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Remarks on vanishing longitudinal cross sections and operator Schwinger terms

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We show that a vanishing total cross section $\sigma_L(\nu, q^2)$ for virtual photons is not compatible with the presence of operator Schwinger terms in the proton-proton matrix element of the equal-time electromagnetic commutator if νW_2 scales.

It has recently been proved,[1] by explicit example, that a vanishing total cross section $\sigma_L(\nu,q^2)$ for virtual longitudinal photons on a hadronic target is compatible with a nonzero value for the one-particle matrix elements of the operator Schwinger term in the equal-time commutator of electromagnetic current components. The example offered, however, does not exhibit scaling, and queries as to the validity of this assertion under scaling—and constant asymptotic $\sigma_T(\nu, q^2)$ at fixed q^2—have been raised. It has further been asserted[1] that the operator Schwinger terms would not appear if $\sigma_L(\nu, q^2)=0$, scaling holds, and usually assumed dispersion representations are valid.

It is our purpose in this note to point out that when $\sigma_L=0$ the Schwinger-term contribution must vanish if νW_2 scales. This follows primarily as a requirement of causality, for, under the condition that the spectral functions $\psi_i(u, s)$ in the Jost-Lehmann-Dyson (JLD) representations of the causal structure functions[2] V_i satisfy

$$\lim_{s\to\infty}\psi_i(u, s)=0\ , \tag{1}$$

causality implies[3,4] that the Fourier transform of the equal-time commutator

$$E_{0k}\equiv\int e^{iq\cdot x}\,\delta(x_0)\langle p\,|\,[J_0(x), J_k(0)]\,|p\rangle\; d^4x \tag{2}$$

has the form

$$E_{0k}=-\vec{p}\cdot\vec{q}\,S_2\,p_k+(S_1+p_0^{\,2}S_2)q_k\ , \tag{3}$$

where S_1 and S_2 are constants. In terms of the structure functions W_i, these are represented by

$$S_1=\frac{1}{2\pi}\int\frac{q_0}{q^2}\left[W_1(\nu, q^2)+\frac{\nu^2}{q^2}\,W_2(\nu, q^2)\right]dq_0\,, \tag{4}$$

$$S_2=-\frac{1}{2\pi}\int\frac{q_0}{q^2}\,W_2(\nu, q^2)dq_0\,. \tag{5}$$

The condition $\sigma_L=0$ reads

$$W_1(\nu, q^2)+\left(\frac{\nu^2}{q^2}-1\right)W_2(\nu, q^2)=0. \tag{6}$$

It immediately follows from (4), (5), and (6) that when $\sigma_L=0$ we must have

$$S_1+S_2=0\,, \tag{7}$$

as noted in Ref. 1.

We shall show[4] that S_2 must vanish whenever νW_2 scales, irrespective of the scaling behavior of W_1. It would then follow that when $\sigma_L=0$ and νW_2 scales, $S_1=S_2=0$, i.e., the Schwinger term vanishes. To show that S_2 vanishes when νW_2 scales, write (5) in the form

$$S_2=\frac{1}{2\pi}\int\frac{x}{\eta^2-x^2}\,W_2(2p_0^{\,2}(x+\alpha),\,4p_0^{\,2}(x^2-\eta^2))\,dx\,, \tag{8}$$

and take the limit $p_0\to\infty$ at fixed α and η:

$$S_2=\lim_{p_0\to\infty}\frac{1}{4\pi p_0^{\,2}}\int\frac{x}{(\eta^2-x^2)(x+\alpha)}\,F_2\!\left(\frac{\eta^2-x^2}{x+\alpha}\right)dx\,.$$

This gives[4]

$$\begin{aligned}S_2&=\lim_{p_0\to\infty}\frac{1}{4\pi p_0^{\,2}}\;\frac{\alpha}{\alpha^2-\eta^2}\int F_2(\omega)\frac{d\omega}{\omega}\\&=0\,.\end{aligned} \tag{9}$$

To obtain the result, we have taken the limit $p_0\to\infty$ inside the integral in (8). Under assumption (1) the existence of this integral and its convergence to the constant value S_2 is guaranteed for all p_0.

As a demonstrative (causal) example, take

$$W_2=\frac{1}{\nu}\,\theta(4\nu^2-q^4) \tag{10}$$

and $\sigma_L=0$ so that W_1 is defined by Eq. (6). The Schwinger term $S=S_1+p_0^{\,2}S_2$, where

$$\begin{aligned}S_1&=-S_2\\&=\frac{1}{2\pi}\int_R\frac{q_0dq_0}{\nu q^2}\,,\end{aligned} \tag{11}$$

and the region R consists of the two intervals (a^-, b^-) and (a^+, b^+) defined by

$$\begin{aligned}a^{\pm}&=-p_0\pm(p_0^{\,2}+\vec{q}^{\,2}+2\vec{p}\cdot\vec{q})^{1/2}\,,\\b^{\pm}&=p_0\pm(p_0^{\,2}+\vec{q}^{\,2}-2\vec{p}\cdot\vec{q})^{1/2}\,.\end{aligned} \tag{12}$$

Since, by causality, S_1 and S_2 are constant they may be calculated in the frame $\vec{p}=\vec{0}$. In this frame R consists of the intervals $(-p_0+A, p_0+A)$, $(-p_0-A, p_0-A)$, where $A=(p_0^2+\vec{q}^2)^{1/2}$ and the integrand is proportional to $(q_0^2-\vec{q}^2)^{-1}$. Explicit integration then gives $S_1=S_2=0$ and the Schwinger term vanishes in agreement with our assertion since νW_2 scales.

On the other hand, in the Creutz example,[1]

$$W_1=-\frac{1}{2\nu}(q^2-\nu^2)\theta(4\nu^2-q^4),$$
$$W_2=\frac{-q^2}{2\nu}\,\theta(4\nu^2-q^4), \qquad (13)$$

$\sigma_L=0$, but νW_2 does not scale. Our theorem does not therefore apply, and, in fact, one finds that $S_1=-S_2=-1/\pi$, so that

$$S=\frac{1}{\pi}(p_0^2-1). \qquad (14)$$

Note that the condition (7) is satisfied.

We finally remark that the requirement of asymptotic behavior (1) is not a stringent condition to satisfy. It may be compared with the statement made in Ref. 1 that under $\sigma_L=0$ and scaling, S vanishes if the usual dispersion relations hold both for finite q^2 and in the scaling limit.

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[1]M. Creutz, Phys. Rev. D 5, 3269 (1972).

[2]These are related to W_i by

$$V_1=\frac{1}{q^2}W_1+\frac{\nu^2}{q^4}W_2, \quad V_2=-\frac{1}{q^2}W_2.$$

[3]J. W. Meyer and H. Suura, Phys. Rev. 160, 1366 (1967).

[4]M. O. Taha, Khartoum report, 1973 (unpublished), and references therein.