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Electromagnetic mass differences, the pion mass, and the ρ-A_1 mass splitting

Kazuo Fujikawa*
Department of Applied Mathematics and Theoretical Physics, University of Cambridge, England

P. J. O'Donnell†
International Centre for Theoretical Physics, Trieste, Italy
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We discuss several problems related to the particle mass and mass differences in the framework of renormalizable gauge theories. We first explicitly calculate the hard-pion and the ρ-meson electromagnetic mass differences based on the chiral SU(2)×SU(2) gauge model of Bardakci. In both cases the neutral particles receive infinite mass shifts, thus rendering the mass differences finite. This is compared with the Schwinger-Yan approach and with the recently proposed model for electromagnetic form factors and the ρ' meson. We also estimate the effects of the ρ' meson on the pion mass based on the soft-pion method and field algebra. We then attempt to obtain the small pion mass by means of an approximate algebraic realization of the chiral symmetry at high energies using a generalization of the Bardakci model. We also discuss the effects of heavier vector and axial-vector mesons on the ρ and A_1 mass splitting. It is shown that heavier mesons help to preserve the ρ-meson universality.

I. INTRODUCTION

Following the present theoretical interest[1] in spontaneously broken gauge theories, many calculations of various electromagnetic mass differences[2-8] and the pion mass[4,9] have been attempted. As for the nucleon mass difference the situation does not appear to be quite as encouraging; either the mass difference diverges or the finite mass difference tends to appear with the wrong sign. Although it may in principle be possible to give a reasonable mass difference for the nucleon based on a gauge model, it seems to be always necessary to include tadpole diagrams.[10] The physical meaning of these tadpoles is not clear and they may just represent an interesting phenomenological observation rather than a basic dynamical scheme. The inclusion of the weak interaction does not change these basic features; in many cases it just provides an effective cutoff for divergent electromagnetic mass shifts.[11] Therefore it gives only a minor contribution (i.e., weak correction in the conventional sense). This is particularly true in view of the precocious scaling in SLAC experiments.[12] The hadronic mass scale seems to be extremely small compared with the hypothetical intermediate-vector-meson masses.

On the other hand the calculation of the pion mass based on the pseudo-Goldstone mechanism proposed by Weinberg[3] is dynamically more attractive. Calculations performed so far, however, either give an unrealistically large pion mass[4] or sometimes even a negative pion mass.[9] Although it is very attractive to imbed the pion-mass-generating mechanism in the unified model of weak and electromagnetic interactions, it is not clear whether this is a good way to generate a pion mass which exhibits almost perfect isospin symmetry; the weak and electromagnetic interactions strongly violate isospin symmetry unless the tadpole diagrams dominate.

Based on these observations we attempt to return to the more conventional approach. We consider a model of electromagnetic interactions which gives a finite mass difference and neglect weak interactions. For this purpose we use the chiral SU(2)×SU(2) gauge model of Bardakci.[8] This model is semirealistic and rich in dynamical content. Every isospin mass splitting in this model is finite.[8] We explicitly calculate the hard-pion and ρ-meson mass differences based on this model. We also generalize the model and discuss the pion mass and ρ-A_1 mass splitting.

II. PION AND ρ-MESON MASS DIFFERENCES

The first reliable calculation of the pion mass difference was performed by Das *et al.*[13] based on the PCAC soft-pion method and current algebra. The result of this calculation gives a fairly good agreement with experiment. When the calculation was extended to include hard-pion effects[14] the result was shown to diverge. In a theory with a cutoff, the numerical value turned out not to be sensitive to the cutoff mass, however. In the advent of the gauge theory, Dicus and Mathur[15] recently have reexamined the soft-pion calculation including the effects of weak interactions.

The calculation of the ρ-meson mass difference has no such reliable physical basis. Nevertheless Kaganovich,[16] for example, recently performed a calculation of the pion and ρ-meson mass differences in the spirit of the effective-Lagrangian method including the effects of weak interactions. He derived conditions for obtaining finite answers. On the other hand, Yan[17] performed those calculations following the suggestion of Schwinger. The finite values were obtained by assuming a rapid decrease of form factors.

A. Chiral model of Bardakci

A renormalizable version of the chiral SU(2) $\times$SU(2) gauge model was given by Bardakci.[8] Every isospin mass splitting in this model is finite.[8] Here we just quote his Lagrangian. We briefly discuss some of the mathematical structure of this model in Sec. III when we discuss the pion mass.

The Lagrangian reads

$$\begin{aligned}\mathcal{L} = &-\tfrac{1}{4}(\partial_\mu \vec{V}_\nu - \partial_\nu \vec{V}_\mu - f\vec{V}_\mu \times \vec{V}_\nu - f\vec{A}_\mu \times \vec{A}_\nu)^2 \\ &-\tfrac{1}{4}(\partial_\mu \vec{A}_\nu - \partial_\nu \vec{A}_\mu - f\vec{A}_\mu \times \vec{V}_\nu - f\vec{V}_\mu \times \vec{A}_\nu)^2 \\ &+\tfrac{1}{2}[\partial_\mu \vec{M}_1 - f(\vec{V}_\mu \times \vec{M}_1 - \vec{A}_\mu M_2)]^2 \\ &+\tfrac{1}{2}(\partial_\mu M_2 - f\vec{A}_\mu \cdot \vec{M}_1)^2 + \tfrac{1}{2}(\partial_\mu N_3)^2 + \tfrac{1}{2}(\partial_\mu N_4)^2 \\ &+\tfrac{1}{2}[\tfrac{1}{2}(f\vec{V}_\mu - e\vec{B}_\mu)N_3 + \tfrac{1}{2}f\vec{A}_\mu N_4]^2 \\ &+\tfrac{1}{2}[\tfrac{1}{2}f\vec{A}_\mu N_3 + \tfrac{1}{2}(f\vec{V}_\mu - e\vec{B}_\mu)N_4]^2 \\ &-\tfrac{1}{4}(\partial_\mu B_\nu - \partial_\nu B_\mu)^2 - V(N,M)\end{aligned} \quad (2.1)$$

and

$$\begin{aligned}-V(N,M) = &\lambda_1(\vec{M}_1^{\,2} + M_2^{\,2}) + \lambda_2(N_3^{\,2} + N_4^{\,2}) \\ &-\lambda_4(\vec{M}_1^{\,2} + M_2^{\,2})^2 - 4\lambda_5(N_3N_4)^2 \\ &-\lambda_6(N_3^{\,2} + N_4^{\,2})^2 \\ &-\lambda_7(\vec{M}_1^{\,2} + M_2^{\,2})(N_3^{\,2} + N_4^{\,2}) \\ &+\lambda_9 M_2(N_3^{\,2} - N_4^{\,2}),\end{aligned} \quad (2.2)$$

where $\vec{V}_\mu, \vec{A}_\mu$ and f are vector and axial-vector fields, and their gauge coupling constant, respectively. $\vec{B}_\mu$ is the photon field $(0,0,B_\mu)$. $\vec{M}_1$ and M_2 correspond to the ordinary $\vec{\pi},\sigma$ system, and N_3 and N_4 are extra scalar mesons required by the Higgs mechanism.[1] Some of the potential terms which do not give rise to new terms have been omitted in Eq. (2.2).

The gauge condition we employed is

$$\vec{N}_1 = \vec{N}_2 = 0, \quad (2.3)$$

namely, the gauge I of Bardakci.[8] The gauge-compensating effective action is given by

$$S_{\text{g.c.}} = (-3i)\delta^4(0)\int d^4x \ln[N_3(x)^2 - N_4(x)^2]. \quad (2.4)$$

In Eqs. (2.1) and (2.2), the fields M_2 and N_3 have the vacuum values

$$\begin{aligned}M_2 &= M_2' + \alpha, \\ N_3 &= N_3' + \beta.\end{aligned} \quad (2.5)$$

The field-mixing-type coupling is obtained from the last three terms in Eq. (2.1):

$$\begin{aligned}\mathcal{L}_{\text{int}} = &-\frac{e}{f}(\tfrac{1}{2}f\beta)^2 V_\mu^3 B^\mu - e(\tfrac{1}{2}f\beta)N_3' V_\mu^3 B^\mu \\ &-e(\tfrac{1}{2}f\beta)N_4 A_\mu^3 B^\mu.\end{aligned} \quad (2.6)$$

The special feature of the gauge in Eq. (2.3) is that it is (global) chiral-invariant, and the fields $\vec{M}_1$ and $\vec{A}_\mu$ mix with each other; M_2 and N_3 generally mix regardless of the gauge condition. This complication, however, does not give rise to any difficulty in the general framework of the gauge theory in the manner of Faddeev and Popov.[18] We summarize the Feynman rules and the definition of the amputated T matrix in this gauge in Appendix A. The gauge (2.3) makes it easier to compare the following calculations with the conventional ones. For the photon field B_μ, we use the Landau gauge

$$\partial_\mu B^\mu(x) = 0. \quad (2.7)$$

The "photon" field B_μ in Eq. (2.1) is not quite massless, but it does not affect the calculations which follow. If an infrared problem arises the complete diagonalization of the (photon) mass matrix is required. The gauge (2.7) helps to make various integrals more convergent.

B. Pion mass difference

We complete the hard-pion calculation of Bardakci.[8] It is easy to see that the ordinary hard-pion calculation of Lee and Nieh[14] is modified in the present gauge by the extra diagrams shown in Fig. 1. (We omit tadpole diagrams which do

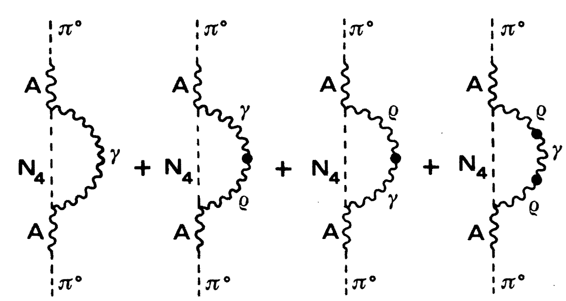

FIG. 1. Scalar-meson contribution to the pion mass difference.

not contribute to the mass difference.) These diagrams give rise to

$$\delta m_{\pi^0}{}^2=\left(\frac{m_A{}^2-m_\rho{}^2}{m_A{}^2}\right)e^2\int\frac{d^4k}{i(2\pi)^4}\,\frac{p^2-(k\cdot p)^2/k^2}{k^2[(k+p)^2-m_N{}^2]}+2e^2\frac{(m_A{}^2-m_\rho{}^2)m_\rho{}^2}{m_A{}^2}\int\frac{d^4k}{i(2\pi)^4}\,\frac{p^2-(k\cdot p)^2/k^2}{k^2(k^2-m_\rho{}^2)[(k+p)^2-m_N{}^2]}$$

$$+e^2\frac{(m_A{}^2-m_\rho{}^2)m_\rho{}^4}{m_A{}^2}\int\frac{d^4k}{i(2\pi)^4}\,\frac{p^2-(k\cdot p)^2/k^2}{k^2(k^2-m_\rho{}^2)^2[(k+p)^2-m_N{}^2]}$$

$$\simeq\frac{3\alpha}{4\pi}m_\rho{}^2\left(\frac{m_\pi{}^2}{m_\rho{}^2}\right)\left\{\tfrac{1}{8}\ln\left(\frac{\Lambda^2}{m_N{}^2}\right)-\tfrac{1}{8}\left(\frac{m_\rho{}^2}{m_N{}^2-m_\rho{}^2}\right)\left[\left(2+\frac{m_\rho{}^2}{m_N{}^2-m_\rho{}^2}\right)\ln\left(\frac{m_N{}^2}{m_\rho{}^2}\right)-1\right]\right\},\qquad(2.8)$$

where m_π and m_N correspond to the mass of the pion and N_4, respectively. The result of Lee and Nieh[14] based on the ordinary chiral model is

$$\delta m_{\pi^\pm}{}^2\simeq\frac{3\alpha}{4\pi}m_\rho{}^2\left\{2\ln 2+\frac{m_\pi{}^2}{m_\rho{}^2}\left[\ln\left(\frac{m_\rho{}^2}{m_\pi{}^2}\right)+\tfrac{19}{4}\ln 2-\tfrac{5}{2}+\tfrac{1}{8}\ln\left(\frac{\Lambda^2}{m_\rho{}^2}\right)\right]\right\},\qquad(2.9)$$

where use has been made of the relation $m_A{}^2=2m_\rho{}^2$. We also confirmed this result in our framework by using a cutoff procedure identical to that used in (2.8); the finite part in general depends on the cutoff procedure. The logarithmic divergence cancels between Eqs. (2.8) and (2.9), as demonstrated by Bardakci.[8] The numerical coefficients of various finite terms in (2.8) are very small even compared with the *hard-pion* part of (2.9) for reasonable values of $m_N{}^2$ in the range $1\le m_N{}^2<\infty$. The only effect of (2.8) is therefore to provide a logarithmic cutoff to (2.9). The numerical value of the mass difference is therefore given by $\delta m_\pi\simeq 6$ MeV.[14] The N_4 meson *strongly* decays into a $\rho\pi$ system as is clear from Fig. 1. Therefore it cannot be η or η'. In the framework of SU(2) or SU(2)×SU(2), the current is purely isovector and we cannot handle problems like η decay or π^0 decay which depend on the interference between isoscalar and isovector electromagnetic currents.

C. ρ-meson mass difference

In this calculation we can also consider two contributions, the conventional part and the part which depends on the scalar mesons. The conventional part contains two diagrams (see Fig. 2) and we get

$$\delta m_{\rho^\pm}{}^2=\frac{\alpha}{4\pi}m_\rho{}^2\times\tfrac{9}{4}$$

$$+\frac{\alpha}{4\pi}m_\rho{}^2\left[-\tfrac{59}{24}+\tfrac{33}{8}\Gamma-\tfrac{3}{4}\ln\left(\frac{\Lambda^2}{m_\rho{}^2}\right)\right]$$

$$=\frac{\alpha}{4\pi}m_\rho{}^2\left[-\tfrac{5}{24}+\tfrac{33}{8}\Gamma-\tfrac{3}{4}\ln\left(\frac{\Lambda^2}{m_\rho{}^2}\right)\right],\qquad(2.10)$$

where $\Gamma=2\pi/\sqrt{27}\approx 1.2$.

The extra contribution from the Higgs scalar meson is given by (see Fig. 3)

$$\delta m_{\rho^0}{}^2=\frac{\alpha}{4\pi}m_\rho{}^2\left[\tfrac{31}{32}\Gamma-\tfrac{35}{48}-\tfrac{3}{4}\ln\left(\frac{\Lambda^2}{m_\rho{}^2}\right)\right]\quad\text{for } M^2=m_\rho{}^2$$

$$=\frac{\alpha}{4\pi}m_\rho{}^2\left[\tfrac{31}{32}\Gamma-\tfrac{5}{48}-\tfrac{3}{4}\ln\left(\frac{\Lambda^2}{M^2}\right)\right]\quad\text{for } M^2\gg m_\rho{}^2,\qquad(2.11)$$

where M is the mass of the N_3' meson.

The divergent parts in Eqs. (2.10) and (2.11) again cancel each other. The contribution from

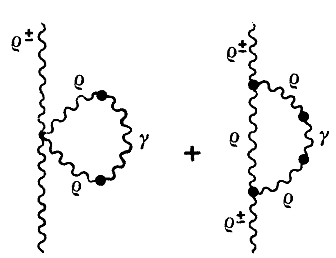

FIG. 2. Conventional diagrams for the ρ-meson mass difference.

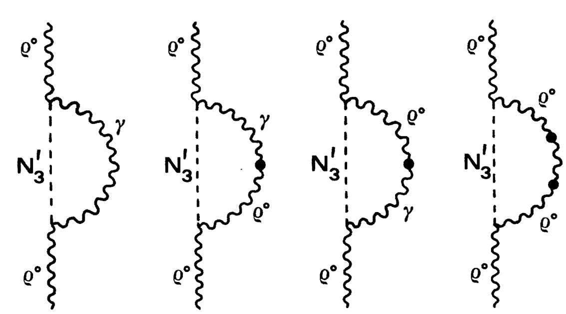

FIG. 3. Scalar-meson contribution to the ρ-meson mass difference.

"spring" diagrams arising from the field mixing $\rho^0 \to \gamma \to \rho^0$ gives rise to

$$\delta m_{\rho^0}{}^2 = \frac{\alpha}{4\pi} m_\rho{}^2 \times \left(\frac{4\pi}{f^2/4\pi}\right), \tag{2.12}$$

where $f^2/4\pi \approx 2.5 \sim 3$.

The final result from Eqs. (2.10)—(2.12) is

$$\delta(m_{\rho^0} - m_{\rho^\pm}) \approx 0.15 \text{ MeV} \quad \text{for } M^2 = m_\rho{}^2 \tag{2.13}$$

and

$$\delta(m_{\rho^0} - m_{\rho^\pm}) \approx 0.66 \text{ MeV} \quad \text{for } M^2 = 10 m_\rho{}^2 . \tag{2.14}$$

The neutral ρ appears to be slightly heavier. The mass difference is, however, very small. The experimental value[19] is

$$\delta(m_{\rho^0} - m_{\rho^\pm}) = 2.4 \pm 2.1 \text{ MeV}. \tag{2.15}$$

Considering the large width of ρ mesons, the theoretical value appears to be consistent with experiment. The choice $M^2 \approx m_\rho{}^2$ is favored by the gauge model for the ρ' meson.[20] As we note in Appendix A, mixing between the σ field and the Higgs scalar field is neglected in this calculation of the ρ-meson mass difference; provided their masses are not too different, the effect of such mixing on the mass differences is small.

D. The ρ' model and the Schwinger-Yan model

The calculation of the pion and ρ-meson mass differences has been performed by Yan[17] based on slightly different assumptions. He took the coupling between γ and ρ to be

$$-i\left(\frac{e}{f}\right) m_\rho{}^2 \left(\frac{M^2}{q^2 - M^2}\right). \tag{2.16}$$

The extra factor $M^2/(q^2 - M^2)$ compared with the conventional field-mixing-type coupling provides an ultraviolet cutoff, thus making the mass difference finite. Equation (2.16) automatically arises in the recently proposed gauge model for the ρ' meson.[20] The mass M^2 corresponds to $m_{\rho'}{}^2$ in this gauge model. The numerical results by Yan indicate (we just extrapolate his results to the point $M^2 \approx 5 m_\rho{}^2$),

$$\delta m_\pi \approx 3 \text{ MeV}, \tag{2.17}$$

$$\delta(m_{\rho^0} - m_{\rho^\pm}) \approx 0.45 \text{ MeV}. \tag{2.18}$$

We note that the ρ-meson mass difference is fairly sensitive to the exact value of $f^2/4\pi$; Eq. (2.18) still seems to be consistent with Eqs. (2.13) and (2.14).

As was noted by Yan, the pion mass difference is substantially modified by the extra form factor. Equation (2.17) can be compared with the experimental value[19]

$$\delta m_\pi \approx 4.6 \text{ MeV} \tag{2.19}$$

and with the conventional chiral model result[14] (this is also the result of Sec. II B)

$$\delta m_\pi \approx 6 \text{ MeV}. \tag{2.20}$$

We note that the Schwinger-Yan approach corresponds to the weak coupling limit, i.e., $f_{\rho'}/f_\rho \ll 1$, in the framework of the chiral gauge theory (in this limit the effects of the heavier axial-vector meson "A_1'" are negligible). An example of the chiral gauge theory with ρ' and A_1' is discussed in Sec. III. In view of the controversial status of the A_1 meson itself, a quantitative discussion of the "A_1'" meson contribution to the pion mass difference is rather difficult at the present stage.

III. PION MASS AND ρ-A_1 MASS SPLITTING

In this section we discuss the effects of the ρ' meson on the masses of the π and the ρ mesons. This investigation is motivated by the phenomenological success of the gauge model[20,21] for the ρ' meson. For this purpose we mainly utilize a generalization of the Bardakci model. Although the field-theoretical approach to strong interactions may not be quite reliable, it is hoped that some of the features we discuss below may be of more general validity than the specific underlying Lagrangian. The success of ordinary vector dominance and its universality, which was suggested by a simple Lagrangian,[22] strengthens this hope. In this respect we would like to comment on the potential terms in the model. These potential terms are strongly restricted by the gauge invariance and they are limited to dimension less than or equal to four by the requirement of renormalizability. Although the requirement of renormalizability may well turn out to be irrelevant, we believe that the scale dimension of four or less together with gauge invariance have a more fundamental meaning. The suppression of the 2π decay mode of the ρ' meson relative to the 4π mode was determined essentially by this scale dimension of interaction terms.[20] The successful description of various form factors,[20] and the absence of any other plausible physical picture for this 2π mode suppression motivate us to investigate the further implications of gauge models[23] for low-lying vector and scalar mesons.

A. Pion mass

Following the gauge model for the ρ' meson,[20] we assume that the ρ' couples to the hadronic system just like the photon does in the chiral model in Eq. (2.1). The effects of the ρ' meson on the pion mass can then be estimated in a fairly model-independent manner by using the PCAC soft-pion method and field algebra. In the following we first present a simple-minded treatment of this problem, and later discuss it in more detail based on the renormalizable gauge theory.

Following the calculation of Dicus and Mather[15] we get (only the neutral-pion case is considered because of isospin symmetry)

$$\begin{aligned}\delta_V m^2 &= \tfrac{1}{2} f'^2 \int \frac{d^4k}{(-i)(2\pi)^4} \frac{(-i)(g^{\mu\nu} - k^\mu k^\nu / m_{\rho'}^2)}{k^2 - m_{\rho'}^2} \int dx\, e^{ikx} \sum_a \langle \pi^0 | T(V_\mu^a(x) V_\nu^a(0)) | \pi^0 \rangle_{\text{connected}} \\ &= -4\left(\frac{f'^2}{4\pi}\right)\left(\frac{i}{f_\pi^2}\right)\frac{1}{(2\pi)^3}\int \frac{d^4k}{m_{\rho'}^2}\left[\int dm^2 \frac{\rho_V(m^2)-\rho_A(m^2)}{m^2} - f_\pi^2\right] \\ &\quad -4\times 3\left(\frac{f'^2}{4\pi}\right)\left(\frac{i}{f_\pi^2}\right)\frac{1}{(2\pi)^3}\int d^4k \frac{1}{k^2 - m_{\rho'}^2}\int dm^2 \frac{\rho_V(m^2)-\rho_A(m^2)}{k^2 - m^2}, \end{aligned} \tag{3.1}$$

where f' is the gauge coupling constant of the ρ' meson. If one approximates ρ_V and ρ_A by single poles at m_ρ^2 and $m_{A_1}^2$, respectively, and uses the Weinberg's first and second sum rules,[15] which are valid in the field algebra model, one gets

$$\delta_V m^2 = \frac{3}{2\pi}\left(\frac{f'^2}{4\pi}\right) m_\rho^2 \frac{m_{A_1}^2}{m_{A_1}^2 - m_{\rho'}^2} \times \left[2\ln 2 - \frac{m_{\rho'}^2}{m_{\rho'}^2 - m_\rho^2}\ln\left(\frac{m_{\rho'}^2}{m_\rho^2}\right)\right], \tag{3.2}$$

where the KSRF (Kawarabayashi-Suzuki-Riazuddin-Fayyazuddin) relation, $m_\rho^2 = 2f_\rho^2 f_\pi^2$ and $m_{A_1}^2 = 2m_\rho^2$, was also used. It is interesting to observe that the Weinberg first spectral sum rule [i.e., the vanishing of the first term in Eq. (3.1)] guarantees the non-Abelian gauge invariance[24] of the mass shift.

In the framework of the gauge model for the ρ' meson,[20] experiment[25] indicates that the coupling constant $f_{\rho'}$ is comparable with f_ρ, as is expected for strongly interacting particles. Equation (3.2) then gives

$$m_\pi^2 \sim \tfrac{1}{2} m_\rho^2. \tag{3.3}$$

This result may indicate that there is an axial-vector spectral weight to cancel the ρ' contribution. In this connection it is interesting to see that

$$\begin{aligned}\delta_A m^2 &= \tfrac{1}{2} f'^2 \int \frac{d^4k}{(-i)(2\pi)^4}(-i)\frac{g^{\mu\nu} - k^\mu k^\nu / m_{A_1'}^2}{k^2 - m_{A_1'}^2}\int dx\, e^{ikx}\sum_a \langle \pi^0 | T(A_\mu^a(x) A_\nu^a(0)) | \pi^0\rangle \\ &= 4\times 3\left(\frac{f'^2}{4\pi}\right)\left(\frac{i}{f_\pi^2}\right)\frac{1}{(2\pi)^3}\int \frac{d^4k}{k^2 - m_{A_1'}^2}\int dm^2 \frac{\rho_V(m^2)-\rho_A(m^2)}{k^2 - m^2}. \end{aligned} \tag{3.4}$$

In this simple-minded treatment the algebraic realization of the chiral symmetry for heavier mesons, $m_{A_1'} = m_{\rho'}$, gives rise to

$$\delta_V m^2 + \delta_A m^2 = 0 \tag{3.5}$$

and the pion stays massless. If one takes the viewpoint that the chiral symmetry breaking is approximated by the asymmetry in the part of vector and axial-vector spectral weights, the present calculation indicates that the symmetry breaking by ρ' without an effective A_1' is too strong. Being a perturbative calculation in strong interactions, these formulas may not be so accurate. [E.g., the numerical value is rather sensitive to whether one uses the "zeroth-order" masses or the physical masses for vector mesons (cf. Sec. III C).] It is, however, still interesting to see that a reasonable choice of heavier vector-meson masses, e.g., [see Eq. (3.14)] $m_{\rho'}^2 \approx 6m_\rho^2$ and $m_{A_1'}^2 \approx 8m_\rho^2$ combined with Eqs. (3.2) and (3.4) give rise to (assuming $f_{\rho'} = f_\rho$)

$$m_\pi^2 \sim \tfrac{1}{20} m_\rho^2. \tag{3.6}$$

Here ρ' and A_1' represent *all* the effects of heavier mesons. If the qualitative feature of this calculation is trusted, it suggests that the chiral symmetry approaches an almost algebraic realization (i.e., $m_V \approx m_A$) at high energies. This is in line with general beliefs about the strong interaction symmetries. The finiteness of the pion mass calculation in Eqs. (3.2) and (3.4) can be understood as the pseudo-Goldstone mechanism[3] in the framework of renormalizable field theories. We can thus justify the naive manipulations in these equations.

B. Chiral SU(2) × SU(2) model with ρ' and A_1' mesons

We discuss a simple example of the chiral SU(2) × SU(2) gauge model with ρ' and A_1' mesons, and comment on why a finite result for the pion mass was obtained in Eqs. (3.2) and (3.4).

Following Bardakci[8] we define the following objects:

$$W_\mu \equiv (\vec{V}_\mu\cdot\vec{\tau}+\vec{A}_\mu\cdot\vec{\tau}\gamma_5)\left(\frac{1+\sigma_3}{2}\right)\left(\frac{1+\Sigma_3}{2}\right),$$

$$M\equiv(\vec{M}_1\cdot\vec{\tau}i\gamma_0\gamma_5+M_2\gamma_0)\left(\frac{1+\sigma_3}{2}\right)\left(\frac{1+\Sigma_3}{2}\right),$$

$$N\equiv[(\vec{N}_1\cdot\vec{\tau}+\vec{N}_2\cdot\vec{\tau}\gamma_5)\sigma_1+(N_3+N_4\gamma_5)\sigma_2]\left(\frac{1+\Sigma_3}{2}\right),$$

$$W'_\mu \equiv (\vec{V}'_\mu\cdot\vec{\tau}+\vec{A}'_\mu\cdot\vec{\tau}\gamma_5)\left(\frac{1-\sigma_3}{2}\right)\left(\frac{1+\Sigma_3}{2}\right), \qquad (3.7)$$

$$M'\equiv(\vec{M}'_1\cdot\vec{\tau}i\gamma_0\gamma_5+M'_2\gamma_0)\left(\frac{1-\sigma_3}{2}\right)\left(\frac{1+\Sigma_3}{2}\right),$$

$$N'\equiv[(\vec{N}'_1\cdot\vec{\tau}+\vec{N}'_2\cdot\vec{\tau}\gamma_5)\Sigma_1+(N'_3+N'_4\gamma_5)\Sigma_2]\left(\frac{1-\sigma_3}{2}\right),$$

$$B_\mu\equiv B_\mu^3\tau^3\left(\frac{1-\sigma_3}{2}\right)\left(\frac{1-\Sigma_3}{2}\right),$$

where W_μ, M, and N stand for the same particles as in Bardakci's model [see Eq. (2.1)], and W'_μ, M', and N' correspond to heavier partners of these particles. The field B_μ corresponds to the photon. In Eq. (3.8) $\vec{\tau}$, $\vec{\sigma}$, and $\vec{\Sigma}$ all obey the algebra of the Pauli matrices. What we are doing is to form a multiple cross product of the chiral SU(2)×SU(2) group by utilizing the projection operators $\frac{1}{2}(1\pm\sigma_3)$ and $\frac{1}{2}(1\pm\Sigma_3)$. See also Ref. 8. Schematically this is shown in Fig. 4. The ordinary chiral symmetry is generated by

$$\vec{\tau}(1\pm\gamma_5). \qquad (3.8)$$

By forming suitable covariant derivatives (the trace covers all the matrix indices)

$$\tfrac{1}{16}\,\mathrm{Tr}(\nabla_\mu M)^2=\tfrac{1}{16}\,\mathrm{Tr}\{\partial_\mu M-i\tfrac{1}{2}f[W_\mu,M]\}^2,$$

$$\tfrac{1}{32}\,\mathrm{Tr}(\nabla_\mu N)^2=\tfrac{1}{32}\,\mathrm{Tr}\{\partial_\mu N-i\tfrac{1}{2}f[W_\mu,N]-i\tfrac{1}{2}f'[W'_\mu,N]\}^2, \qquad (3.9)$$

$$\tfrac{1}{16}\,\mathrm{Tr}(\nabla_\mu M')^2=\tfrac{1}{16}\,\mathrm{Tr}\{\partial_\mu M'-i\tfrac{1}{2}f'[W'_\mu,M']\}^2,$$

$$\tfrac{1}{32}\,\mathrm{Tr}(\nabla_\mu N')^2=\tfrac{1}{32}\,\mathrm{Tr}\{\partial_\mu N'-i\tfrac{1}{2}f'[W'_\mu,N']-i\tfrac{1}{2}e[B_\mu,N']\}^2,$$

and

$$-\tfrac{1}{32}\,\mathrm{Tr}\{\partial_\mu W_\nu-\partial_\nu W_\mu-i\tfrac{1}{2}f[W_\mu,W_\nu]\}^2,$$

$$-\tfrac{1}{32}\,\mathrm{Tr}\{\partial_\mu W'_\nu-\partial_\nu W'_\mu-i\tfrac{1}{2}f'[W'_\mu,W'_\nu]\}^2,$$

it is straightforward to write down a "renormalizable" Lagrangian. The potential terms (see Appendix B) which have dimension less than or equal to four can directly mix M and M', for example, through the terms

$$\mathrm{Tr}(M)^2\,\mathrm{Tr}(M')^2. \qquad (3.10)$$

These terms may be treated as renormalization counterterms. In that case we take Fig. 4 as the basic coupling scheme.

We make vector mesons, W_μ and W'_μ, massive by giving vacuum values to N_3 and N'_3 [see Eq. (3.9)]. The pion multiplet which arises from M can be made massless by assigning a vacuum value to M_2, but one may give a chiral-invariant *positive* mass term to the M' multiplet. We then break the chiral symmetry explicitly by a potential term (i.e., chiral-time component)

$$\mathrm{Tr}\left[M'N'\left(\frac{1-\sigma_3}{2}\right)\left(\frac{1-\Sigma_3}{2}\right)\gamma_0 N'\right]. \qquad (3.11)$$

This is analogous to the λ_9 term in Eq. (2.2). Due to this term the degeneracy among W'_μ multiplet and also among M' multiplet is removed. It is easy to check that the explicit pion mass term in the potential is prohibited (a sort of pseudo-Goldstone mechanism[3]) if one imposes the extra reflection symmetry

$$(\vec{M}_1,M_2)\to(-\vec{M}'_1,-M_2) \qquad (3.12)$$

on the Lagrangian. This is so because all the allowed potentials then contain the M field in the combination $M^2\propto\vec{M}_1{}^2+M_2{}^2$; the elimination of the σ tadpole simultaneously removes the pion mass term. Potential terms are listed in Appendix B. The reflection symmetry (3.12) is broken spontaneously, but the Lagrangian and all the Feynman amplitudes preserve it [the gauge we defined in Eq. (2.3) is also invariant under this symmetry].

The effects of Eq. (3.11), of course, propagate to the pion multiplet via vector-meson interactions (the symmetric potentials do not propagate such symmetry breaking). The lowest-order perturbation then gives rise to the result in Sec. III A. See Appendix C. In this calculation σ tadpole diagrams introduce some complications (i.e., renormalization counterterms) for the axial-vector meson contribution. The gauge defined in Eq. (2.3) simplifies this calculation.

If one removes M' and all the terms proportional to γ_5 in W'_μ and N' from Eq. (3.7), one gets a model

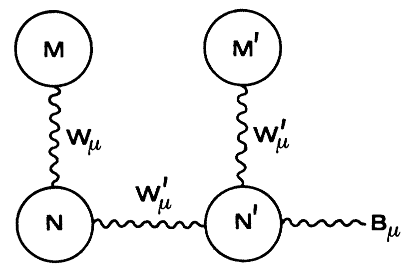

FIG. 4. Coupling scheme for the model in Eq. (3.7).

where the ρ' triplet is added to the Bardakci model. It is again not difficult to check that the reflection symmetry (3.12) still prohibits the zeroth-order pion mass term. (See the potential terms in Appendix B.) This explains the finite result in Eq. (3.2).

C. ρ and A_1 mass splitting

As we explained elsewhere,[20] the ordinary chiral SU(2)×SU(2) field algebra model[26,8] may have a difficulty related to the ratio $f_{\rho\pi\pi}/f_{\rho\gamma}$ if the m_ρ and m_{A_1} difference is entirely generated by the vacuum value of the σ field. The present model offers a possible solution to this problem.

Due to the field mixing between W_μ and W'_μ in the second term in Eq. (3.9), the heavier mesons ρ' and A'_1 influence the masses of ρ and A_1. The mass splitting among ρ' and A'_1 also induces further mass splitting among ρ and A_1. The physical particle masses are given by

$$m_\rho^2, m_{\rho'}^2 = \tfrac{1}{2}(\hat{M}_\rho^2 + \hat{M}_{\rho'}^2 \mp \{(\hat{M}_\rho^2 + \hat{M}_{\rho'}^2)^2 - 4[\hat{M}_\rho^2 \hat{M}_{\rho'}^2 - (\tfrac{1}{2}f\beta)^2(\tfrac{1}{2}f'\beta)^2]\}^{1/2}),$$
$$m_{A_1}^2, m_{A'_1}^2 = \tfrac{1}{2}(\hat{M}_{A_1}^2 + \hat{M}_{A'_1}^2 \mp \{(\hat{M}_{A_1}^2 + \hat{M}_{A'_1}^2)^2 - 4[\hat{M}_{A_1}^2 \hat{M}_{A'_1}^2 - (\tfrac{1}{2}f\beta)^2(\tfrac{1}{2}f'\beta)^2]\}^{1/2}), \quad (3.13)$$

where

$$\hat{M}_\rho^2 = (\tfrac{1}{2}f\beta)^2,$$
$$\hat{M}_{\rho'}^2 = (\tfrac{1}{2}f')^2(\beta^2 + \beta'^2),$$
$$\hat{M}_{A_1}^2 = (\tfrac{1}{2}f)^2(\beta^2 + 4\alpha^2),$$
$$\hat{M}_{A'_1}^2 = (\tfrac{1}{2}f')^2(\beta^2 + \beta'^2 + 4\alpha'^2),$$

with α, β, α', and β' the vacuum values of M_2, N_3, M'_2, and N'_3 in Eq. (3.7), respectively. The vacuum value α' may be generated by the explicit chiral symmetry breaking [see Eq. (3.11)]. It can be checked that the zeroth-order masses $\hat{M}_\rho^2 \approx 2m_\rho^2$, $\hat{M}_{\rho'}^2 \approx 5m_\rho^2$, $\hat{M}_{A_1}^2 \approx 3m_\rho^2$ and $\hat{M}_{A'_1}^2 \approx 7m_\rho^2$ together with $f \approx f'$ give rise to a reasonable mass spectrum;

$$m_\rho^2 = m_\rho^2, \quad m_{\rho'}^2 \approx 6m_\rho^2,$$
$$m_{A_1}^2 \approx 2m_\rho^2, \quad m_{A'_1}^2 \approx 8m_\rho^2, \quad (3.14)$$

Here ρ' and A'_1 represent all the effects of heavier vector mesons.

The on-shell coupling constant $f_{\rho\pi\pi}$ is given by

$$f_{\rho\pi\pi} \simeq \left(1 - \frac{1}{2}\frac{\hat{M}_{A_1}^2 - \hat{M}_\rho^2}{\hat{M}_{A_1}^2}\frac{m_\rho^2}{\hat{M}_\rho^2}\right) f. \quad (3.15)$$

This is valid when the mixing between the pion and A'_1 is small, as is the case in our model. See also Ref. 26. If one accepts the above mechanism of generating the masses of the ρ and the A_1 mesons the deviation of $f_{\rho\pi\pi}$ from f is small, and Eq. (3.15) combined with the ρ' meson propagator gives an almost identical result for the ratio of $f_{\rho\pi\pi}$ to $f_{\rho\gamma}$ as our previous study.[20] Namely

$$\frac{1}{4\pi}f_{\rho\pi\pi}^2 : \frac{1}{4\pi}f_{\rho\gamma}^2 \approx 1 : \left(\frac{m_{\rho'}^2 - m_\rho^2}{m_{\rho'}^2}\right)^2. \quad (3.16)$$

We can therefore maintain a satisfactory agreement with experiment.[20]

If one accepts this mechanism the electromagnetic form factor we discussed previously,[20] i.e.,

$$F_\pi(q^2) = \frac{m_\rho^2}{q^2 - m_\rho^2}\frac{m_{\rho'}^2}{q^2 - m_{\rho'}^2} \quad (3.17)$$

is also correctly described by the present gauge model in the small q^2 region. If $f_{\rho\pi\pi}$ substantially deviates from f, the gauge condition necessitates a direct coupling between the pion and the ρ' and also between the pion and the photon. A double pole form factor (3.17) cannot be described by the present gauge model in such cases. The ρ meson universality is also significantly modified in such cases. Note that the deviation of $f_{\rho\pi\pi}$ from the universal value f is caused by the field mixing between π and A_1.

Finally we comment on the generalization of the model (3.7) by adding further multiplets W''_μ, M'', and N'' etc. [one can obviously continue the procedure we used in Eq. (3.7)]. See Fig. 5. If one assumes that the particles which are connected by a verticle line in Fig. 5 are approximately degenerate in mass, this scheme may simulate some of the features of the dual-resonance model.[27] The massless pion due to the spontaneous chiral symmetry breaking and an algebraic realization of heavier mesons give a particularly simple model.

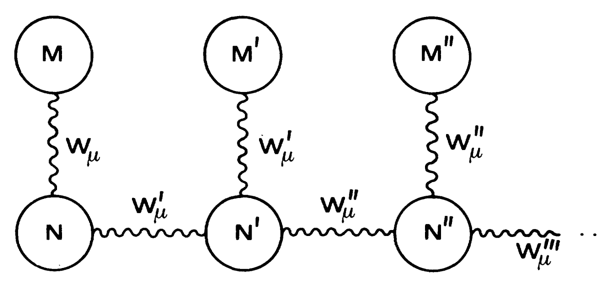

FIG. 5. Coupling scheme for the model with many vector and scalar mesons.

IV. CONCLUSION

In the present note we investigated the implications of renormalizable gauge models for low-lying vector and scalar mesons. Our study indicates that this approach provides a consistent description of those low-lying mesons and their electromagnetic properties in the low-energy region. One of the interesting results of the present study is that the heavier vector and axial-vector mesons help to restore the ρ-meson universality. This universality was partly sacrificed in the past[26] when one tried to explain the small pion mass as a consequence of the spontaneous breaking of chiral symmetry.

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APPENDIX A

We summarize the Feynman rules and the definition of the amputated T matrix. The quadratic part of the Lagrangian in Eq. (2.1) is given by

$$\mathcal{L}_{\text{quad}} = -\tfrac{1}{4}(\partial_\mu \vec{V}_\nu - \partial_\nu \vec{V}_\mu)^2 - \tfrac{1}{4}(\partial_\mu \vec{A}_\nu - \partial_\nu \vec{A}_\mu)^2 + \tfrac{1}{2}(\partial_\mu \vec{M}_1)^2 + \tfrac{1}{2}(\partial_\mu M_2')^2 + \tfrac{1}{2}(f\alpha)^2 \vec{A}_\mu \cdot \vec{A}^\mu$$
$$+ (f\alpha)\partial_\mu \vec{M}_1 \cdot \vec{A}^\mu + \tfrac{1}{2}(\partial_\mu N_3')^2 + \tfrac{1}{2}(\partial_\mu N_4)^2 + (\tfrac{1}{2}f\beta)^2(\vec{A}_\mu \cdot \vec{A}^\mu + \vec{V}_\mu \cdot \vec{V}^\mu) - V_{\text{quad}}\,, \tag{A1}$$

where

$$-V_{\text{quad}} = -\left(4\lambda_4\alpha^2 + \lambda_9 \frac{\beta^2}{2\alpha}\right)(M_2')^2 - \lambda_9\left(\frac{\beta}{2\alpha}\right)(\vec{M}_1)^2 - 4\lambda_6\beta^2(N_3')^2 - (4\lambda_5\beta^2 + 2\lambda_9\alpha)(N_4)^2 - 2\lambda_9\alpha(\vec{N}_2)^2$$
$$+ 2\lambda_9\beta(\vec{M}_1\cdot\vec{N}_2) - (4\lambda_7\alpha\beta - 2\lambda_9\beta)N_3'M_2' + \tfrac{1}{2}\delta\mu_1^2[2\alpha M_2' + (M_2')^2 + (\vec{M}_1)^2]$$
$$+ \tfrac{1}{2}\delta\mu_2^2[2\beta N_3' + (N_3')^2 + (\vec{N}_1)^2 + (\vec{N}_2)^2 + (N_4)^2]\,. \tag{A2}$$

In Eq. (A2) we wrote explicitly $\vec{N}_1$ and $\vec{N}_2$ fields and also tadpole counterterms (in our gauge $\vec{N}_1 = \vec{N}_2 = 0$). To simplify the situation we adjust λ_7 in the following such that

$$4\lambda_7\alpha\beta - 2\lambda_9\beta = 0 \tag{A3}$$

and remove the mixing between M_2' and N_3'. There are several ways to quantize Eq. (A1). One way is to treat homogeneous terms as free parts and field mixing terms as interactions (i.e., summation of "spring" diagrams). We, however, employ the path integral method.[18] This method offers a systematic treatment of the amputated T matrix. Equivalence of these two methods is important because algebraic properties are usually discussed based on the first method.[8]

A straightforward calculation gives the following propagators:

$$\vec{A}_\mu:\ (-i)\ \frac{g^{\mu\nu} - k^\mu k^\nu/m_A^2}{k^2 - m_A^2} + i\,\frac{k^\mu k^\nu}{k^2 - m_\pi^2}\left(\frac{m_A^2 - m_\rho^2}{m_A^2 m_\rho^2}\right), \tag{A4}$$

$$\vec{M}_1:\ \left(\frac{m_A^2}{m_\rho^2}\right)\frac{i}{k^2 - m_\pi^2}\,, \tag{A5}$$

and

$$\vec{A}_\mu \text{ to } \vec{M}_1 \text{ transition: } \frac{(m_A^2 - m_\rho^2)^{1/2}}{m_\rho^2}\,\frac{k_\mu}{k^2 - m_\pi^2}\,, \tag{A6}$$

where

$$m_A^2 = m_{A_1}^2 \equiv (\tfrac{1}{2}f\beta)^2 + (f\alpha)^2\,,$$
$$m_\rho^2 \equiv (\tfrac{1}{2}f\beta)^2\,, \tag{A7}$$
$$m_\pi^2 \equiv \lambda_9(\beta^2 + 4\alpha^2)/\alpha\,.$$

All other propagators have the standard form. It should be noted that the apparent mass term for $\vec{M}_1$ in Eq. (A2) and the actual pion mass in Eq. (A7) are different.

Equation (A5) indicates that a two-point $\vec{M}_1$ amputated amplitude, for example, is given by[18]

$$\lim_{k^2 \to m_\pi^2}\left(\frac{m_\rho}{m_A}\right)^2\left(\frac{k^2 - m_\pi^2}{i}\right)^2 G_2 = \frac{1}{(\sqrt{z}\,)^2}\,T_2\,, \tag{A8}$$

with

$$z^{1/2} \equiv m_A/m_\rho\,, \tag{A9}$$

where T_2 is the amputated T matrix in the *ordinary* sense and G_2 the Green's function. It is sufficient just to insert a wave-function normalization factor for external $\vec{M}_1$ particle legs.

Although the above simple gauge is sufficient in our applications, one may also use a slightly more complicated gauge such as the R_ξ gauge.[24]

APPENDIX B

We list all the allowed potentials for the model discussed in Sec. III. See Eq. (3.7).

The first group contains only the M and N fields

$$\begin{aligned}&\mathrm{Tr}(M^2),\quad \mathrm{Tr}(N^2),\\&\mathrm{Tr}(M^2)\mathrm{Tr}(N^2),\quad \mathrm{Tr}(M^2N^2),\\&[\mathrm{Tr}(M^2)]^2,\quad [\mathrm{Tr}(N^2)]^2,\\&\mathrm{Tr}(M^4),\quad \mathrm{Tr}(N^4).\end{aligned} \tag{B1}$$

The second group contains only the M' and N' fields

$$\begin{aligned}&\mathrm{Tr}(M'^2),\quad \mathrm{Tr}(N'^2),\\&\mathrm{Tr}(M'^2)\mathrm{Tr}(N'^2),\quad \mathrm{Tr}(M'^2N'^2),\\&[\mathrm{Tr}(M'^2)]^2,\quad [\mathrm{Tr}(N'^2)]^2,\\&\mathrm{Tr}(M'^4),\quad \mathrm{Tr}(N'^4).\end{aligned} \tag{B2}$$

The third group contains the mixing between the M and N fields and the M' and N' fields

$$\begin{aligned}&\mathrm{Tr}(M^2)\mathrm{Tr}(M'^2),\quad \mathrm{Tr}(M^2)\mathrm{Tr}(N'^2),\\&\mathrm{Tr}(N^2)\mathrm{Tr}(M'^2),\quad \mathrm{Tr}(N^2)\mathrm{Tr}(N'^2),\\&\mathrm{Tr}(N^2N'^2),\quad \mathrm{Tr}(N^2M'^2).\end{aligned} \tag{B3}$$

It should be noted that we can generate all the necessary vacuum values without the terms in Eq. (B3). All these potentials in Eqs. (B1)–(B3) are invariant under the reflection symmetry (3.12). The potential (3.11) is also invariant under this symmetry.

Finally we have the term

$$\mathrm{Tr}(MNM'N). \tag{B4}$$

This violates the symmetry (3.12). This term has an effect on the subsystem M, N, and W_μ of fields similar to the λ_9 term in Eq. (2.2).

APPENDIX C

We perform the lowest-order perturbation calculation of the pion mass using the model discussed in Secs. II and III. As we observed in the text, the symmetric potentials do not propagate the symmetry-breaking effects. This is due to the fact that the M field appears always in the combination $M^2 \propto \vec{M}_1{}^2 + M_2{}^2$. This property combined with the derivative mixing between $\vec{A}$ and $\vec{\pi}$ in Eq. (A7) suggests that we should use the Landau gauge[24] for the heavier vector mesons, ρ' and A_1'. We then eliminate all the derivative couplings and the $\vec{\pi}$-$\vec{A}_1$ mixing together with unphysical scalar[24] contributions from our *soft-pion* calculation.

The relevant interaction terms for ρ' and A_1' are given by [see the second term in Eq. (3.9), and also the last three terms in Eq. (2.1)],

$$\begin{aligned}\mathcal{L}_{\rm int} &= \tfrac{1}{2}[\tfrac{1}{2}(f\vec{V}_\mu - f'\vec{V}_\mu')(N_3'+\beta) + \tfrac{1}{2}(f\vec{A}_\mu - f'\vec{A}_\mu')N_4]^2 + \tfrac{1}{2}[\tfrac{1}{2}(f\vec{V}_\mu - f'\vec{V}_\mu')N_4 + \tfrac{1}{2}(f\vec{A}_\mu - f'\vec{A}_\mu')(N_3'+\beta)]^2\\ &= -\frac{f'}{f}m_\rho{}^2(\vec{V}_\mu'\cdot\vec{V}^\mu + \vec{A}_\mu'\cdot\vec{A}^\mu) + \left(\frac{f'^2\beta}{4}\right)N_3'[(\vec{V}_\mu')^2 + (\vec{A}_\mu')^2] + \cdots.\end{aligned} \tag{C1}$$

In the following calculations N_3' and M_2' stand for the *shifted parts* of N_3 and M_2, respectively [see Eq. (2.5)]. The last two terms in Eq. (C1) do not contribute to the pion mass. The N_3' tadpole contribution from these terms is absorbed by the renormalization counterterm in Eq. (A2), and it causes an infinite mass shift for the N_3' and N_4 fields. All other necessary couplings are found in Eqs. (2.1) and (2.5).

$$\mathcal{L}' = \tfrac{1}{2}f^2(\vec{V}_\mu\times\vec{M}_1)^2 - f^2\alpha(\vec{A}_\mu\times\vec{V}_\mu)\cdot\vec{M}_1 + \tfrac{1}{2}f^2(\vec{A}_\mu\cdot\vec{M}_1)^2 + f^2\alpha(\vec{A}_\mu)^2M_2'. \tag{C2}$$

The last term in (C2), combined with the tadpole counterterm in Eq. (A2), gives rise to an effective interaction

$$\left(-\frac{1}{2\alpha}\right)f^2\alpha(\vec{A}_\mu)^2(\vec{M}_1)^2 = -\tfrac{1}{2}f^2(\vec{A}_\mu)^2(\vec{M}_1)^2 \tag{C3}$$

in our calculation.

The ρ' contribution is given by [see also Eq. (A8)]

$$\delta_V m^2 = \left(\frac{m_{A_1}{}^2}{m_\rho{}^2}\right)\int \frac{d^4k}{(-i)\,(2\pi)^4}\left[(2if^2)\,(-i)^3\frac{g^{\mu\nu}}{(k^2-m_\rho{}^2)^2}\,\frac{(g_{\mu\nu}-k_\mu k_\nu/k^2)}{k^2-m_{\rho'}{}^2}\left(-i\frac{f'}{f}\,m_\rho{}^2\right)^2 \right.$$
$$\left. +\,2(-if^2\alpha)^2\,(-i)^4\frac{1}{k^2-m_{A_1}{}^2}\,\frac{g^{\mu\nu}}{(k^2-m_\rho{}^2)^2}\,\frac{(g_{\mu\nu}-k_\mu k_\nu/k^2)}{k^2-m_{\rho'}{}^2}\left(-i\frac{f'}{f}\,m_\rho{}^2\right)^2\right]$$
$$=6f'^2 m_\rho{}^2 m_{A_1}{}^2\int\frac{d^4k}{(-i)\,(2\pi)^4}\,\frac{1}{(k^2-m_\rho{}^2)\,(k^2-m_{A_1}{}^2)\,(k^2-m_{\rho'}{}^2)}\,. \qquad \text{(C4)}$$

The factor $m_{A_1}{}^2/m_\rho{}^2$ comes from Eqs. (A7) and (A9); Eq. (A7) gives $(m_{A_1}/m_\rho)^4$ and Eq. (A9) gives $(m_\rho/m_{A_1})^2$. ($\lambda_9=0$ in the present case.) Equation (C4) agrees with Eq. (3.2) if one uses the KSRF relation.

The A_1' contribution is given by

$$\delta_A m^2 = \left(\frac{m_{A_1}{}^2}{m_\rho{}^2}\right)\int \frac{d^4k}{(-i)\,(2\pi)^4}\left\{-(2if^2)\,(-i)^3\frac{g^{\mu\nu}}{(k^2-m_{A_1}{}^2)^2}\,\frac{g_{\mu\nu}-k_\mu k_\nu/k^2}{k^2-m_{A_1'}{}^2}\left(-i\frac{f'}{f}\,m_\rho{}^2\right)^2 \right.$$
$$\left. +\,2(-if^2\alpha)^2\,(-i)^4\frac{1}{k^2-m_\rho{}^2}\,\frac{g^{\mu\nu}}{(k^2-m_{A_1}{}^2)^2}\,\frac{g_{\mu\nu}-k_\mu k_\nu/k^2}{k^2-m_{A_1'}{}^2}\left(-i\frac{f'}{f}\,m_\rho{}^2\right)^2\right\}$$
$$=-6f'^2 m_\rho{}^2 m_{A_1}{}^2\int\frac{d^4k}{(-i)\,(2\pi)^4}\,\frac{1}{(k^2-m_\rho{}^2)\,(k^2-m_{A_1}{}^2)\,(k^2-m_{A_1'}{}^2)}\,. \qquad \text{(C5)}$$

Finally the explicit evaluation of Eq. (C4) gives

$$\delta_V m^2 = \frac{3}{2\pi}\left(\frac{f'^2}{4\pi}\right)\frac{m_\rho{}^2 m_{A_1}{}^4}{(m_{A_1}{}^2-m_\rho{}^2)\,(m_{A_1}{}^2-m_{\rho'}{}^2)}\left[\ln\left(\frac{m_{A_1}{}^2}{m_\rho{}^2}\right)-\left(\frac{m_{A_1}{}^2-m_\rho{}^2}{m_{A_1}{}^2}\right)\frac{m_{\rho'}{}^2}{m_{\rho'}{}^2-m_\rho{}^2}\ln\left(\frac{m_{\rho'}{}^2}{m_\rho{}^2}\right)\right]. \qquad \text{(C6)}$$

We note that all the masses in these equations are the "zeroth-order" masses. See Sec. III C.

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‡Present address: The Department of Physics, University of Toronto, Toronto, Ontario M5S 1A7, Canada.

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Galilean subdynamics and the dual resonance model*

L. C. Biedenharn
Physics Department, Duke University, Durham, North Carolina 27706

H. van Dam
Physics Department, University of North Carolina, Chapel Hill, North Carolina 27514
(Received 13 August 1973)

Dynamics in Minkowski space is discussed in terms of an eight-parameter extended Galilei group, a subgroup of the Poincaré group. This method—Galilean subdynamics—is developed and discussed in detail and applied to the construction of two explicit models having four-point amplitudes of the Veneziano type. There are no difficulties with unphysical states.

I. INTRODUCTION

The present paper is an application of the "method of Galilean subdynamics," to the problem of constructing a consistent basis for the dual resonance model.

Galilean subdynamics, developed in connection with our interpretation of Dirac's positive-energy wave equation,[1,2] is another attempt to describe relativistic dynamics. Dynamics in Minkowski space has been conventionally described as the change with time of a configuration given at one instant of time in a particular reference frame in Minkowski space. There have always been difficulties with this approach for both classical theory and quantum theory. One of the difficulties is the proper description of a composite particle. As a suitable reference frame, one might choose, for example, a rest frame of the particle. If the constituents of the particle move slowly then one can use nonrelativistic mechanics for the description—at least approximately. In the general case, however, this clearly does not work, and it seems unlikely that proceeding in this way one can ever find a relativistic description of a hadron.

Much more successful have been attempts at an over-all Minkowski space-time viewpoint. At the classical level such theories do allow one to describe, and to develop models for, composite systems.[3] At the quantum level there are still difficulties despite the remarkable results of, say, the dual resonance model within the S-matrix framework.

There exists a potentially important alternative to this over-all space-time viewpoint. For classical physics this alternative was introduced by Dirac[4] and designated the *front form* of dynamics. Dirac proposed to consider a family of parallel tangent spaces to the light cone instead of the usual family of parallel spaces at various instants of time (called by Dirac the *instant form* of dynamics). Superficially, for classical theory, it is not clear that the front form of dynamics is any better than the instant form of dynamics. However, the quantum version of the front form shows an important distinction from the classical case. This follows from the fact that of the three coordinates in the front—x_+, x_1, x_2, (with x_- being the coordinate specifying the front)—the coordinate x_+, unlike x_1 and x_2, has the nature of a time, that is, the momentum conjugate to x_+ (denoted by P_-) has a spectrum confined to the open positive half-line. It is accordingly not permitted in the quantum version to assume a kinematics based upon specifying a point within the front. A way out of this fundamental difficulty is afforded by the fact that there exists an eight-parameter subgroup of the Poincaré group which adjoins the operator P_- to the seven generators that leave the front invariant. This subgroup has the group structure of nonrelativistic (Galilean) dynamics in two space dimensions, together with a scaling operator. The momentum