Are bootstraps of low-spin particles meaningful?

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We argue that for spin-$\frac{1}{2}$ and spin-1 particles bootstraps may be trivial. In many instances such particles can be embedded in a renormalizable Lagrangian formalism in which they may Reggeize. In such instances a correctly carried out calculation will produce bootstraps for any values of mass and coupling constant.

We have recently discussed[1] the possibility that the elementary gauge vector mesons and fermions present in a large class of renormalizable Lagrangian models Reggeize in the sense of Gell-Mann *et al.*[2] That is, if one calculates the scattering amplitudes in such Lagrangian models, one finds that these particles may lie on Regge trajectories. Our calculations were at the level of checking certain necessary factorization properties of the Born approximation and represent a weak-coupling approach to the problem, but they have certain implications for hadron physics that should be emphasized: If Reggeization indeed takes place old-fashioned bootstrap calculations for spin-$\frac{1}{2}$ or spin-1 particles,[3] if done without drastic mutilations of the unitarity equations and of analyticity, may lead to trivial results; these particles will bootstrap for any values of mass and coupling constant. The fact that Reggeization implies trivialization of certain bootstraps is implicit in Mandelstam's work[4] and is also suggested by Gell-Mann *et al.*[2] Gell-Mann and Zachariasen[5] have discussed it for the particular case of the fermion bootstrap although at the time it was felt to be of little relevance to hadron physics. We discuss it because we know now many examples of Reggeization in models which might describe real hadrons.[6]

The argument is essentially the following: Consider a conventional calculation which claims to bootstrap vector mesons or spin-$\frac{1}{2}$ fermions. We have in mind a calculation of scattering amplitudes where some of the forces are produced by the exchange of such particles and the $J=1$ or $J=\frac{1}{2}$ partial-wave amplitudes are required to have poles with positions and residues equal to the masses and coupling constants of these exchanged particles. Such a calculation may in general make two kinds of predictions: The first kind is group-theoretical and states that for only certain internal-symmetry groups and representations is a bootstrap possible.[7] We do not dispute the validity of such predictions. The other kind claim to actually determine numerical values for masses and coupling constants. However, if we can write down a renormalizable Lagrangian describing the vector mesons and spin-$\frac{1}{2}$ fermions and demonstrate that they Reggeize then the bootstrap equations will have a solution for any value of mass and coupling constant since these enter as arbitrary parameters in the Lagrangian. We believe Reggeization takes place in certain theories describing gauge mesons (which acquire mass through the Higgs-Kibble mechanism[8]) coupled in a gauge-invariant manner to other particles.[1]

We illustrate the argument by considering the ρ bootstrap in π-π scattering.[3] A suitable Lagrangian is obtained by first writing down a Yang-Mills theory for an SU(2) local group, coupling in a complex doublet of scalar mesons and a pion triplet. After spontaneous symmetry breaking we have an isospin-one massive ρ triplet, a pion triplet, and a scalar-isoscalar meson σ. The relevant part of the interaction Lagrangian is

$$L_I = -g\partial_\mu\vec{\rho}_\nu \times \vec{\rho}_\mu \cdot \vec{\rho}_\nu - \tfrac{1}{4}g^2(\vec{\rho}_\mu \times \vec{\rho}_\nu)^2 + \tfrac{1}{2}gm\sigma\vec{\rho}_\mu{}^2 + \tfrac{1}{8}g^2\sigma^2\vec{\rho}_\mu{}^2 + g\vec{\rho}_\mu \times \vec{\pi}\cdot\partial_\mu\vec{\pi} + \tfrac{1}{2}g^2(\vec{\rho}_\mu \times \vec{\pi})^2 \,. \tag{1}$$

The Born approximation at $J=1$, $I=1$ factorizes[1,2]; the 5×5 matrix of helicity amplitudes describing transitions between the $\pi\pi$ state and the four $\rho\rho$ states (one of which is "nonsense" at $J=1$) has rank one. The ρ is expected to Reggeize. What this means is that if one computes in the complex angular momentum plane the $\pi\pi\to\pi\pi$ partial-wave amplitude using unitarity, analyticity, coupling to the $\rho\rho$ channel and ρ, π, and σ exchange, the amplitude will agree at $J=1$ with that computed directly from the Lagrangian (in perturbation theory say). In particular, the $I=1$ amplitude will have a direct-channel pole which can be identified as the ρ pole. The bootstrap condition will be an identity valid for any values of m and g. Indeed, a crude coupled-channel N/D calculation which takes N equal to the dominant part of the Born approximation (ρ, π, and σ exchange in the u and t channels) and neglects higher-order effects gives the $I=1$ $\pi\pi$ scattering amplitude near $J=1$

$$T(s,J)=\tfrac{2}{3}g^2\,\frac{s-4\mu^2}{s-m^2}\,\frac{\alpha(s)-1}{J-\alpha(s)}\,, \tag{2}$$

$$\alpha(s)=1+\frac{g^2}{2\pi^2}\,(s-m^2)\int_{4m^2}^{\infty}\frac{ds'}{s'-s}\,[s'(s'-4m^2)]^{-1/2}\,. \tag{3}$$

At $J=1$, $T=-\frac{2}{3}g^2(s-4\mu^2)(s-m^2)^{-1}$ and the calculation "predicts" in this approximation an output zero-width ρ meson with mass m and coupling g equal to the (arbitrary) input values. A more sophisticated calculation would modify the above form and in particular give the ρ a width, but should still produce a trajectory $\alpha(s)$ passing through 1 at the ρ mass. We note that in the absence of σ exchange, a cutoff would be needed in the dispersion integral for D. But with the specific σ coupling of Eq. (1) no cutoff is needed; σ exchange provides a natural softening of the high-energy behavior in ImD.

The complex J-plane calculation whose results are given above involves coupling the physical ("sense") $\pi\pi$ and $\rho\rho$ helicity states to the nonsense $\rho\rho$ state (helicity two at $J=1$). An output ρ trajectory is produced then by the potential in the nonsense channel [which goes like $(s-m^2)(J-1)^{-1}$ near $J=1$ and dominates all others] and appears in the $\pi\pi$ channel through its coupling to the nonsense one.

A similar calculation could be done at $J=1$. One need not consider the nonsense state but should still consider the coupled $\pi\pi$ and (sense) $\rho\rho$ channels. (It is difficult to justify a truncated version of the coupled channel problem which ignores the $\rho\rho$ channel where large forces are present, even though it gives a nontrivial bootstrap.) Again with σ exchange present, one subtraction (which normalizes D) is sufficient to render the dispersion integrals convergent. The crucial point, however, as emphasized by Mandelstam[4] is that the equations should be solved with due regard to the various threshold and conspiracy conditions that the amplitudes must satisfy[9] (as they should in the complex-J calculation also). It is the essence of his argument that if this is done the relevant solution (minimal number of CDD poles) of the dynamical equations will have no free parameters and must agree with the one obtained in the complex-J calculation or *the one computed from a renormalizable Lagrangian* (insofar as we believe that the latter is analytic, unitary and satisfies the kinematical constraints). Therefore, the solution will again have an output ρ pole with position and residue equal to the input parameters for any value of these parameters since in the Lagrangian they can be arbitrary.

Similar comments apply to a spin-$\frac{1}{2}$ fermion bootstrap.[2,5] We have in mind bootstrapping the fermion by looking at fermion–vector-meson and fermion-pion scattering and exchanging fermions and mesons.[1] This is different from the reciprocal bootstrap.[10] The Δ may emerge from the above calculation but in this approach it is not the main agent for producing the N.

One may ask if bootstrap conditions for the scalar mesons have some content. At $J=1$ the basic argument is that because of kinematical constraints the solutions of the dynamical equations cannot differ from the one computed from the Lagrangian. At $J=0$ this is not the case.[9] The solution of the dynamical equations is not uniquely determined by the constraints and could differ from the one computed from the Lagrangian. Parameters which have arbitrary values in the latter might be determined by imposing additional constraints on the amplitude. For instance, one can demand factorization of the Born approximation at $J=0$; in certain models we have studied this takes place only for certain couplings and masses.[1] Or one can demand better high-energy behavior than required by unitarity. Such demands may provide some nontrivial bootstrap conditions but do not appear to modify our conclusions regarding the $J=1$ or $J=\frac{1}{2}$ bootstraps.

We conclude with some general comments: In our models the vector mesons must have isospin (or "color"[11]). Abelian gauge meson theories do not Reggeize as readily as non-Abelian ones and they may not trivially bootstrap if indeed they bootstrap at all.[7]

Some bootstrap calculations in the literature involve couplings which would lead to unrenormalizable Lagrangians.[12] If such couplings can be induced in higher order by starting with renormalizable ones we expect the bootstraps to still be trivial. But there exist anomalous couplings (e.g., $\rho\pi\omega$) which can be induced only through fermion loops. For such situations the Jackiw-Bell-Adler anomaly[13] is expected to cause renormalizability difficulties in the gauge theories[14] and it may also destroy Reggeization of the vector mesons and fermions. It is not clear what happens to the bootstraps, but we do question the validity of calculations which now involve highly divergent dispersion integrals and require arbitrary cutoff procedures.

We wish to comment also on the role of the scalar mesons. The factorization of the Born approximation and presumably the Reggeization of the vector meson require that the scalar meson be exchanged wherever a vector meson can be exchanged between vector mesons. It softens the bad high-energy behavior that vector exchange produces in much the same way as this behavior would be softened by exchanging ρ-Reggeons instead of elementary spin-one ρ's. One might speculate that a fully Reggeized bootstrap, which exchanges a ρ trajectory and produces a ρ trajectory would also be trivial in the neighborhood of $J=1$. This suggests the possibility that neither the slope nor position of Regge trajectories are determined by a bootstrap unless additional input is provided by requiring for instance better high-energy behavior than unitarity requires (superconvergence for example[15]). Otherwise, the masses and couplings of the spin-$\frac{1}{2}$ and spin-1 particles would have to be fixed by hand, the rest of the hadron spectrum being then uniquely determined. Renormalizable Lagrangian theory could be used to describe spin-$\frac{1}{2}$ and spin-1 particles, and all others would emerge as conventional bound states and resonances.[16]

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