in a manner described by the strong statistical bootstrap. As an application of this model, we study the production of three and five charged pions in pion-induced reactions on hydrogen, deuterium, and heavier nuclei. In particular, the size of the cross section for diffractively producing five charged pions is related uniquely, through factorization, to that for producing three charged pions. The results, including mass distributions, are in good agreement with the available data.

I. INTRODUCTION

The nature of the multipion enhancements diffractively produced on hydrogen and complex nuclei has been a subject much discussed in recent years. There have been several experiments,¹⁻⁵ but a comparatively small amount of theoretical work to explain them.^{6,7}

Recently, it has been proposed that these enhancements might be considered as the result of a statistical superposition of a continuum of pion resonances or fireballs.^{8,9} The validity of this description is expected to improve as the mass of the fireball increases. While the A_1 three-pion enhancement is on the borderline of validity for statistical considerations, the A_3 three-pion en-

hancement, and the five-pion enhancement, being at higher excitation should be better described by such a treatment. Higher-multiplicity pion systems should then be ideal for study using statistical resonance superposition. Some aspects of the model used here to describe these phenomena have already been presented in Refs. 8, 9, and 10.

Specifically, we assume that a superposition of resonances is excited peripherally in a high-energy hadronic reaction through Pomeranchukon exchange. The fireballs then decay into the final state under consideration, e.g., 3π , 5π , according to branching ratios determined essentially by the statistical bootstrap. The spectral shape is determined by this program. The absolute cross-section normalization of course depends on Pomer-

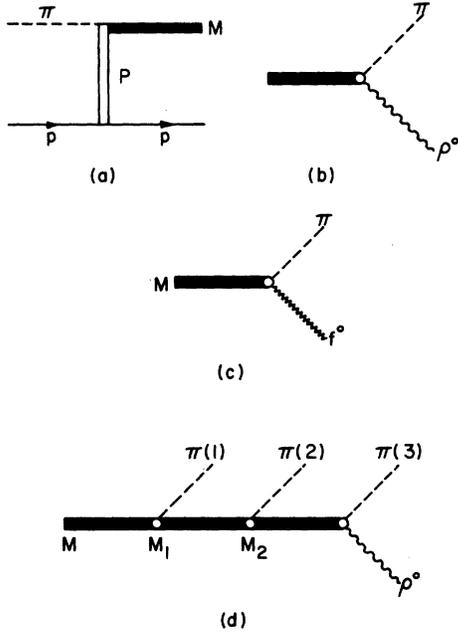


FIG. 1. (a) Dynamical mechanism for the diffractive excitation of a heavy fireball of mass M . (b) The dominant fireball decay vertex for three-pion production in the A_1 region. (c) Same as (b), in the A_3 region. (d) The dominant linear decay chain for 5π production.

anchukon-exchange parameters. However, using factorization, we can relate the cross sections for various enhancements one to another. Furthermore, the knowledge of production amplitudes for diffractive processes on protons can be used to calculate coherent production on nuclei,¹¹ as outlined below.

II. DESCRIPTION OF FIREBALL DECAY, AND DIFFRACTIVE PRODUCTION ON PROTONS

The proposed picture of peripheral, diffractive production of charged 3π and 5π modes is shown in Fig. 1. We assume that the relevant density of pion resonances is given by a continuum, together with the discrete π^+ , π^- , and ρ^0 states.

The fireball decay is described by a linear chain, as indicated by the strong statistical bootstrap solution for the density of states.¹²⁻¹⁴ In this model, an ideal high-mass resonance will produce n particles in its first-generation decay, with the probability distribution¹² $P(n) = (\ln 2)^{n-1} / (n-1)!$, independent of the initial fireball mass. It is easily seen that two-body decays dominate over three- and four-body ones [$P(2) = 69\%$, $P(3) = 24\%$, $P(4) = 6\%$, ...], with higher-multiplicity channels giving negligible contributions (in fact, the average number of decay products is 2.4). Moreover, it is found that the dominant contributions come from

configurations in which one heavy resonance is produced, together with a number of light particles (bearing in mind the above probability distribution), except at the end of the chain where only light particles are produced. The length of the decay chain is proportional to the mass of the parent fireball.

We emphasize that these properties hold, in an average sense, for very massive ideal fireballs: To describe the nonasymptotic situation considered here, we will make use of them in a slightly modified form, motivated by experiment.

Our description of the fireball decay chain is then as follows: At each vertex, except the last one, a single (charged) pion is emitted, together with a heavy resonance which we describe statistically. The chain terminates by the emission of a pion and a ρ^0 meson (we take the emission of correlated pion pairs to be dominated by the discrete ρ resonance, as is strongly indicated by experiment). Note also that we consider only two-body decays involving a pion and a fireball at each vertex because of the limited masses of the parent resonances under consideration: For high fireball masses ($M \geq 5$ GeV) the contribution of light particles other than pions (in general, η , K , ρ , ω , ...) being emitted earlier in the decay chain should presumably be included in order to achieve asymptotic bootstrap consistency. Finally, for the case of A_3 production, the experimental fact of the dominance of the $f^0\pi$ mode requires the inclusion of the f^0 as a discrete state. As far as the 5π enhancement is concerned, however, a possible $f^0\pi$ contribution is small compared to the dominant mechanism of chain decay described above and illustrated in Fig. 1(d).

We write for the differential cross section for the processes of Fig. 1

$$\begin{aligned} d\sigma = & \frac{(2\pi)^4}{2[\lambda(s, m_p^2, m_\pi^2)]^{1/2}} \frac{d^3 p_f}{(2\pi)^3 2E_f} \\ & \times \prod_{i=1}^n \frac{d^3 q_i}{2E_i} \delta^4(p_1 + p_2 - p_f - \sum q_i) \\ & \times R(s, t, M) \omega(M, m_1 \cdots m_n, q_1 \cdots q_n). \end{aligned} \quad (1)$$

In Eq. (1), $R(s, t, M)$ is the Pomernchukon-exchange amplitude squared for the diagram of Fig. 1(a), corresponding to a total center-of-mass energy squared s , and four-momentum transfer squared t , M being the mass of the produced fireball. The momenta and energies are labeled as follows: p_1 , p_2 , and p_f are the four-momenta of the incident pion and proton, and outgoing proton, respectively (E_f is the outgoing proton energy). The q_i and E_i are the four-momenta and energies of the n decay products (π, ρ, f) of mass m_i of the produced fireball.

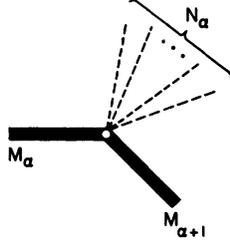


FIG. 2. General decay vertex of a fireball of mass M_α into a fireball of mass $M_{\alpha+1}$ and n_α light particles.

At high s , $[\lambda(s, m_p^2, m_\pi^2)]^{1/2} \rightarrow s$, and we have $d^3p_f/2E_f \rightarrow \pi dt dM^2/2s$, so that

$$\frac{d\sigma}{dM^2} = \frac{\pi^2}{2s^2} \int dt R(s, t, M) \times \left[\prod_{i=1}^{n_\alpha} \frac{d^3q_i}{2E_i} \delta^4(p_1 + p_2 - p_f - \sum q_i) \times \omega(M, m_1 \cdots m_n, q_1 \cdots q_n) \right]. \quad (2)$$

The quantity $\omega(M, m_1 \cdots m_n, q_1 \cdots q_n)$ is the branching ratio for the decay of an excited meson

$$Q = \frac{\int \cdots \int d^3K_{\alpha+1} \prod_{i=1}^{n_\alpha} d^3q_i \delta(M_\alpha - E_{\alpha+1} - \sum E_i) \delta^3(\vec{K}_{\alpha+1} + \sum \vec{q}_i)}{\int \cdots \int d^4K_{\alpha+1} \delta^+(K_{\alpha+1}^2 - M_{\alpha+1}^2) \prod_{i=1}^{n_\alpha} d^4q_i \delta^+(q_i^2 - m_i^2) \delta^4(K_\alpha - K_{\alpha+1} - \sum q_i)} \quad (3')$$

Here, K_α and $K_{\alpha+1}$ label the four-momenta of the decaying and produced fireballs of mass M_α and $M_{\alpha+1}$, respectively. The q_i and E_i represent the four-momenta and energies of the n_α light particles, as before.

The form (3) involves the ratio of the density of states of the *particular* decay mode within the parent fireball to the *total* density of states of the parent fireball. This may be viewed as the explicit

state of mass M into the particular channel under consideration; its form is determined by the statistical assumption outlined above and in Refs. 8 and 9. In particular, it factorizes into the contributions of the successive decay vertices of the treelike chain; explicitly, at each vertex, we write the following expression for the branching ratio of a fireball of mass M_α to decay into a fireball of mass $M_{\alpha+1}$, plus n_α light particles (Fig. 2),

$$\omega_\alpha(M_\alpha, M_{\alpha+1}; m_1 \cdots m_{n_\alpha}) = \left[\frac{V}{(2\pi)^3} \right]^{n_\alpha} \frac{\rho(M_{\alpha+1})}{2M_{\alpha+1}} \times \frac{Q(M_\alpha, M_{\alpha+1}; m_1 \cdots m_{n_\alpha})}{\rho(M_\alpha)}. \quad (3)$$

We denote by $\rho(m)$ the density of hadronic states at mass m , and by V a characteristic hadron volume, both quantities to be specified later.

The function $Q(M_\alpha, M_{\alpha+1}; m_1 \cdots m_{n_\alpha})$ is the ratio of noninvariant to invariant phase space, *evaluated in the rest frame of the decaying fireball*. Explicitly, we have

it realization of a competition mechanism between all available decay channels of the original excited meson state.¹⁶ The ratio of noninvariant to invariant phase space appears in (3) since the density of states in the statistical bootstrap is formulated in terms of noninvariant phase space, and we require an invariant matrix element.

As an example, the diagram of Fig. 1(d) leads to

$$\frac{d\sigma}{dM^2} = \frac{\pi^2}{2s^2} \int dt R(s, t, M) \left[\prod_{i=1}^4 \frac{d^3q_i}{2E_i} \delta^4(p_1 + p_2 - p_f - \sum q_i) \left(\prod_{\alpha=1}^2 \frac{d^3K_\alpha}{2K_\alpha^0} dM_\alpha^2 \right) \delta^4(K_1 - K_2 - q_2) \delta^4(K_2 - q_3 - q_4) \times \omega(M, m_1 \cdots m_4, q_1 \cdots q_4) \right], \quad (4)$$

where K_α is the four-momentum of the intermediate fireball of mass M_α (the factor in large parentheses, with associated delta functions, allows for summation over intermediate fireball momenta). For $\omega(M, m_1 \cdots m_4, q_1 \cdots q_4)$ we have the factorized form

$$\omega(M, m_1 \cdots m_4, q_1 \cdots q_4) = \omega_0(M, M_1; m_1) \omega_1(M_1, M_2; m_2) \times \omega_2(M_2; m_3, m_4), \quad (5)$$

where

$$\omega_0(M, M_1; m_1) = \frac{V}{(2\pi)^3} \frac{M^4 - (M_1^2 - m_1^2)^2}{M^2 \rho(M)} \frac{\rho(M_1)}{2M_1}, \quad (5')$$

$$\omega_1(M_1, M_2; m_2) = \frac{V}{(2\pi)^3} \frac{M_1^4 - (M_2^2 - m_2^2)^2}{M_1^2 \rho(M_1)} \frac{\rho(M_2)}{2M_2}, \quad (5'')$$

$$\omega_2(M_2; m_3, m_4) = \frac{V^2}{(2\pi)^6} \frac{M_2^4 - (m_3^2 - m_4^2)^2}{M_2^2 \rho(M_2)}. \quad (5''')$$

In the specific case of the decay chain shown in Fig. 1(d) we have $m_1 = m_2 = m_3 = m_\pi$ and $m_4 = m_\rho$.

It remains to specify $R(s, t, M)$. This is determined by the nature of the Pomeranchukon-exchange mechanism. We adopt the simple form

$$R(s, t, M) = \beta(t) s^2 / M^3 \quad (6)$$

appropriate for *PPR* triple-Regge coupling. A possible *PPP* contribution could be added without appreciably changing the results below. Such a parametrization was first used by Hwa in his diffractive excitation model,¹⁷ and while it is not expected to be valid for general M , it should provide a good description of the fireball mass distribution for the limited range of M we are considering. Here, $\beta(t)$ is a slowly varying function of t (for small t) which essentially reduces to a constant in the case of coherent reactions; further, the precise form of $\beta(t)$ is unimportant for our study of reactions on hydrogen since we integrate over t , and the only t dependence comes from this function.

III. COHERENT PRODUCTION ON NUCLEI

The coherent production of a pion fireball on a nucleus involves in general a coupling of channels for all excited states that couple diffractively to the initial pion. An optical-model scheme for calculating the amplitude for production in any channel has been described in detail by several authors.^{18,19} We argue here that because of the large number of states available for diffractive coupling to any pion fireball mass, statistical assumptions should be involved. Our statistical picture of Sec. II implies random phases⁸ for the various two-body diffractive amplitudes on a nucleon. As a result, it follows that coherent production of any fireball should be dominated by a one-step process in which the incident pion undergoes damping through the nucleus, making a diffractive transition on one nucleon, as in Sec. II; the diffractively produced fireball proceeds to go through the nucleus, being damped in its outgoing motion (the damping would correspond to dominantly incoherent nondiffractive processes).

The amplitude for such coherent production on heavy nuclei ($A \geq 10$) is given by¹¹

$$F(q^2) = f_{\text{hyd}}(0) 2\pi A \int_{-\infty}^{\infty} dz \int_0^{\infty} db b \exp(iq_L z) J_0(q_T b) \\ \times \rho(b, z) \exp[-\frac{1}{2}\sigma'_1 T(b)] \\ \times \exp[-\frac{1}{2}(\sigma'_2 - \sigma'_1) T(b; z)], \quad (7)$$

where

$$T(b) \equiv T(b; -\infty) = \int_{-\infty}^{\infty} A \rho(b, z') dz',$$

$$T(b; z) = \int_z^{\infty} A \rho(b, z') dz',$$

$$\sigma'_k = \sigma_k (1 - i\alpha_k).$$

We have put $f_{\text{hyd}}(0)$ for the forward production amplitude on hydrogen. Here, $\rho(b, z)$ is the single nucleon density in the nucleus [$\int \rho(\vec{r}) d^3 r = 1$], A is the number of nucleons; σ_1 and σ_2 are the total cross sections of particles 1 and 2 on nucleons, and α_1 and α_2 are the ratios of real to imaginary parts of the forward amplitudes for the elastic scattering of particles 1 and 2 on nucleons. In the present case, the indices 1 and 2 refer to the incident pion and outgoing fireball, respectively. The (b, z) are cylindrical coordinates for the projectile and the produced fireball. Finally, q_T and q_L are the transverse and longitudinal momentum transfers given by

$$q_T \approx p_{\text{lab}} \Theta,$$

$$q_L \approx (M^2 - M_0^2) / 2p_{\text{lab}},$$

where M_0 and M are the masses of the incident and outgoing particles, respectively, p_{lab} being the incident particle momentum in the laboratory.

Formula (7) has been used^{3,19} to analyze the coherent production on nuclei of 3π systems in the A_1 region, and of 5π systems. The resulting A dependence leads to a total cross section³ $\sigma_{A_1} \approx 26$ mb and $\sigma_{5\pi} \approx 17$ mb rather independent of the mass, within the region of the enhancements considered. The A_1 enhancement produced on nuclei looks much like that on protons with the A_2 cut away. We propose then that the 5π enhancement seen in nuclei is the same as the diffractive component on hydrogen, cut off more sharply at the high-mass end due to longitudinal momentum-transfer effects. This is consistent with a statistical resonance-superposition description of the enhancement on hydrogen.

Note that the resonances or fireballs produced in a one-step process as described above would decay predominantly outside the nucleus since they are moving so swiftly. In fact, the Lorentz contraction factor for a 15-GeV/ c resonance with a width of, say, 200 MeV assures this. The fact that the 5π fireballs have a large apparent mean free path in nuclei could then be a real effect, i.e., perhaps $\sigma_{5\pi}^{\text{tot}}$ is small; further experiments would be of interest in this connection.

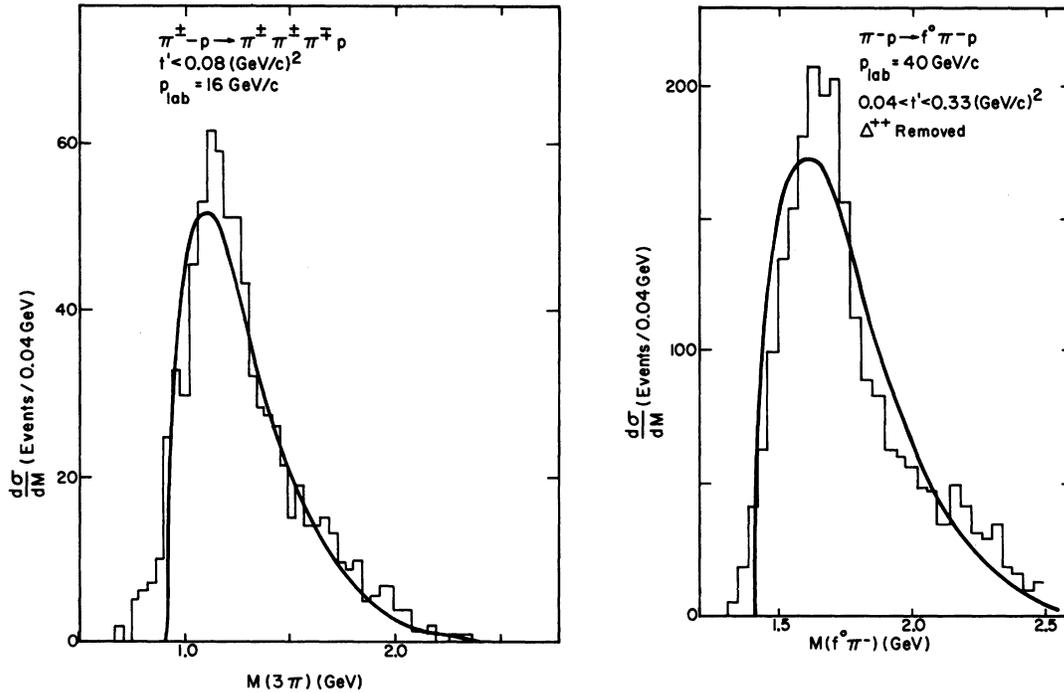


FIG. 3. (a) Combined data for the A_1 mass spectrum from the reactions $\pi^\pm p \rightarrow \pi^\pm \pi^\pm \pi^\mp p$ at 16 GeV/c (Ballam *et al.*, Ref. 2) compared with the contribution of the diagram of Fig. 1(b). Only events for which $t' < 0.08$ (GeV/c) 2 are included ($t' \equiv |t - t_{\min}|$). (b) The A_3 mass distribution ($f^0\pi$) in the reaction $\pi^- p \rightarrow \pi^+ \pi^+ \pi^- p$ at 40 GeV/c [$0.04 < t' < 0.33$ (GeV/c) 2 , Δ^{++} removed], compared with the contribution of diagram 1(c). Data from CIBS collaboration, Ref. 2.

IV. RESULTS OF CALCULATIONS AND DISCUSSION

We have calculated the mass distributions for the processes (i) $\pi^\pm p \rightarrow \pi^\pm \pi^\pm \pi^\mp p$; (ii) $\pi^\pm p \rightarrow \pi^\pm \pi^\pm \pi^\mp \pi^\mp p$; and (iii) $\pi^\pm A \rightarrow \pi^\pm \pi^\pm \pi^\mp \pi^\mp A$ (coherent production; A stands for a variety of nuclear targets), using the formalism developed above and applying it to the dominant diagrams illustrated in Fig. 1(b) and (c) contribute to 3π production in the A_1 and A_3 regions, respectively, and (d) is the dominant contribution to the 5π enhancement]. As mentioned above, we have obtained the spectral shapes for coherent production on nuclei by correcting the results on hydrogen for t_{\min} effects, in a manner to be described later in this section.

We have taken for the density of hadronic states the form^{12,20}

$$\rho(m) \approx cm^{-3} e^{m/T}, \quad (8)$$

with $T = 160$ MeV. This corresponds to the volume V appearing in Eq. (5) being that of a sphere of radius $r = 1.1$ F.

Figure 3 shows the results of the calculation of the $A_1(\rho\pi)$ and $A_3(f^0\pi)$ mass spectra on hydrogen, using the above methods along with experimental data from Ref. 2.

In Fig. 4, we give the prediction of the present

model for the spectrum of five charged pions diffractively produced on hydrogen in arbitrary units. However, given the absolute cross section for $\rho\pi$ production, the absolute value for 5π production can be computed. We do this below in the case of reactions on Freon, for which data of this type are available.⁴

Figures 5, 6, and 7 show the results of our calculations of coherent 5π spectra on heavy nuclei,³

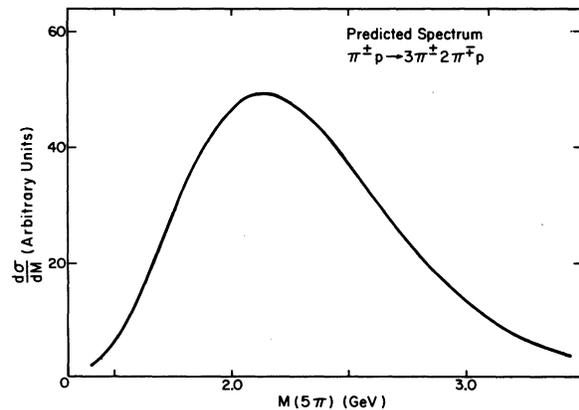


FIG. 4. Model prediction for the charged 5π mass spectrum diffractively produced on hydrogen.

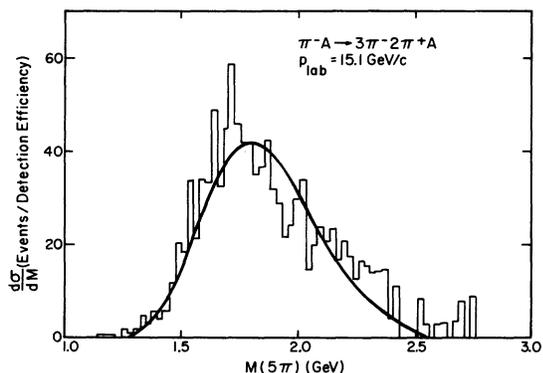


FIG. 5. The coherent five-pion spectrum observed by Bemporad *et al.* (Ref. 3), on complex nuclei, compared with the contribution of diagram 1(d). t_{\min} effects are included as explained in the text.

Freon,⁴ and deuterium,⁵ respectively. We have used for these calculations the results on hydrogen, modified by a multiplicative factor of $e^{-R^2 q_L^2/4}$ to account for nuclear t_{\min} effects. The values of the radius parameter squared R^2 used for various targets are listed, along with the reactions considered, in Table I. The normalization for heavy nuclei relative to hydrogen is provided by Eq. (7).

Absolute values for the cross sections for coherent production of three and five charged pions on Freon (C_2F_5Cl) at 16 GeV/c have been given by Huson *et al.*⁴ Assuming that one-step coherent production dominates as outlined in Sec. III, and that the $\rho\pi$ channel dominates the 3π cross section as is borne out by experiment, we can consider the ratio of production cross section $\sigma_{3\pi}/\sigma_{5\pi}$ on Freon as being determined by the same ratio on hydrogen, modified by the t_{\min} effects described above (this assumes comparable mean free paths in Freon for 3π and 5π fireballs). There are two

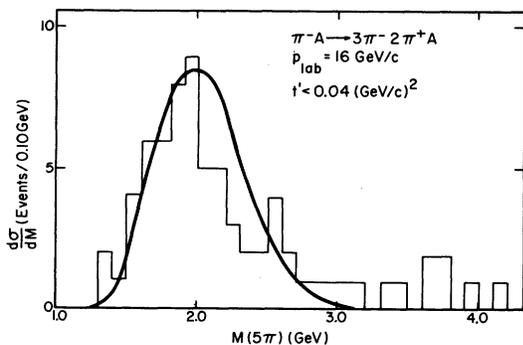


FIG. 6. The coherent five-pion spectrum observed by Huson *et al.*, on a freon (C_2F_5Cl) target, compared with the contribution of diagram 1(d), including t_{\min} effects. Only events with $t' < 0.04$ (GeV/c)² are included (Ref. 4).

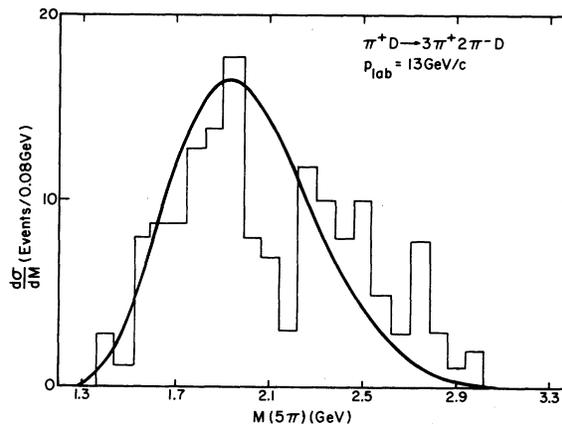


FIG. 7. The data of Paler *et al.* on coherent 5π production on deuterium (Ref. 5) is compared with the contribution of diagram 1(d).

distinct diagrams of the type shown in Fig. 1(d), corresponding to pions 2 and 3 being interchanged (we assume that no pion fireballs carry exotic quantum numbers). Moreover, isospin couplings at the vertices introduce a factor of $\frac{1}{2}$ for the diagram shown in Fig. 1(d) relative to that in 1(b). Taking all this into account, we find the result

$$\frac{\int [d\sigma(3\pi)/dM] dM}{\int [d\sigma(5\pi)/dM] dM} = \frac{\sigma_{3\pi}}{\sigma_{5\pi}} = 46 \quad (\text{theoretical}),$$

on Freon at 16 GeV/c incoming pion momentum. Experiment⁴ yields the result

$$\sigma_{3\pi}/\sigma_{5\pi} = 40_{-14}^{+26} \quad (\text{experimental}).$$

The agreement between theory and experiment is thus seen to be excellent.

Finally, we remark that there is a substantial dependence on energy of the 5π mass distribution. The peak is displaced to higher masses as the energy increases for a given target, and to lower masses as the target mass number increases for a given laboratory energy.

TABLE I. List of reactions on nuclei, with target rms radius $\langle r^2 \rangle_{\text{nuc}}^{1/2}$; this is related to the radius parameter R appearing in the t_{\min} factor $e^{-R^2 q_L^2/4}$ by $\langle r^2 \rangle_{\text{nuc}} = \frac{3}{2} R^2$.

Target	Mean mass number	$\langle r^2 \rangle_{\text{nuc}}^{1/2}$ (F)	Reaction (lab momentum)	Ref.
D	2	2.8	$\pi^+D \rightarrow 3\pi^+2\pi^-D$ (13 GeV/c)	5
Freon (C_2F_5Cl)	19	3.0	$\pi^-Fr \rightarrow 3\pi^-2\pi^+Fr$ (16 GeV/c)	4
Complex nuclei (Be, C, Al, Si, Ti, Cu, Ag, Ta, Pb)	84	5.0	$\pi^-A \rightarrow 3\pi^-2\pi^+A$ (15.1 GeV/c)	3

V. CONCLUSIONS

We have presented a statistical picture of 3π and 5π diffractive production on hydrogen and coherent production on nuclei in which a continuum of resonances is peripherally produced via Pomeron exchange. It seems clear that a suitable extension of the methods described above will permit calculation of diffractive enhancements for other projectiles and for higher mass excitations involving higher multiplicities of secondaries.

The results above indicate that statistical methods are very successful in describing the high-mass meson continuum (mass ≥ 1.2 GeV). It should be pointed out that the present model should suc-

cessfully describe the baryon continuum as well. Also, the photoproduction of heavy vector mesons can be treated similarly, both on hydrogen and nuclear targets.⁹ Further experiments, especially at the NAL and Serpukhov facilities would clearly be of considerable interest.

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