[10]For reasons outlined in the next paragraph, we insist that the line corresponding to the external fermion be on the boundary of the graph.

[11]One of us (R.R.) wishes to acknowledge a conversation with J. Rosner and an enlightening correspondence with N. Nakanishi about this question.

[12]See, e.g., J. W. Essam and Michael E. Fisher, Rev. Mod. Phys. 42, 271 (1970).

[13]There is undoubtedly a reason for this, but we have not discovered it. It is not true for graphs whose td have loops.

[14]M. Levine, U.S.A.E.C. Report No. CAR-882-25, 1971 (unpublished), and in proceedings of the Third Colloquium on Advanced Computing Methods in Theoretical Physics, Marseilles, June, 1973 (to be published).

[15]A. C. Hearn, Stanford Artificial Intelligence Project Memo No. AIM-133, 1970 (unpublished). See also A. C. Hearn, in *Interactive Systems for Experimental Applied Mathematics*, edited by M. Klerer and J. Reinfields (Academic, N.Y., 1968), pp. 79–90.

[16]M. Veltman, CERN report, 1967 (unpublished).

PHYSICAL REVIEW D VOLUME 9, NUMBER 2 15 JANUARY 1974

Finiteness of radiative corrections to semileptonic decays in unified gauge theories

Rabindra N. Mohapatra* and Susumu Sakakibara
Center for Theoretical Physics, Department of Physics and Astronomy, University of Maryland, College Park, Maryland 20742
(Received 27 August 1973)

Using current algebra and Bjorken-Johnson-Low techniques we exhibit the finiteness of second-order radiative corrections to strangeness-changing and strangeness-preserving semileptonic weak processes in the context of an $SU(2)_L \times U(1)$ gauge model (with the hadronic extension as suggested by Glashow, Iliopoulos, and Maiani) with strong interactions accounted for to all orders.

It is important[1] to estimate the higher-order weak and electromagnetic radiative corrections to weak semileptonic decays to check the Cabibbo form of universality. In the current-current[2] [or universal Fermi interaction (UFI)] or old intermediate-vector-boson[1] (IVB) models of weak-interaction calculations, the second-order electromagnetic corrections lead in general to divergent results. With the recent progress in spontaneously broken gauge theories,[3] there appears to have emerged a concrete possibility of constructing unified theories of weak and electromagnetic interactions, wherein all higher-order corrections are finite. In fact, it has been shown that, in such models, unitary-gauge calculations of second-order (g^2) radiative corrections to purely leptonic processes like $\mu^- \to e^- + \bar{\nu} + \nu'$, $\nu + \bar{\nu} \to \nu + \bar{\nu}$ yield finite answers.[4] However, the corresponding calculations for semileptonic decays of hadrons are complicated due to the presence of strong interactions, which must, of course, be treated nonperturbatively. The aim of the present paper is to summarize the results of an investigation into this problem (for both strangeness-conserving and strangeness-changing semileptonic processes) in the context of $SU(2)_L \times U(1)$ gauge theories[5] with hadronic part given by Glashow, Iliopoulos, and Maiani[6] (GIM). The techniques of the Bjorken-Johnson-Low expansion and current algebra have been employed to isolate the divergent parts in loops involving the strong vertices in a manner which treats strong interactions nonperturbatively. It is found that by working in the unitary gauge, after mass, wave-function, and vertex renormalizations have been performed, the remaining contributions have both quadratic as well as logarithmic divergences. The residual divergences cancel among themselves, yielding a *finite answer for the second-order radiative corrections to $\Delta S = 1$ and $\Delta S = 0$ semileptonic processes in this class of theories*. It is worth noting that the process of isolating quadratic divergences depends only on a knowledge of the equal-time commutation relations between weak currents and is therefore independent of the details of strong interactions. However, in order to isolate the logarithmic divergences, we assume that (i) the generalized Bjorken-Johnson-Low expansion[7] for the Fourier transform of the T product of currents is valid; and (ii) the strong interactions are mediated by a neutral gluon or by an SU(3)″ octet of vector mesons as in a class of recently proposed renormalizable, non-Abelian gauge models.[8,9]

The weak-interaction Lagrangian for the class of $SU(2)_L \times U(1)$ theories described above is (in the unitary gauge)

$$\mathcal{L}_{\text{int}} = \mathcal{L}_{\text{int}}^{\text{leptons}} + \mathcal{L}_{\text{int}}^{\text{hadrons}} + \mathcal{L}_{\text{int}}(W_\mu^\pm, \sigma, A_\mu, Z_\mu)\,, \tag{1}$$

$$\begin{aligned}\mathcal{L}_{\text{int}}^{\text{lepton}} = & -\frac{g}{2\sqrt{2}}\,\bar{l}\gamma^\mu(1-\gamma_5)\nu W_\mu + \text{H.c.} + e\bar{l}\gamma^\mu l A_\mu + g\sec\phi\,\bar{l}\left[\tfrac{1}{2}\cos 2\phi\,\tfrac{1}{2}(1+\gamma_5) - \sin^2\phi\,\tfrac{1}{2}(1-\gamma_5)\right]\gamma^\mu l Z_\mu \\ & -\tfrac{1}{4}g\sec\phi\,\bar{\nu}\gamma^\mu(1-\gamma_5)\nu Z_\mu - \left(\frac{g m_l}{2m_W}\right)\bar{l}l\sigma\,,\end{aligned} \tag{2}$$

where $\phi = \tan^{-1}(g'/g)$ is the Weinberg-Salam angle, $e = g\sin\phi$, l stands for negatively charged lepton, and

$$\mathcal{L}_{\text{int}}^{\text{hadron}} = -\frac{g}{2\sqrt{2}}(J_W^{\mu\dagger} W_\mu + J_W^\mu W_\mu^+) - ej_{\text{em}}^\mu A_\mu - \tfrac{1}{4}g\sec\phi\, J_Z^\mu Z_\mu - \tfrac{1}{2}gS\sigma\,, \tag{3}$$

where

$$J_W^\mu = \bar{\psi}C\gamma^\mu(1-\gamma_5)\psi\,, \quad j_{\text{em}}^\mu = \bar{\psi}Q\gamma^\mu\psi\,,$$

$$\begin{aligned}J_Z^\mu &= \bar{\psi}C_0\gamma^\mu(1-\gamma_5)\psi - 4\sin^2\phi\, j_{\text{em}}^\mu \\ &= I^\mu - 4\sin^2\phi\, j_{\text{em}}^\mu\,,\end{aligned}$$

$$S = \frac{1}{m_W}\bar{\psi}M\psi\,,$$

where

$$C = \begin{pmatrix} 0 & 0 & -\sin\theta & \cos\theta \\ 0 & 0 & \cos\theta & \sin\theta \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad Q = \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 0 & \\ & & & 0 \end{pmatrix}, \quad M = \begin{pmatrix} m_{\mathcal{P}'} & & & \\ & m_{\mathcal{P}} & & \\ & & m_{\mathfrak{N}} & \\ & & & m_\lambda \end{pmatrix}, \quad \psi = \begin{pmatrix} \mathcal{P}' \\ \mathcal{P} \\ \mathfrak{N} \\ \lambda \end{pmatrix},$$

and

$$C_0 = [C, C^\dagger] = \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix}. \tag{4}$$

We also note that $m_W = m_Z\cos\phi$, and

$$\begin{aligned}\mathcal{L}_{\text{int}}(W_\mu^\pm, \sigma, A_\mu, Z_\mu) = & \, ig(\cos\phi\, Z^\nu + \sin\phi A^\nu)[W^\mu(\partial_\mu W_\nu^+ - \partial_\nu W_\mu^+) - W^{\mu+}(\partial_\mu W_\nu - \partial_\nu W_\mu) + \partial^\mu(W_\mu W_\nu^+ - W_\nu W_\mu^+)] \\ & + g m_W W_\mu^+ W^{-\mu}\sigma\,.\end{aligned} \tag{5}$$

It has been pointed out earlier[10] that, in the class of gauge models we are interested in, there is an allowable counterterm ($\mathcal{L}_c$) which has to be added to the above Lagrangian in order to get finite results in order g^2 as well as to maintain the parity and strangeness selection rules observed in nature. $\mathcal{L}_c$ will play a crucial role in canceling some divergences arising out of corrections to the hadronic vertex.

The diagrams arising in order g^4 are shown in Figs. 1 to 6. The diagrams, that provide pure renormalization of the lepton lines or the W-boson lines have been omitted. Let us first consider the divergent contributions[4] of Fig. 1. One can write the modification to the propagator as follows:

$$i\pi^{\mu\nu}(k) = (k^2 g^{\mu\nu} - k^\mu k^\nu)(ak^4 + bk^2 + c) + g^{\mu\nu}(dk^2 + e)\,, \tag{6}$$

where a, b, c, d, and e are divergent constants.

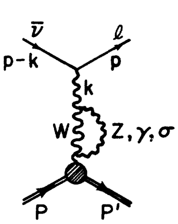

FIG. 1. Modification of the W-boson propagator (shaded blob represents the hadronic vertex).

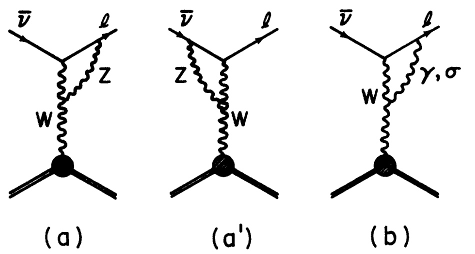

FIG. 2. Corrections to the lepton vertex.

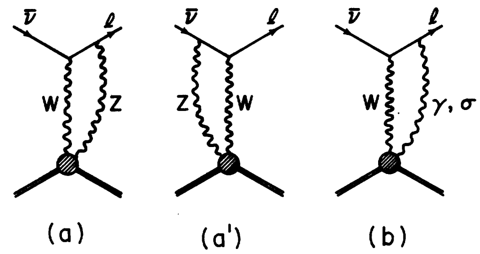

FIG. 3. Box graphs due to two-boson exchange.

Introducing the W-boson mass renormalization counterterm

$$i\delta m_W^2 = a m_W^6 + b m_W^4 + (c+d) m_W^2 + e, \tag{7}$$

and wave-function renormalization

$$Z_3 = 1 - i(3a m_W^4 + 2b m_W^2 + c + d), \tag{8}$$

one is left with the following residual divergent contribution to the amplitude:

$$M^{(1)} = +\frac{g^2}{8} M^{\alpha\beta}\left[g_{\alpha\beta}[a(k^2 + 2m_W^2) + b] - \frac{k_\alpha k_\beta}{m_W^2}\left(a m_W^2 - \frac{d}{m_W^2}\right)\right], \tag{9}$$

where

$$M^{\alpha\beta} = \bar{u}\gamma^\alpha(1-\gamma_5)v\langle P'|J_W^\beta|P\rangle. \tag{10}$$

The contribution of the Z, γ, and σ exchanges shown in Figs. 1(a), 1(b), and 1(d) are the following:

$$a^Z = -\frac{g^2\cos^2\phi}{12 m_Z^2 m_W^2}\ln\Lambda^2,$$

$$a^\gamma = 0 = a^\sigma,$$

$$b^Z = \frac{g^2\cos^2\phi}{4m_Z^2 m_W^2}[\Lambda^2 - \tfrac{7}{3}(m_W^2 + m_Z^2)\ln\Lambda^2],$$

$$b^\gamma = -\frac{g^2\sin^2\phi}{m_W^2}\,\tfrac{5}{6}\ln\Lambda^2,$$

$$b^\sigma = 0,$$

$$d^Z = \frac{g^2\cos^2\phi}{4m_Z^2 m_W^2}(m_Z^2 - m_W^2)^2\ln\Lambda^2,$$

$$d^\gamma = -\frac{1}{2}\,\frac{g^2\sin^2\phi}{m_W^2}\, m_W^2\ln\Lambda^2,$$

$$d^\sigma = \tfrac{1}{4} g^2\ln\Lambda^2,$$

where we will always denote

$$\int\frac{d^4k}{k^2(2\pi)^4} = \Lambda^2$$

and (11)

$$\int\frac{d^4k}{k^4(2\pi)^4} = \ln\Lambda^2.$$

Next, we concentrate on the modification to the lepton vertex:

$$-i\Gamma^\mu = \frac{g}{2\sqrt{2}}\bar{u}\gamma_\alpha(1-\gamma_5)v[(k^2 g^{\alpha\mu} - k^\alpha k^\mu)(fk^2 + t) + g^{\alpha\mu}(rk^2 + s)]. \tag{12}$$

Introducing the lepton vertex renormalization constant (arising from this diagram)

$$\frac{1}{Z_1^l} - 1 = f m_W^4 + (t+r) m_W^2 + s, \tag{13}$$

the residual divergent contribution to the amplitude coming from Eq. (12) can be written as

$$M^{(2)} = -\tfrac{1}{8} g^2 M^{\alpha\beta}\left\{[t + r + f(k^2 + m_W^2)] g_{\alpha\beta} - \frac{k_\alpha k_\beta}{m_W^2}(r + f m_W^2)\right\}, \tag{14}$$

$$f^Z = -\frac{g^2\cos^2\phi}{12 m_Z^2 m_W^2}\ln\Lambda^2, \quad f^\gamma = f^\sigma = 0,$$

$$t^Z = \frac{g^2\cos^2\phi}{4m_Z^2 m_W^2}\{\Lambda^2 - \tfrac{8}{3}(m_Z^2 + m_W^2)\ln\Lambda^2 + [\tan^2\phi(m_Z^2 - m_W^2) + \tfrac{1}{2}\sec^2\phi\, m_l^2]\ln\Lambda^2\},$$

$$t^\gamma = -\tfrac{7}{6}\frac{g^2\sin^2\phi}{m_W^2}\ln\Lambda^2,$$

$$t^\sigma = +\frac{g^2}{4m_W^2}\ln\Lambda^2, \tag{15}$$

$$r^Z = -(m_Z^2 - m_W^2)\frac{g^2\sin^2\phi}{4m_Z^2 m_W^2}\ln\Lambda^2,$$

$$r^\sigma = -\frac{g^2}{4m_W^2}\ln\Lambda^2,$$

$$r^\gamma = \frac{g^2\sin^2\phi}{4m_W^2}\ln\Lambda^2.$$

We will next consider the box graphs depicted in Fig. 3. Here the loop involves the strong vertex.

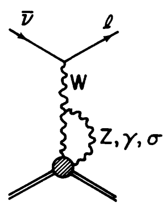

FIG. 4. Modifications of the strong vertex.

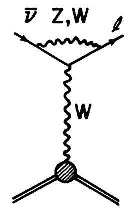

FIG. 5. This graph provides pure renormalization of the lepton vertex.

To isolate its divergent parts, we will use Ward-Takahashi identities (WTI), current algebra, and symmetry properties of the strong-interaction Hamiltonian depicted earlier and the Bjorken technique. We will illustrate this procedure for the Z contribution and only quote the results in the other cases.

$$M^{(3a)}=\frac{-g^4 i}{32\cos^2\phi}\int\frac{d^4q}{(2\pi)^4}\,\frac{g_{\lambda\rho}-(k-q)_\lambda(k-q)_\rho/m_W^{\,2}}{(k-q)^2-m_W^{\,2}}\,\frac{g_{\mu\nu}-q_\mu q_\nu/m_Z^{\,2}}{q^2-m_Z^{\,2}}$$

$$\times\frac{\bar{u}(p)\gamma^\mu[\frac{1}{2}\cos 2\phi(\not{p}-\not{q})-m_l\sin^2\phi]\gamma^\rho(1-\gamma_5)v(k-p)}{(p-q)^2-m_l^{\,2}}M^{\nu\lambda}(q), \qquad (16)$$

where

$$M^{\nu\lambda}(q)=\int d^4x\, e^{iq\cdot x}\langle P'|T(J_Z^\nu(x)J_W^\lambda(0))|P\rangle. \qquad (17)$$

Let us look at most divergent terms in the integral, that arise out of the "longitudinal" parts of two propagators. Using the current commutation relations,

$$[J_Z^0(x), J_W^\nu(0)]\delta(x_0)=4\cos^2\phi\,\delta^4(x)J_W^\nu, \qquad (18)$$

and WTI, we can write

$$q_\nu(k-q)_\lambda M^{\nu\lambda}(k,q)$$
$$=+i(k-q)_\lambda 4\cos^2\phi\langle P'|J_W^\lambda|P\rangle$$
$$+\int d^4x\, e^{iq\cdot x}\langle P'|\{\delta(x_0)[\partial_\nu J_Z^\nu(x), J_W^0]$$
$$-T(\partial_\nu J_Z^\nu(x)\partial_\lambda J_W^\lambda(0))\}|P\rangle. \qquad (19)$$

Only the last term in Eq. (19) depends on the loop momentum q and its asymptotic behavior can be determined via the Bjorken technique,[7] after which we have isolated all the divergent parts in the expression concerned and the coefficients of the divergences are expressed purely in terms of equal-time commutators of currents J^μ and $\partial_\lambda J^\lambda$'s. In order to evaluate $\partial_\lambda J^\lambda$, we note that

$$\partial_\lambda J^\lambda(y)=-i\left[\int d^3x\, J^0(x), H_{\text{strong}}+H_{\text{mass}}\right]. \qquad (20)$$

Since H_{strong} is $U(4)_L\times U(4)_R$ symmetric in the models of interest J^0 commutes with it, and the known structure of H_{mass} (the quark mass term) then gives

$$\partial_\lambda J_Z^\lambda=-2i\bar{\psi}MC_0\gamma_5\psi$$

and $\qquad (21)$

$$\partial_\lambda J_W^\lambda=i\bar{\psi}[MC(1-\gamma_5)-CM(1+\gamma_5)]\psi.$$

From Eq. (21), using canonical anticommutation relation of ψ fields, we find $[\partial_\mu J_Z^\mu(x), \partial_\lambda J_W^\lambda(0)]_{\text{ET}}=U^0$, where U^0 is the time component of U^μ defined below:

$$U^\mu=2\bar{\psi}[C_0M^2, C]\gamma^\mu(1-\gamma_5)\psi+4\bar{\psi}MCM\gamma^\mu(1+\gamma_5)\psi. \qquad (22)$$

Using these techniques, we find that the divergent contributions of Fig. 3 to the amplitude are the

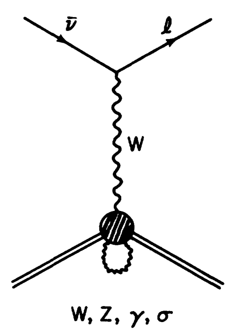

FIG. 6. This graph provides pure renormalization of the strong vertex.

following:

$$\begin{aligned} M_Z^{(3)} &= M^{(3a)} + M^{(3a')} \\ &= \frac{g^4 \cos^2\phi M^{\alpha\beta}}{32 m_Z^2 m_W^2}\{g_{\alpha\beta}[\Lambda^2 - (\tfrac{1}{3}k^2 + 3m_Z^2 + 3m_W^2)\ln\Lambda^2] + \tfrac{4}{3}k_\alpha k_\beta \ln\Lambda^2\} \\ &+ \frac{g^4 M^{\alpha\beta}}{32 m_Z^2 m_W^2}(\tfrac{1}{2}m_l^2 g_{\alpha\beta} - k_\alpha k_\beta)\ln\Lambda^2 + \frac{ig^4\tan^2\phi}{64 m_Z^2 m_W^2}\bar{u}\not{k}(1-\gamma_5)v\int d^3x \langle P'|[\partial_\nu J_Z^\nu(\vec{x},0), J_W^0(0)]|P\rangle \ln\Lambda^2 \\ &+ \frac{g^4}{128 m_Z^2 m_W^2}\bar{u}\gamma_\alpha(1-\gamma_5)v\langle P'|U^\alpha|P\rangle\ln\Lambda^2; \end{aligned} \tag{23}$$

the photon contribution,

$$M_\gamma^{(3)} = -\frac{g^4\sin^2\phi}{8m_W^2}g_{\alpha\beta}M^{\alpha\beta}\ln\Lambda^2;$$

the σ contribution,

$$M_\sigma^{(3)} = -\frac{g^4 m_e}{32 m_W^3}\bar{u}(1-\gamma_5)v\int d^3x\langle P'|[S(x,0), J_W^0(0)]|P\rangle\ln\Lambda^2. \tag{24}$$

Note that none of the above divergences can be absorbed as renormalization counterterms. The diagrams of Fig. 4 can also be treated in a similar way. As in the case of diagram 2, we can write the modifications of the strong vertex as

$$-i\langle P'|\Lambda^\mu|P\rangle = \frac{g}{2\sqrt{2}}\langle P'|J_{W\alpha}|P\rangle[(k^2g^{\alpha\mu} - k^\alpha k^\mu)(f'k^2 + t') + g^{\alpha\mu}(r'k^2 + s')] - \frac{1}{2\sqrt{2}}R^\mu, \tag{25}$$

where R^μ stands for the remainder. After renormalization, the residual divergent contribution to the amplitude can be written as

$$M^{(4)} = -\frac{g^2}{8}M^{\alpha\beta}\left\{[t' + r' + f'(k^2 + m_W^2)]g_{\alpha\beta} - \frac{k_\alpha k_\beta}{m_W^2}(r' + f'm_W^2)\right\} + \frac{g^2}{8}R'_\mu \frac{g^{\mu\nu} - k^\mu k^\nu/m_W^2}{k^2 - m_W^2}\bar{u}\gamma_\nu(1-\gamma_5)v, \tag{26}$$

$$\begin{aligned} &f'^Z = -\frac{g^2\cos^2\phi}{12 m_Z^2 m_W^2}\ln\Lambda^2, \quad f'^\gamma = f'^\sigma = 0, \\ &t'^Z = \frac{g^2\cos^2\phi}{4m_Z^2 m_W^2}[\Lambda^2 - \tfrac{5}{3}(m_Z^2 + m_W^2)\ln\Lambda^2], \\ &t'^\gamma = -\frac{2}{3}\frac{g^2\sin^2\phi}{m_W^2}\ln\Lambda^2; \quad t'^\sigma = 0, \\ &r'^Z = -\frac{g^2\cos^2\phi}{4}\left(\frac{1}{m_Z^2} + \frac{1}{m_W^2}\right)\ln\Lambda^2, \quad r'^\gamma = -\frac{g^2\sin^2\phi}{4m_W^2}\ln\Lambda^2, \quad r'^\sigma = 0. \end{aligned} \tag{27}$$

We will split the various contributions to R_μ into two parts: $R_\mu = R'_\mu + R''_\mu$, where $(R''^Z_\mu + R''^\sigma_\mu + R''^\gamma_\mu)$ can be absorbed into vertex renormalization and R_μ is the residual divergence appearing in Eq. (26).

$$\begin{aligned} R'^Z_\mu &= -\frac{g^2}{16m_Z^2 m_W^2}(k^2 g_{\mu\alpha} - k_\mu k_\alpha - m_W^2 g_{\mu\alpha})\langle P'|U^\alpha|P\rangle\ln\Lambda^2 \\ &+ k_\mu\left[-\frac{ig^2}{8m_W^2}\int d^3x\langle P'|[J_Z^0(\vec{x},0), \partial_\mu J_W^\mu(0)]|P\rangle + \frac{ig^2}{8m_Z^2}\int d^3x\langle P'|[J_W^0(\vec{x},0), \partial_\mu J_Z^\mu(0)]|P\rangle\right]\ln\Lambda^2, \end{aligned} \tag{28}$$

and

$$\begin{aligned} R''^Z_\mu &= \frac{g^2}{4m_W^2}\ln\Lambda^2\int d^3x\left\langle P'\left|\left[J_{W\mu}(x), \frac{\partial}{\partial t}\partial_\lambda J_Z^\lambda(0)\right]_{\mathrm{ET}}\right|P\right\rangle + \frac{g^2}{4m_Z^2}\ln\Lambda^2\int d^3x\left\langle P'\left|\left[J_{W\mu}(x), \frac{\partial}{\partial t}\partial_\lambda J_Z^\lambda(0)\right]_{\mathrm{ET}}\right|P\right\rangle \\ &+ \frac{g^2}{16m_W^2}\ln\Lambda^2\langle P'|U_\mu|P\rangle, \end{aligned} \tag{29}$$

$$R'^{\gamma}_{\mu}=-\frac{ig^2\sin^2\phi}{2m_W^2}k_\mu\int d^3x\langle P'|[j^0_{em}(\vec{x},0),\partial_\mu J^\mu_W(0)]|P\rangle\ln\Lambda^2, \tag{30}$$

$$R''^{\gamma}_{\mu}=-\frac{g^2\sin^2\phi}{m_W^2}\int d^3x\left\langle P'\left|\left[j_{em,\mu}(\vec{x},0),\frac{\partial}{\partial t}\partial_\mu J^\mu_W(0)\right]\right|P\right\rangle\ln\Lambda^2, \tag{31}$$

$$R'^{\sigma}_{\mu}=-\frac{g^2}{4m_W}k_\mu\int d^3x\langle P'|[S(\vec{x},0),J^0_W(0)]|P\rangle\ln\Lambda^2, \tag{32}$$

$$R''^{\sigma}_{\mu}=-\frac{ig}{8m_W}\ln\Lambda^2\int d^3x\langle P'|L_\mu|P\rangle, \tag{33}$$

where

$$L_0=[S(\vec{x},0),\partial_\mu J^\mu_W(0)]=-\frac{2i}{m_W}\bar{\psi}MCM\gamma_0(1+\gamma_5)\psi. \tag{34}$$

We first observe from Eq. (22) that on adding the last term of Eq. (29) and R''^{σ}_{μ} the wrong-chirality [i.e., $\frac{1}{2}(1+\gamma_5)$] currents cancel and the remainder can be absorbed into vertex renormalization. Similarly, the sum of the first and second term of R''^{Z}_{μ}, and R''^{γ}_{μ} also have the right chiral structure and can be absorbed as renormalization constant,[11] as remarked earlier.

At this point, we observe that in the sum of the residual divergences coming from various graphs the Z, γ, and σ exchange contributions *separately* add up to zero, i.e.,

$$\begin{aligned}&M^{(1)}_Z+M^{(2)}_Z+M^{(3)}_Z+M^{(4)}_Z=0,\\&M^{(1)}_\gamma+M^{(2)}_\gamma+M^{(3)}_\gamma+M^{(4)}_\gamma=0,\\&M^{(1)}_\sigma+M^{(2)}_\sigma+M^{(3)}_\sigma+M^{(4)}_\sigma=0.\end{aligned} \tag{35}$$

Now, we only have to look at the diagrams in Fig. 5 and Fig. 6. The divergent contribution of the diagram in Fig. 5 can be completely absorbed by vertex renormalization. A similar remark holds also for the divergent contribution of Fig. 6. But we will illustrate the latter a little more. Let us look at the W contribution (Fig. 6)

$$-i\langle P'|\Lambda^\mu|P\rangle_W=\left(\frac{g}{2\sqrt{2}}\right)^3\int\frac{d^4q}{(2\pi)^4}\frac{g^{\alpha\beta}-q^\alpha q^\beta/m_W^2}{q^2-m_W^2}\times M^\mu_{\alpha\beta}(q,k), \tag{36}$$

where

$$M^\mu_{\alpha\beta}(q,k)=\int d^4x\,d^4y\,e^{iq(x-y)}\times\langle P'|T(J^\dagger_{W\alpha}(x)J_{W\beta}(y)J^\mu_W(0))|P\rangle. \tag{37}$$

Using the Bjorken technique[7] and Ward-Takahashi identity, we can write the divergent part of this as

$$\begin{aligned}\frac{g^3}{16\sqrt{2}}\Big\langle P'\Big|\Big\{&-\ln\Lambda^2\int d^4y\,d^3x\,T\left(\left[\frac{\partial}{\partial t}J^\dagger_{W\alpha}(\vec{x},y_0),J^\alpha_W(y)\right]J^\mu_W(0)\right)\\&+\frac{\Lambda^2}{m_W^2}\int d^3x\,d^4y\,T([J^0_W(\vec{x},y_0),\partial_\alpha J^{\alpha\dagger}_W(y)]J^\mu_W(0))-\frac{\ln\Lambda^2}{m_W^2}\int d^3y\,d^3x\,[J^\alpha_W(x),[J^\dagger_{W\alpha}(y),J^\mu_W(0)]_{ET}]_{ET}\\&-\frac{\ln\Lambda^2}{m_W^2}\int d^4y\,d^3x\,T([\partial_\lambda J^\lambda_W(x,y_0),\partial_\alpha J^{\alpha\dagger}_W(y)]J^\mu_W(0))-\frac{\ln\Lambda^2}{m_W^2}\int d^3x\,[\partial_0\partial_\alpha J^\alpha_W(x),I^\mu(0)]_{ET}\\&+\frac{\ln\Lambda^2}{m_W^2}\int d^3x\,d^3y\,[[J^\mu_W(0),\partial_\alpha J^{\alpha\dagger}_W(x)]_{ET},\partial_\lambda J^\lambda_W(y)]_{ET}\Big\}\Big|P\Big\rangle.\end{aligned} \tag{38}$$

The important point to note here is that the first, second, and fourth terms cancel against the contribution of counterterms in $\mathcal{L}_c$.[10] The third, fifth, and sixth terms can be absorbed into vertex renormalization. Similar things happen for the photon contribution and the *sum* of the Z contribution and σ contribution. Within the above set of assumptions, we have shown that second-order, radiative corrections to both $\Delta S=0$ and $\Delta S=1$ semileptonic decays are finite in $SU(2)_L\times U(1)$ gauge theories with GIM[6] mechanism. Details of these considerations will be published in a longer article now under preparation.

*Work supported by National Science Foundation Grant No. NSF GP 20709.

[1]See R. E. Marshak, Riazuddin, and C. P. Ryan, *Theory of Weak Interactions in Particle Physics* (Wiley, New York, 1969), for a general review. Also, A. Sirlin, in *Proceedings of the Topical Conference on Weak Interactions, CERN, 1969* (CERN Geneva, Switzerland), p. 409.

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[9]Note that, in this class of models, the Hamiltonian to zeroth order in (g, e) can be written as $H = H_{\text{strong}} + H_{\text{mass}}$, where H_{strong} is $U(4)_L \times U(4)_R$-invariant and H_{mass}, denoting the quark mass term, breaks this symmetry in $(4^*, 4) + (4, 4^*)$ manner.

[10]R. N. Mohapatra and P. Vinciarelli, Phys. Rev. D 8, 481 (1973); R. N. Mohapatra, J. C. Pati, and P. Vinciarelli, *ibid.* 8, 3652 (1973). Explicitly,

$$\mathcal{L}_c = g^2 \int \frac{d^4k}{k^4(2\pi)^4} \int d^3x \left\{ \frac{1}{32\cos^2\phi} [J^{\mu}_{Z}(x), J_{\mu Z}(0)]_{\text{ET}} + \tfrac{1}{8} [J^{\mu}_{W}(x), J_{\mu W}(0)]_{\text{ET}} - \frac{1}{32 m_Z^2} [\partial_\mu \dot{J}^{\mu}_{Z}(0),\ \partial_\alpha J^{\alpha}_{Z}(0)]_{\text{ET}} - \frac{1}{8 m_W^2} [\partial_\mu \dot{J}^{\mu}_{W}(x),\ \partial_\alpha J^{\alpha}_{W}(0)]_{\text{ET}} + \tfrac{1}{8} [\dot{S}(x), S(0)] \right\}.$$

[11]In order to evaluate commutators like $[J^{\mu}_{W}(\vec{x}, 0),\ (\partial/\partial t) \times \partial_\lambda J^{\lambda}_{Z}(0)]$ etc. we need the explicit structure for the Hamiltonian. Only, here we will use the gluon model or the model of Ref. 8. On evaluating the above commutator and on covariantizing it, we get a structure like $\bar{\psi}\gamma^{\mu}(1-\gamma_5)K\gamma_\lambda \partial^\lambda \psi$ which upon using the Dirac equation reduces to a form which can be absorbed into vertex renormalization.