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Tridents produced by 1-4-GeV positrons*

Y. K. Lim
Physics Department, University of Singapore, Singapore 10

K. Fukui
Air Force Cambridge Research Laboratories, Bedford, Massachusetts 01730

P. S. Young
Physics Department, Mississippi State University, Mississippi State, Mississippi 39762
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The mean free paths for trident production in nuclear emulsions by positrons of mean energies 0.84, 1.71, and 3.42 GeV have been determined to be 103 ± 25, 73 ± 16, and 44 ± 9 cm, respectively. Two methods of background corrections have been used. Our values are found to be consistent with the theoretical total cross sections predicted by quantum electrodynamics.

I. INTRODUCTION

The trident process refers to the direct creation of an electron-positron pair by a fast charged particle in the Coulomb field of a nucleus or an atomic electron. Four Feynman diagrams may be employed for analyzing the process, two of spacelike momentum transfers and two of timelike momentum transfers. Four additional diagrams arise from the exchange effect if the incident particle is an electron or a positron. The process was first studied theoretically by Bhabha[1] and Racah.[2] The former used the Weizsäcker-Williams method and considered the field of the moving particle as a superposition of γ rays in the rest frame of the incident particle. This treatment took account of the two spacelike diagrams and the results were then Lorentz-transformed back to the laboratory frame. Racah employed the perturbation theory and used the first four of the Feynman diagrams to obtain a total cross section which is about half that given by Bhabha. Block *et al.*[3] modified Bhabha's calculations by retaining all the terms in the expression for the differential cross section, making approximations only after all integrations had been performed. This brought Bhabha's total cross section into close agreement with that given by Racah.

Murota *et al.*[4] made a detailed calculation of the process by the Feynman-Dyson method, in which the incident charged particle is treated quantum dynamically, while the target particle is regarded as a fixed Coulomb field. Using the spacelike diagrams, but considering also the contributions from the timelike diagrams (and the exchange effect when an electron or a positron is used as the incident particle), they concluded that Bhabha's total cross section is valid for trident production if the incident electron has an energy exceeding 10 GeV. Thus there is a basic agreement among the theories for electrons above this energy, while at lower energies Murota *et al.* set limits for the contributions due to the timelike momentum transfers, interference, and the exchange effect which had to be added to Bhabha's total cross section.

In most experiments performed to study this process, electrons have been used as the incident particles, although observations have also been made using muons[5] and pions.[6] In the experiments where nuclear emulsions and bubble chambers have been used, mean free paths and consequently total cross sections have been measured, while in the counter experiments differential cross sections have been determined.

The earliest experiments on electron-produced tridents were carried out using cloud chambers for energies less than 10 MeV.[7] Most of the later measurements have been made in nuclear emulsions.[3,8-14] The data below 0.8 GeV, obtained mainly using electrons from accelerators,[8-10] show good agreement with the modified Bhabha curve or the Racah curve. Using cosmic-ray electrons in the energy range from 0.1 to 10 GeV with a mean value of 0.4 GeV, Block et al.[3] obtained a cross section also in agreement with theory. The results at higher energies,[11-13] which were obtained using cosmic-ray electrons, on the other hand, show cross sections that were two to four times those given by the theories.

In measuring the trident mean free paths, a complication arises from the presence of bremsstrahlung (BS) pairs located close to the primary tracks. Since they are indistinguishable from the trident, they are referred to as pseudotridents. A particular method employed to correct for this kind of background may seriously affect the results. For instance, in a study of 13.75-GeV electron interactions in nuclear emulsions, Cary *et al.*[14] reported a trident mean free path of 31.5 ± 2.7 cm, in agreement with the theory of Murota *et al.*, when they used an extrapolation method for background correction. However, they obtained a value of 10.5 ± 0.48 cm when the number of pseudotridents was actually calculated. Employing a method of background correction quite similar to that used by emulsion workers, Böhm *et al.*[15] obtained the total trident cross section at 4 GeV in a propane bubble chamber, which agreed with the theoretical value within one standard deviation (30%).

In counter experiments the outgoing positrons were counted. The background correction was based on the fact that the trident-production rate varies linearly with target thickness while the probability of bremsstrahlung pair production changes quadratically. Examples of this kind of experiment are those done by Camac[16] at 230 MeV, by Criegee[17] at 31.5 MeV, and by Grossetête *et al.*[18] at 500 MeV. The last work has been reported to agree with the theory of Murota *et al.* through a suitable choice of the undetermined constants contained in the formula for total cross-section calculations.

It has been suggested[19] that the trident process can be used to test the validity of quantum electrodynamics at a distance of about 0.2 fermi. Although the experimental evidence currently available does not indicate any breakdown of quantum electrodynamics, a detailed check on the theories of trident production is still of interest because discrepancies exist between the cosmic-ray data and the theories, and also between the cosmic-ray data and the bubble-chamber and counter data. Furthermore, discrepancies between experimental data and theory have been noted in experiments performed to study the muon and pion tridents.[5,6] With these facts in mind, we have performed our trident experiment by use of the positron beam in the GeV region.

II. EXPERIMENTAL PROCEDURE AND RESULTS

A set of 12 in.×7 in. 600-μ-thick G5 nuclear emulsion plates was exposed to the positron beams at the Cambridge Electron Accelerator at the energies 1, 2, 4, and 4.5 GeV. After processing, the plates were line-scanned under a 100× oil-emersion objective. The track-following of the primary tracks started at approximately 1 mm from the entry point of the incident beam and continued either for a maximum length of 1.5 cm, or up to the point where any one of the following events was observed: The track had gotten into a cluster of tracks and become indistinguishable from the others, it had suffered a large-angle deflection, it had branched out into three minimum-ionizing tracks showing the typical formation of a trident, or a bremsstrahlung pair had appeared nearby on the same focal plane.

For electrons and positrons in the GeV region radiation loss predominates. Based on a 2.97-cm radiation length in emulsions and the mean track length, the mean energy was calculated for each group of the primary positrons. The mean energies for the 4- and 4.5-GeV positrons turned out to be nearly equal and these have been treated together as a single group. As a result, we have three groups of positrons, which have mean energies of 0.84, 1.71, and 3.42 GeV, respectively.

When an electron pair produced through the bremsstrahlung process appears close to a beam track within the limits of resolution, which are 0.2μ in the focal plane and 0.4μ in depth in emulsions, the pair and the incident particle become indistinguishable from a true trident. In our experiment two methods have been employed to correct for these pairs.

Method I. In this method, we first used the maximum-likelihood calculation as used by Block *et al.*[3] to correct the bias on the number of apparent tridents due to the finite lengths scanned. Then we followed the scheme of Piron *et al.*[20] to calculate the number of bremsstrahlung pairs per unit length inside a tube of radius 0.2 μ around the trajectory of the incident positron. In these calculations, the pairs with energy lower than 10 MeV were not included because they are hardly detectable due to their large opening angle. The mean free paths for trident production thus estimated are listed in Table I.

TABLE I. Trident mean free paths in emulsion using method I for pseudotrident correction.

Mean positron energy (GeV)	0.84 ± 0.14	1.71 ± 0.25	3.42 ± 0.57
Total scanned length (cm)	3183	3608	2703
Pseudotridents per cm	5.49×10^{-3}	12.6×10^{-3}	26.1×10^{-3}
Apparent tridents (true + pseudotridents)			
Number observed	49	93	122
Number from the maximum-likelihood calculation	47.9 ± 7.0	91.8 ± 9.7	120.8 ± 11.1
Number of true tridents	30.4 ± 8.2	46.3 ± 11.8	50.3 ± 13.9
Trident mean free path (cm)	105 ± 28	78 ± 20	54 ± 15

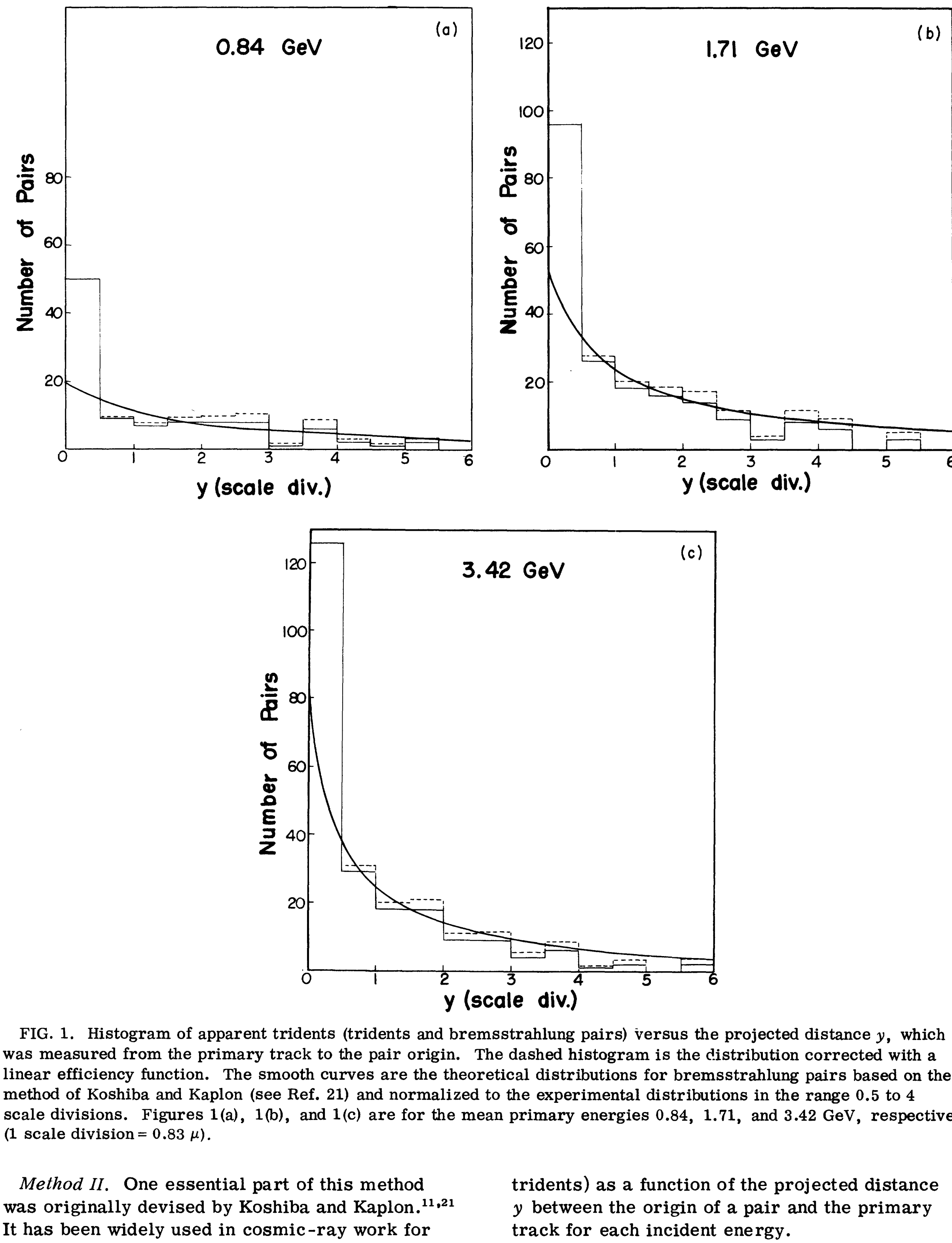

FIG. 1. Histogram of apparent tridents (tridents and bremsstrahlung pairs) versus the projected distance y, which was measured from the primary track to the pair origin. The dashed histogram is the distribution corrected with a linear efficiency function. The smooth curves are the theoretical distributions for bremsstrahlung pairs based on the method of Koshiba and Kaplon (see Ref. 21) and normalized to the experimental distributions in the range 0.5 to 4 scale divisions. Figures 1(a), 1(b), and 1(c) are for the mean primary energies 0.84, 1.71, and 3.42 GeV, respectively. (1 scale division = 0.83 μ).

Method II. One essential part of this method was originally devised by Koshiba and Kaplon.[11,21] It has been widely used in cosmic-ray work for background correction. Following their method, we have calculated the theoretical differential distribution of bremsstrahlung pairs (i.e., pseudo-tridents) as a function of the projected distance y between the origin of a pair and the primary track for each incident energy.

In order to obtain the "corrected" number of true tridents, we first plotted the number of apparent tridents (after the maximum likelihood

TABLE II. Trident mean free paths in emulsion using method II for pseudotrident correction.

Mean positron energy (GeV)	0.84 ± 0.14	1.71 ± 0.25	3.42 ± 0.57
Apparent tridents with $y \leq 0.5$ div. (i.e., 0.4 μ)			
Number observed	50	96	126
Number from the maximum-likelihood calculation	48.9 ± 7.1	94.8 ± 9.8	124.8 ± 11.2
Number of true tridents	31.7 ± 8.2	52.4 ± 11.8	71.6 ± 13.4
Trident mean free path (cm)	101 ± 27	69 ± 15	38 ± 7

correction) versus y. These are shown in Fig. 1 as the solid-line histogram. Since our scanning efficiency for finding a pair decreases as y increases and since the most distant pairs observed in this work were located at about $y = 10$ μ (divided into 12 divisions), we arbitrarily assumed a linear-scanning efficiency function $f = 1 - (y/y_0)$, where y_0 has been set to 12 divisions. The histogram modified by this efficiency function is shown in Fig. 1 by dashed lines. It may be seen that the efficiency modification has not appreciably changed the distributions.

The theoretical differential distribution is now normalized to the corresponding modified experimental data such that the total numbers of pairs in the range $y = 0.5$ to 4 div. are equal. The difference between the observed number of apparent tridents and the extrapolated number of the pairs in the range $y = 0$ to 0.5 div. (namely, 0.4 μ) then gives the number of true tridents. The results are summarized in Table II.

From Tables I and II we see that the agreement between the corrected numbers of tridents is good for the lower-energy groups and there is a difference of 1.5 standard deviations at the energy of 3.42 GeV. We also notice that the second method consistently led to a higher number of the true tridents. In the calculation of pseudotridents using the scheme of Piron *et al.*, account has been taken of all bremsstrahlung pairs with energies exceeding 10 MeV. The mean opening angle of the pairs created by 10-MeV photons is about 0.05 radian. Pairs of such large opening angles are easily missed. One might expect, therefore, that method I sets an upper limit for the number of pseudotridents and consequently a lower limit for the number of true tridents. But in method II, the corrected number of true tridents turned out to be an upper limit. It is because the calculated number of BS pairs in the range $y = 0$ to 0.5 div. was lower than the actual value. This was caused by the extrapolation of the normalized theoretical distribution. Due to the scanning difficulties, we could not avoid missing those BS pairs which are too far away from the primary tracks and out of focus. Consequently, the normalization based on the experimental BS pairs has lowered the theoretical distribution in the range $y = 0.5$ to 4 div. and the extrapolation of such a distribution naturally yielded the number of BS pairs in the shorter range as smaller than the actual value. This has made the correction smaller and the number of tridents higher. Based on the above reasoning, we think the best values for the mean paths would be the averages of the two sets of results. The results are given in Table III.

III. DISCUSSION

Since positrons and electrons are identical except for the sign of their charges, and the theories of Bhabha[1] and Racah[2] make no distinction between them, our data can be directly compared with those obtained by use of electrons as incident particles. In Fig. 2 we plotted our data along with three theoretical curves and also with the data of other investigators using emulsions.[3,8–14] Solid curve I was obtained from the Bhabha theory modified by Block *et al.*,[3] while solid curve II was due to the theory of Racah.[2] Both curves apply to the complete screening assumption. The third curve (the dashed curve) was based on the Bhabha theory with the assumption of no screening effect. In the calculation of these three curves we assumed an effective charge of $Z = 21.38$ for the emulsion nuclei and have replaced Z^2 by $Z(Z+1)$ to include the contribution of the atomic electrons.

From Fig. 2 we notice that our mean free paths are somewhat lower than theoretical values. Nevertheless, a fair agreement still exists, es-

TABLE III. Mean free paths for trident production averaged over two results.

Mean positron energy (GeV)	0.84 ± 0.14	1.71 ± 0.25	3.42 ± 0.57
No. of true tridents (averaged over two results)	31.0 ± 7.6	49.4 ± 0.7	61.0 ± 12.3
Mean free paths (averaged over two results)	103 ± 25	73 ± 16	44 ± 9

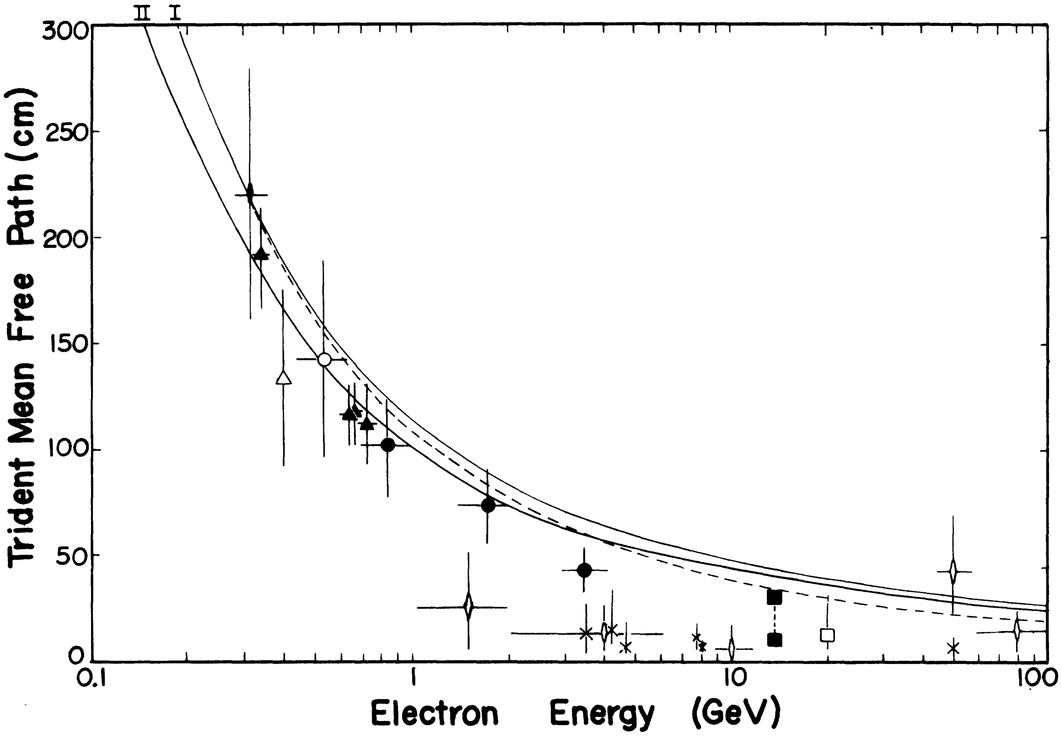

FIG. 2. Electron trident mean free paths in nuclear emulsions as function of primary energy, measured by Block *et al.*, (Ref. 3) (△), Cary *et al.* (Ref. 14) (■), Gailloud and Piron (Ref. 9) (▲), Koshiba and Kaplon (Ref. 11) (X), Leonard (Ref. 10) (◆), Loeffler (Ref. 8) (O), Naugle and Freier (Ref. 12) (◇), Tumanyan *et al.* (Ref. 13) (□), and this experiment (●). Curve I is based on Bhabha's theory (Ref. 1) modified by Block *et al.* (Ref. 3) for complete screening (solid curve) and for nonscreening (dashed curve). Curve II is from Racah's theory (Ref. 2).

pecially compared with the Racah curve (solid curve II), even though our values are lower by 20–30% compared with Bhabha's screened mean free paths (solid curve I). We could account for this amount of discrepancy by considering the uncertainties involved in the theories. As we have mentioned in the Introduction, Murota *et al.*[4] have set a lower limit of 10 GeV on the incident energy for Bhabha's formula for total cross sections to be valid. It should be pointed out that the incident energies in this experiment lie between 1 and 4 GeV. In addition, the theoretical formulas contain undetermined constants of the order unity. For example, in the expression for Bhabha's nonscreening total cross section, $\sigma \simeq \ln^3(k\gamma)$, where γ is the Lorentz factor of the primary electron and k is undetermined. Therefore if k changes from 1 to 1.7, the total cross section would have to increase, and consequently the mean free path decrease, by 20% at our energy range.

As a whole, the emulsion data appear to favor agreement with the theories, except for the data[11–13] using cosmic rays as the source of primary electrons in the energy range from 1 to 12 GeV. It should be pointed out that these cosmic-ray experiments involved low statistics and uncertain energy measurements. One interesting point was demonstrated by the two values of Cary *et al.* at 13.75 GeV, one being much closer to the theoretical prediction than the other. Since the difference in the two values was caused by different methods of background correction, this example indicated that data could be brought into agreement with the theory in a reasonable way. Therefore we do not have reason to cast any doubts on the validity of quantum electrodynamics in view of the study of the electron-trident process carried out up to the present stage.

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Measurements of the differential cross section of the reaction $pp \rightarrow d\pi^+$ from 3.0 to 5.0 GeV/c*

H. L. Anderson, D. A. Larson,† and L. C. Myrianthopoulos
The Enrico Fermi Institute, The University of Chicago, Chicago, Illinois 60637

L. Dubal,‡ C. K. Hargrove, E. P. Hincks,§ R. J. McKee, and H. Mes||
The National Research Council of Canada, Ottawa, Canada K1A OR6

D. Kessler and A. C. Thompson**
Carleton University, Ottawa, Canada K1S 5B6
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A measurement of the complete differential cross section for the reaction $pp \rightarrow d\pi^+$ at 3.00, 3.20, 3.43, 3.65, 3.83, 4.00, 4.20, and 5.05 GeV/c incident proton momentum has been made in an attempt to establish the role of the $\Delta(1950)$ in this region. The data show that the previously observed enhancement in the forward cross section between 3 and 4 GeV/c due to this isobar is an effect which damps out quickly as the production angle departs from zero degrees, in contrast with the well-known enhancement at 1.35 GeV/c, which is evident at all angles. In particular, the one-pion-exchange model is in poor agreement with the extended set of data. A detailed description is given of a novel proportional-wire-chamber system which facilitated the selection of this rather rare reaction from a very high competing background.

I. INTRODUCTION

This is a report on a comprehensive series of measurements of the complete angular distribution and total cross section for the reaction

$$pp \rightarrow d\pi^+ \tag{1}$$

over the range of incident momenta from 3.00 to 5.05 GeV/c. These measurements were carried out at the Bevatron of the Lawrence Berkeley Laboratory as an extension of previous work of our collaboration on reaction (1).[1]

The low-energy region,[2] below 2.3 GeV/c for reaction (1), has been successfully described in terms of the excitation of the (3, 3) resonance.[3] Between 2.3 and 4.3 GeV/c the forward cross section shows another structure which may be ascribed to the (7, 3) resonance.[4–8] The purpose of