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Partial-wave analysis of the low-mass $\pi^+\pi^+\pi^-$ system produced by incident π^+ mesons at 13 GeV/c*

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Results are presented of a spin-parity analysis of the $(\pi^+\pi^+\pi^-)$ system formed opposite a proton by incident 13-GeV/c π^+ mesons in a bubble chamber exposure of 750 000 pictures in hydrogen. Relative proportions of the contributing J^P states are given for $(3\pi)^+$ masses between 0.9 and 1.4 GeV/c^2 and their dependence on $t'_{\pi\to3\pi}$ is also discussed. Measurement of the phase between interfering production amplitudes gives information about the resonance status of the A_1, A_2 mesons. The method employs the University of Illinois three-body partial-wave analysis program.

INTRODUCTION

A maximum-likelihood[1-4] spin-parity analysis has been applied to the $(\pi^+\pi^+\pi^-)$ system produced opposite a proton by 13-GeV/c incident π^+ mesons in the SLAC 82-in. bubble chamber.

The analysis assumes that the event distribution probability may be written in the form

$$W=\sum M_{M_1\eta}^{J_1P_1}\,\rho_{M_1,M_2,\eta}^{J_1P_1,J_2P_2}\,M_{M_2\eta}^{J_2P_2*},$$

where $\rho_{M_1M_2\eta}^{J_1P_1,J_2P_2}$ is the production density matrix expanded in terms of states of definite spin-parity J_P, z component M, and reflection parity η.[5] In a t-channel exchange model, $\eta=+1(-1)$ approximately projects out the natural (unnatural) parity exchange contributions.

The decay amplitude, $M_{M\eta}^{JP}$, for the 3π system is expressed as a function of the Dalitz-plot coordinates s_1, s_2 $[s_{1,2}=m^2(\pi^+_{1,2}\pi^-)]$; θ and ϕ, the Gottfried-Jackson polar and azimuthal angles of the π^-; and γ, the azimuthal angle of rotation of the $(\pi^+\pi^+)$ plane about the π^- direction in the 3π rest frame, measured from the π^-/incident π^+ plane. Typical distributions in $m(\pi^+\pi^-)$ and the angles θ, ϕ, γ are shown in Fig. 1. The decay is assumed to proceed by an intermediate $\epsilon\pi$, $\rho\pi$, or $f\pi$ state except for an uncorrelated phase space background having isotropic distribution in all variables. Any $\pi^-\pi^+$ rescattering is neglected. In an assumption of coherence the decay amplitude for a given spin-parity state is expanded as a sum over the intermediate two-body states, each having definite orbital angular momentum l between the π^+ and $\pi^+\pi^-$ systems,

$$M_{M\eta}^{JP}=\sum_l C_l^{JP}\,M_{M\eta}^{JPl}\,.$$

The C_l^{JP} coefficients are complex numbers relating the phase and magnitude of the different angular momentum decay amplitudes.

For the purpose of this analysis with $0.9\le M(3\pi)\le 1.40$ GeV/c^2 total spin J and l have been limited to ≤ 2. These restrictions are the only additional assumptions made, although some states have been omitted after experimentally determining that they were negligible. For example, throughout the mass region considered, the fits required that $\eta=-1$ states hardly contribute, maximum errors being of the order 1.5% (cf. to $\eta=-1$ levels of 1 to 3% found in A_2^- production at 5 and 7.5 GeV/c).[2]

EXPERIMENTAL DATA

The data sample consists of 24 832 events satisfying fitting criteria to the reaction $\pi^+p \to \pi^+\pi^+\pi^-p$.

The observed $(3\pi)^+$ mass spectrum is shown in Fig. 2 for events with $1.12\le M(p\pi^+_{\text{slow}})\le 1.32$ GeV/c excluded. It has been observed at 13 GeV/c that if the positive pions are ordered in longitudinal phase space, the $\Delta^{++}(1231)$ is always formed with the "slow" π^+, there being no mass enhancement in the $(p\pi^+_{\text{fast}})$ system and no indication from angular distributions that p-wave decay is present.[6] Accordingly, $(p\pi^+_{\text{slow}})$ has been cut in the stated interval both for the experimental data and the theoretical expression. This cut amounts to less than 8%. There is no appreciable contamination from other N^*'s.

PARTIAL-WAVE ANALYSIS

The parameters in the fit are the C_l^{JP} and $\rho_{MM'\eta}^{JP,JP}$ both presumed to be functions of the 3π mass, m, and the momentum transfer squared, $t'=t-t_{\min}$, from the beam to the 3π system. Small (m, t') bins were selected in order to perform an energy-

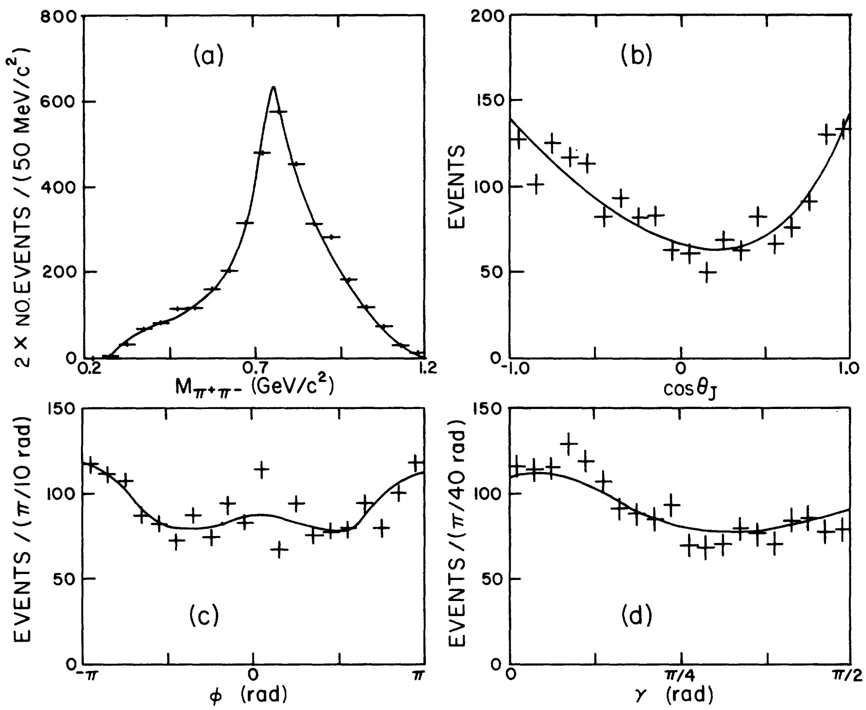

FIG. 1. Comparison of the fit to the data for events of reaction (1) in the intervals $1.25 \leq M_{\pi^+\pi^+\pi^-} \leq 1.35$ GeV/c^2, $0.0 \leq |t'| \leq 0.5$ GeV^2. (a) $\pi^+\pi^-$ mass spectrum, two entries per event, (b) $\cos\theta$, (c) ϕ, (d) γ (see text for definitions of angles). The solid line is the result of a Monte Carlo event generation using the fitted parameters.

independent analysis. Initially a dependence $e^{At'}$ was imposed on the density matrix elements in each such bin. The slope parameter A decreases as m increases from ~10 to ~6 GeV^{-2} in the region $0.9 \leq M(3\pi) \leq 1.40$ GeV/c^2.

In order to specify the intermediate resonant dipion states, Particle Data Tables[7] were used for the f^0 mass and width and the ρ^0 mass. Small variations in the likelihood for a given fit then determined the values $\Gamma(\rho) = 135$ MeV/c^2,

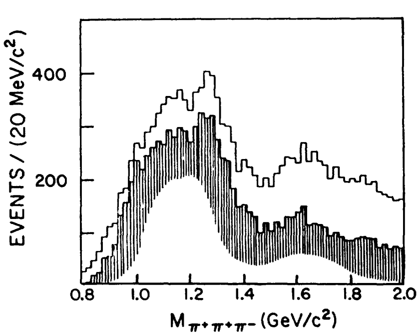

FIG. 2. $\pi^+\pi^+\pi^-$ mass spectrum for events from the reaction $\pi^+p \to \pi^+\pi^+\pi^-p$ where events with $112 \leq M(p\pi^+_{slow}) \leq 1.32$ GeV/c^2 are excluded. The shaded histograms have events in which at least one $\pi^+\pi^-$ combination satisfies $0.66 \leq M_{\pi^+\pi^-} \leq 0.86$ GeV/c^2.

$M(\epsilon) = 785$ MeV/c^2, $\Gamma(\epsilon) = 400$ MeV/c^2 for all future fits.

The quantum numbers of the 3π states found, by trial fits, to contribute in the low-mass region are given in Table I. The state labeled $|\text{flat}\rangle$ is taken to be an entirely uncorrelated amplitude representative of a (3π) phase space background in that it gives rise to a uniform Dalitz plot density and isotropic distribution in the angles θ, ϕ, γ. In all fits except at the higher masses, where contamination from higher-spin states probably enters, this isotropic state is consistent with zero. This gives support for the model's assumption of an

TABLE I. Quantum numbers of the 3π found to contribute in the low-mass region.

State	J	P	l	M	η	Decay
1			$\lvert\text{flat}\rangle$			3π
2	0	−	S	0	+	$\epsilon\pi$
3	0	−	P	0	+	$\rho\pi$
4	1	+	S	0	+	$\rho\pi$
5	1	+	S	1	+	$\rho\pi$
6	1	+	P	0	+	$\epsilon\pi$
7	1	+	P	1	+	$\epsilon\pi$
8	2	−	P	0	+	$\rho\pi$
9	2	−	D	0	+	$\epsilon\pi$
10	2	+	D	1	+	$\rho\pi$

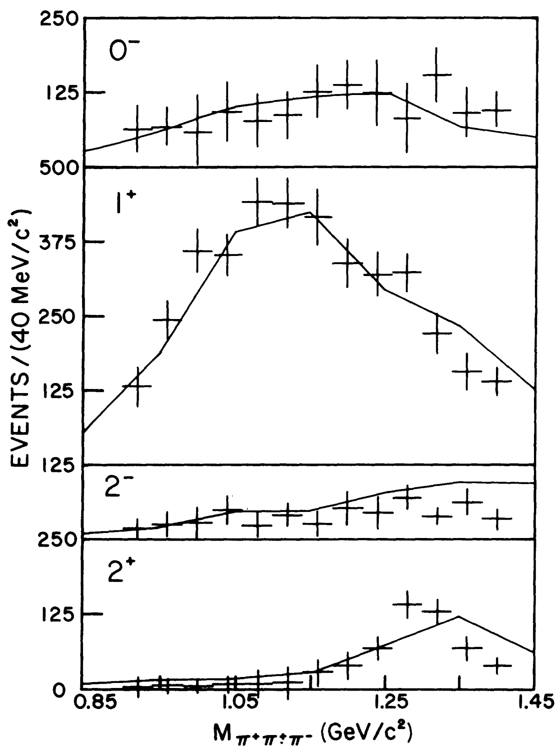

FIG. 3. Events in the principal J^P states. The solid line shows the results from the reaction $\pi^- p \rightarrow \pi^-\pi^-\pi^+ p$ (Ref. 4) in the same energy region normalized to the same total of events in the 3π mass region considered.

intermediate dipion state with no rescattering.

With the exception of the 2^+ wave, states of nonzero spin component (M) have small contributions. Throughout, all 1^- states and 2^+ states with $M \neq 1$ had very small matrix elements (< 2%) and were thus excluded from further consideration, enabling the number of parameters to be held below an acceptable level of ~ one per ten events, for most mass and t' binned data samples. To obtain a clear picture of the mass and t' behavior of the production probability of a given state, the corresponding diagonal density matrix element is multiplied by the total number of events in a (m, t') bin to give what is referred to as the "events" of that state.

Thus the events in the four principal J^P states are shown as a function of (3π) mass in Fig. 3. The solid line in this diagram shows the equivalent results from the $\pi^- p$ analysis[4] normalized to the same total of events. The density of spin states appears to be independent of the charge of the pion.

The three-pion $1^+_{M=0}$ state events produced opposite the proton with $|t'| < 0.5$ (GeV)^2 are shown in 40-MeV/c^2 mass intervals in Fig. 4. The solid line is a fit to a simple nonrelativistic Breit-Wigner, with parameters $M = 1128 \pm 8$ MeV/c^2 and $\Gamma = 367 \pm 30$ MeV/c^2, presumably to be identified as the A_1, though some 50 MeV/c^2 higher in mass ($\sim \frac{1}{8}\Gamma$). These parameters give a total cross section (of 305 ± 25 μb) for the entire t' range, or 106 ± 16 μb in the range $1.0 < M(3\pi) < 1.2$, in close agreement with the A_1^- cross section.[3] In this region the ratio of P wave 1^+ (ϵ decay) to S wave (ρ decay), i.e., $|C_P^{1^+}|^2 / |C_S^{1^+}|^2$ averages 11%, frequently compatible with zero.

This simple resonance picture is, however, not compatible with the phase variation of the amplitude. It is, of course, impossible to obtain the absolute value of the phase of any single amplitude but the interference density matrix element between two states, by definition, gives their relative phase. From the evidence of Fig. 3 the 0^- has a constant amplitude and this state also has a large interference with the 1^+ amplitude. In Fig. 5 the phase difference ψ = (phase 1^+S) − (phase 0^-S) is plotted. It is clear that from threshold to about 1.2 GeV/c^2, i.e., covering the entire peak area of the A_1, the relative phase is almost constant, rising from ~190° to only ~200°. This behavior is still consistent with a description by a pole partially shielded from the physical region by a cut[8] but a simple resonance interpretation is apparently completely ruled out.

As the (3π) mass increases the relative phase appears to undergo some rapid variation in the region of 1.2 GeV/c^2. Some further fits in 20-MeV/c^2 bins have been added to Fig. 9 (below) here to elucidate this behavior which might possibly be associated with an $A_{1.5}$ resonance. This

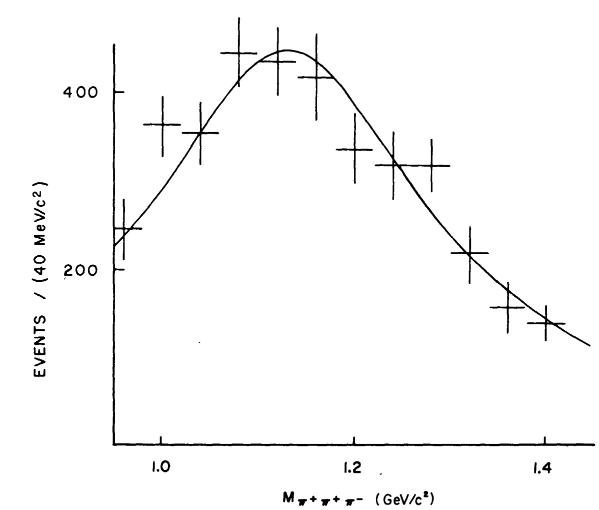

FIG. 4. $1^+_{M=0}$ events as function of $M_{\pi^+\pi^+\pi^-}$. The solid line is a nonrelativistic Breit-Wigner fit presuming no background.

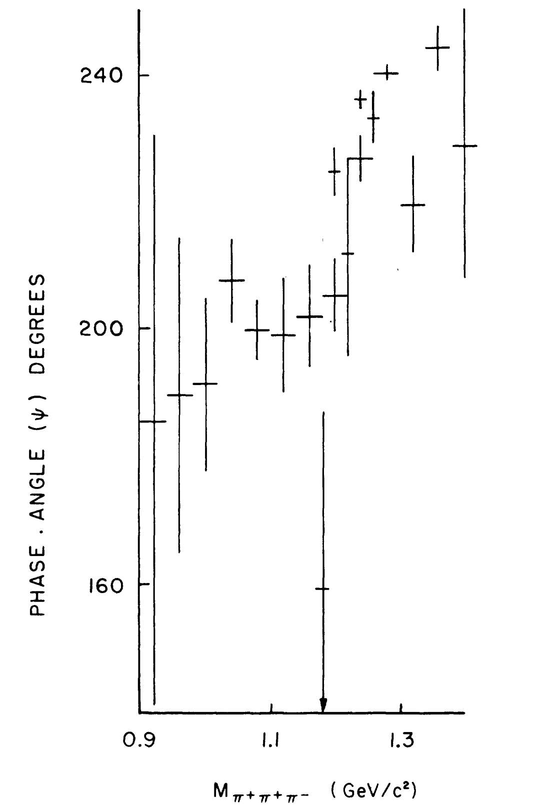

FIG. 5. Phase angle of 1^+S amplitude as compared to 0^-S amplitude. An increase in angle ψ corresponds to a clockwise rotation in the Argand diagram of the 1^+ state.

has been proposed as a narrow $\rho\pi$ structure at 1.19 GeV/c^2 (Ref. 9) but ρ cuts do not enhance any possible evidence in the mass spectra of Fig. 2. As the total phase variation between 1.1 and 1.3 GeV/c^2 is only $\sim 50°$ at most, and no structure appears in Fig. 9 (below) it would seem that if any part of the 1^+ wave is resonant here, it must be superimposed on a large nonresonant background.

The only contribution from the normal spin-parity series is the 2^+ wave which appears to decay entirely by D wave ($\rho\pi$). There are no $\eta = -1$ states and the $M=2$ states constitute ($1.1 \pm 2.3\%$) of the reaction in the region of 1.3 GeV/c^2 and so have been ignored. The fraction of the events in $2^+_{M=1}$ are fitted to a simple Breit-Wigner in Fig. 6, parameterized by $M(A_2) = 1298 \pm 7$ MeV/c^2 and $\Gamma(A_2) = 110 \pm 23$ MeV/c^2, with a total cross section corrected for t' behavior (see later) of 37.7 ± 8.3 μb.

The confirmation of the simple resonance behavior is demonstrated in Fig. 7 using the $2^+/1^+$ interference, which, it can be seen, is almost maximal, the full circle being the positivity limit. A large phase variation is observed, most rapid

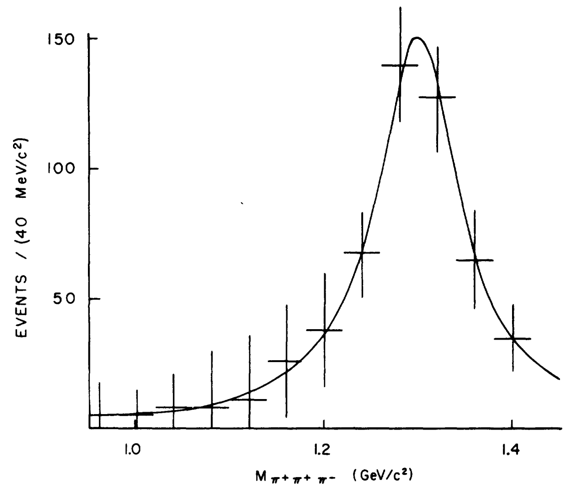

FIG. 6. $2^+_{M=1}$ events as function of $M_{\pi^+\pi^+\pi^-}$. The solid line is a nonrelativistic Breit-Wigner fit presuming no background.

in the area of the A_2 peak. It is meaningless to go below 1.24 GeV/c^2 because of the variation of the 1^+ amplitude.

The above results on the A_2^+ are completely consistent with the behavior of the A_2^- at slightly lower energies.[2]

t' BEHAVIOR

Having imposed the mass behavior of the density matrix from this analysis, it is possible to chose a rather wide interval of 100 MeV/c^2 around the A_2 mass ($1.25 < M_{\pi^+\pi^+\pi^-} < 1.35$ GeV/c^2) to study its t' behavior. The differential cross section $d\sigma/dt'(2^+)$ computed from the diagonal element is displayed in Fig. 8, where the cross section is normalized to that of the A_2. The fit is to the

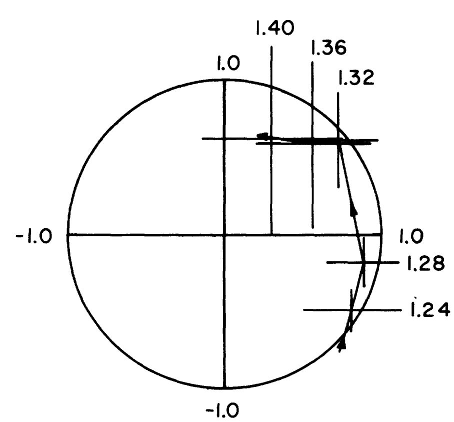

FIG. 7. Argand diagram of the interference term between 2^+ and 1^+ amplitudes. The full circle is the positivity-limit boundary.

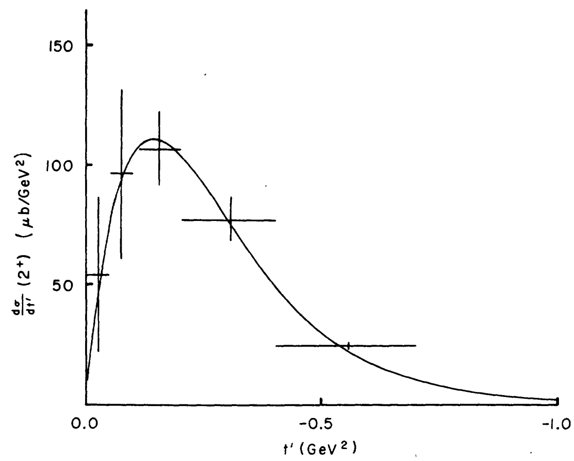

FIG. 8. $d\sigma/dt'$ for 2^+ events in $1.25 < M_{\pi^+\pi^+\pi^-} < 1.35$ GeV/c^2, normalized to the total A_2 cross section and fitted to $Bt'e^{At'}$.

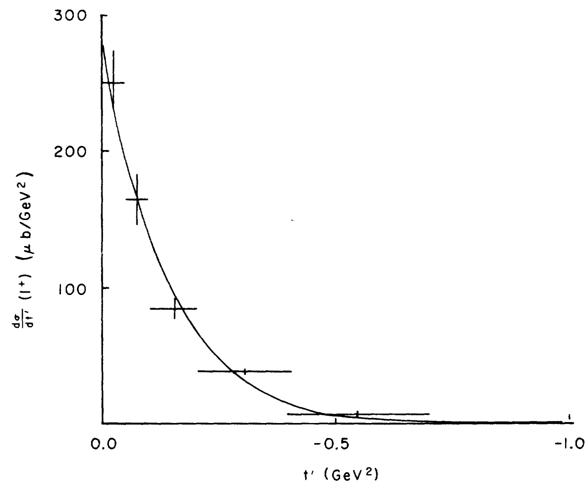

FIG. 10. $d\sigma/dt'$ for 1^+ events in $1.25 < M_{\pi^+\pi^+\pi^-} < 1.35$ GeV/c^2 fitted to $Be^{At'}$.

function $Bt'e^{At'}$ with slope parameter, $A = 7.16 \pm 0.20$ GeV^{-2}. Of a number of elementary functions which were attempted this gave the best confidence level, and improved the χ^2 by five standard deviations from a simple exponential fit, greatly favoring a dip to zero rather than a forward peak.

The differential cross sections for 0^- and 1^+ events in the same mass band, and normalized to the respective cross sections in this area, are shown in Figs. 9 and 10. A simple exponential fit, $e^{At'}$, with $A = 9.5 \pm 2.8$ (GeV/c)$^{-2}$ is only marginally preferred by $d\sigma/dt'(0^-)$, but the 1^+ events prefer this formula to the forward dip by eighteen standard deviations and fit $A = 7.18 \pm 0.32$ GeV^{-2}. It may be seen that all the slope factors are compatible within errors. They are also in agreement with slope factors for $(3\pi)^-$ production in the same mass interval up to 40 GeV/c.[10]

s- AND t-CHANNEL HELICITY CONSERVATION

If the 1^+ structure at 1.1 GeV/c may be still rescued as an A_1 resonant state the steep exponential slope and lack of polarization suggest that it is diffractively produced. An investigation of the polarization in the 1^+ submatrix can then give information on the conservation of the incident π^+ zero helicity. In Fig. 11 the variation of ρ_{00} is shown as a function of t' in the mass interval $1.05 < M_{\pi^+\pi^+\pi^-} < 1.15$ GeV/c^2. As has already been noted, when this is evaluated in the usual Gottfried-Jackson (t-channel) system there

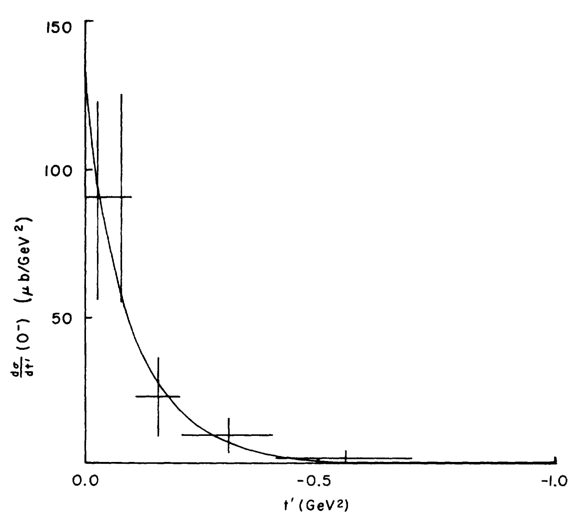

FIG. 9. $d\sigma/dt'$ for 0^- events in $1.25 < M_{\pi^+\pi^+\pi^-} < 1.35$ GeV/c^2, fitted to $Be^{At'}$.

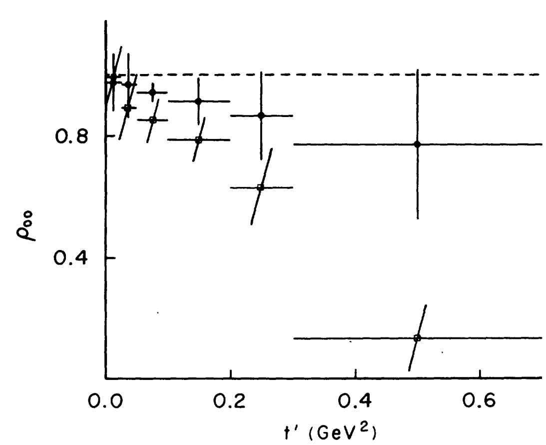

FIG. 11. ρ_{00} as a function of t' in the 1^+ submatrix evaluation in the t-channel (full circles) and s-channel (open squares) systems ($1.05 < M_{\pi^+\pi^+\pi^-} < 1.15$ GeV/c^2).

is only a small $M=1$ component and so $\rho_{00}\sim1.0$ decreasing very slowly. However, when the z axis is defined as the direction of motion of the $(\pi^+\pi^+\pi^-)$ system in the over-all c.m. system (s-channel axes), ρ_{00} falls off considerably at large t'. This analysis, therefore, significantly favors conservation of helicity in the t channel, although neither system is perfect. This is contradiction to ρ photoproduction results which show pure s-channel helicity conservation but in agreement with other diffractive dissociative reactions,[11] and A_1^- production.[3]

CONCLUSION

An essentially model-independent partial wave analysis has been made of the low-mass $(\pi^+\pi^+\pi^-)$ system. The A_1 region has been shown to be composed principally of a $1^+_{M=0}$ state decaying via an S wave into $\rho\pi$. A Breit-Wigner dependence describes the 1^+ intensity in the neighborhood but lack of phase variation between the 1^+S and 0^-S states rules out a simple resonance interpretation. Near 1.2 GeV/c^2, on the contrary, there is a narrow signal in phase but no significant change in intensity. Assuming a diffractive dissociation process for the A_1, t-channel helicity conservation is favored.

Production of the 2^+ state is 100% polarized with $M=1$. A simple Breit-Wigner is found to fit the A_2 region and explains a rapid phase variation between 2^+ and 1^+ amplitudes. The differential cross section has a pronounced dip in the forward direction.

It is hoped that partial-wave analyses will contribute toward an understanding of the diffractive components of the 3π spectra. It would seem that no present model can adequately describe even such gross physical features like the mass dependence of the exponential slope in t', simultaneously with any of the more subtle details of the data presented.[12]

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[1]This analysis utilizes the Illinois University spin-parity program [described in D. V. Brockway, Ph.D. thesis, 1970, U. S. Atomic Energy Commission Report No. COO-1195-197 (unpublished)] which formed the basis of recent studies of the $(\pi^-\pi^-\pi^+)$ system (Refs. 2, 3, and 4).

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