Scaling in unified gauge theories*

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(Received 11 April 1973)

The new unified gauge theories of strong, weak, and electromagnetic interactions scale in low-order perturbation theory with a nonvanishing W_1 and up to a log in νW_2. This is in contrast to "old" Yang-Mills models, and resurrects the possibility of data interpretation in terms of bosonic (hadronic) constituents.

I. INTRODUCTION

The recent gauge theory of hadrons[1] is successful in describing low-energy hadron physics. This theory is essentially a renormalizable realization of the Yang-Mills ideas[2] for massive vector mesons. Its most striking progress has come with the very natural extension to a unified gauge theory of strong, weak, and electromagnetic interactions.[3] In addition however, mechanisms have been proposed within this framework for calculation of hadron symmetry breaking[4,5] and the pion mass.[6,7] The elegance of the hadron theory is further enhanced by its known connection with dual models,[8] and the presumably related fact that (at least) the vector mesons of the theory lie on Regge trajectories.[9] Thus, it is surprising but conceivable that the theory is viable beyond the low-energy regime. It is our purpose in this note to report a different advance toward high energies: In terms of electroproduction form factors, the *unified theories scale* (up to ln$s$), in contrast to "old" Yang-Mills models. Our scaling is then no worse than the gluon model in perturbation theory.[10]

The scaling mechanism, a matter of correct current dimension and renormalizability, takes place primarily in the *boson* system. The mechanism works qualitatively in a manner independent of the size of the hadronic gauge group. For simplicity then, we report here on the very simplest models of this type; we calculate electroproduction from a ρ^0 target in the Abelian and SU(2) models of the hadrons. In a lowest-order perturbative calculation, we obtain finite W_1 and νW_2 (up to a log). Extension to more realistic cases, with SU(3) $\otimes$ SU(3) and perhaps higher symmetry vector mesons[7] is certainly possible along the same lines. Because ours is a perturbative calculation, we cannot seriously compare our results with data. Nevertheless, the structure functions are suggestively realistic, and lead us to sketch the possible foundations of a "gauge-parton" model. Our mechanism appears also to describe the known "point-like" features of e^+e^- annihilation.

"Old" Yang-Mills theories failed to scale in a very bad way. Callan and Gross[11] argued, and their conclusions are roughly borne out (in low-order perturbation theory), that $W_1/\nu W_2 \sim O(1/q^2)$ in the scaling limit always. Sometimes this is realized by $W_1 \to 0$, νW_2 finite, sometimes by W_1 finite and W_2 blowing up *linearly*, depending on which graphs are considered. It is easy to suspect that these problems are connected with both (a) the nonrenormalizability of the models and (b) the fact that the currents do not have dimension three. Indeed this is the case; the Higgs-Kibble mechanism of the "new" models fixes (a) and (b) giving "new" field-current identities which, as conjectured in Ref. 3, give finite W_1 and W_2 (up to ln$s$).

The over-all picture here is important: Because "old" Yang-Mills theories are bad, deep-inelastic scattering data and e^+e^- annihilation, etc., have been persistently interpreted in terms of fermionic-constituent partons. However, our results show that the W_1 contribution from bosonic constituents is large enough so that it would be a mistake to interpret data only in terms of fermionic constituents. Thus, our results make possible a reinterpretation in terms of bosonic constituents or "ordinary" hadrons. We feel that these two languages—fermionic partons vs bosonic (hadronic) constituents—are complementary. In any case, a future scaling dual model will certainly have one interpretation in terms of hadron parameters.

II. SIMPLE MODELS

We begin our discussion by giving the Lagrangian for a simplified world, namely a U(n) gauge theory of hadrons coupled to a U(1) photon B, a lepton l, and a "baryon" q:

$$\mathcal{L} = -\tfrac{1}{4} F^{\alpha}_{\mu\nu} F^{\mu\nu}_{\alpha} + \tfrac{1}{4}\,\mathrm{Tr}(\nabla_\mu M^\dagger \nabla^\mu M) + V(M) + \bar{q}(i\not{\nabla} - M_q)q + \bar{l}(i\not{\partial} + g\not{B} + m_l)l\,. \tag{2.1}$$

Here M is an $n\times n$ complex matrix of scalars, V

$\equiv \frac{1}{2}\vec{\lambda}\cdot\vec{V}$ are the hadronic vector mesons ($\frac{1}{2}\lambda^\alpha$ are the $n\times n$ representation of the generators), $F^\alpha_{\mu\nu}$ are the usual Yang-Mills covariant derivatives while

$$\nabla_\mu M = \partial_\mu M - if V_\mu M + igMB_\mu Q\,. \tag{2.2}$$

Further

$$\nabla_\mu q = \partial_\mu q - if V_\mu q \tag{2.3a}$$

if q is "quark," or

$$\nabla_\mu q = \partial_\mu q - if[V_\mu, q] \tag{2.3b}$$

if q is a multiplet of baryons, say in the adjoint representation [SU(3) octet]. Q is the representation of the charge operator. Here we are exercising an option for simplicity in not coupling B directly to the baryons. Further, for SU(3) quarks or baryons, we could *not* couple B directly.[12] In any case, the point we are making here is that scaling will be achieved *without* fermionic constituents (no coupling of B to q).

The form of ∇M is characteristic of the unified models. The M's are in position both to make massive hadronic vector mesons, and to couple strong with nonstrong interactions. Indeed this is the case on spontaneous breakdown $\langle M^0\rangle = 2\kappa$.

Our calculation will be restricted to U(1) for simplicity, but we shall also comment on non-Abelian cases by taking SU(2) as an example. In both of these groups, in the unitary gauge, we are left with one scalar (M^0), massive vectors $\vec{\rho}$, and a massless universal photon γ. In terms of the hadronic coupling f and the B coupling g, the electron charge is

$$e = \frac{gf}{(g^2+f^2)^{1/2}} < g, f\,. \tag{2.4}$$

It is worth mentioning that, as always in the unified approach, the fine-structure constant e is necessarily smaller than the vector coupling constant f.

There are two equivalent ways of looking at Feynman graphs. We can use B and V with an explicit mixing, or the diagonalized ρ^0, γ (see Fig. 1). In this latter case ρ^0 has an explicit coupling to the lepton ($-e^2/f$). In practice, we used the ρ^0, γ graphs, but from the point of view of currents it is more convenient to use B. To lowest order in e, the current probed in electroproduction is

$$\begin{aligned} J_\mu &= \frac{\delta \mathfrak{L}^{(M)}}{\delta B^\mu} \\ &= \tfrac{1}{4} f (M^0 + 2\kappa)^2 V_\mu \\ &= \frac{m^2}{f} V_\mu + (\text{operator terms})\,. \end{aligned} \tag{2.5}$$

This is the new field-current identity discussed in Ref. 3 and 4. Note that the current has asymptotic dimension 3—due to the Higgs-scalar correction terms. We remark also that this is *not* just the electromagnetic current (coupling to diagonalized γ), but includes ρ^0 current as well.

We now begin to discuss the simplest scaling calculation in these models, namely electroproduction from a ρ^0 target. We will return later to discuss the possible relevance of this calculation in a more general parton picture.

The relevant graphs for electroproduction on a ρ^0 target (to order f^2) are shown in Fig. 2. A little discussion of their features is in order. The contact term in the $2M^0$ intermediate state is most directly related to the dimension of the current, as it displays exactly the free-field constitution of the current. This graph contributes a constant W_1 and a $\nu W_2 \to \nu$ in the scaling limit. The other terms in the $2M^0$ intermediate state cancel the diverging piece of νW_2 leaving both finite.

This "longitudinal" suppression is the entrance of the renormalizability of the theory. Indeed one can show that an increasing νW_2 of this form would demand a subtraction not allowed by the initial gauge invariance.

The story is the same with each of the other intermediate states. Most striking is the $\rho^+\rho^-$ intermediate state, in the SU(2) version, where the

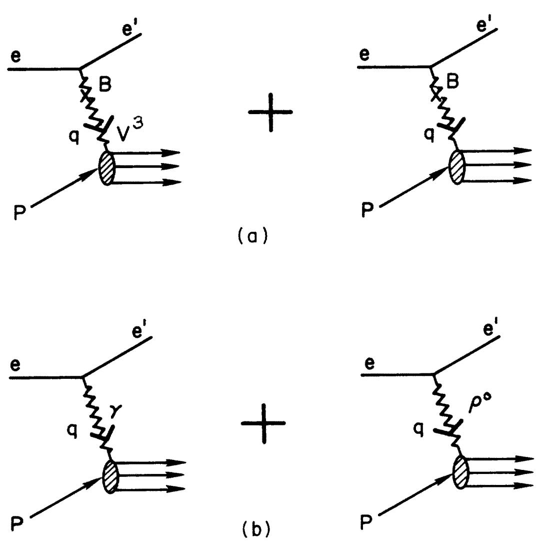

FIG. 1. Kinematical variables shown for deep-inelastic electron scattering. (a) Approach with undiagonalized B, V^3. (b) Approach with diagonalized photon (γ) and neutral ρ (ρ^0).

Higgs-scalar contribution is canceling graphs that are part of a pure "old" Yang-Mills calculation. The way this works is instructive: The old Yang-Mills graphs, though intrinsically more singular, are suppressed by the fact that they go through *both* ρ^0 and γ. The minus sign in the ρ^0 coupling (or, equivalently, the fact that only the double pole in the B-V formalism is contributing) knocks these graphs down by an extra $(q)^{-2}$ before we get into the heart of the problem. Then, they are on an equal footing with the direct-channel M^0 term, and cancellation occurs.

The actual calculation is algebraically quite involved, although it is never very hard to see cancellation of singular νW_2 terms. A few helpful details are included in the Appendix. In the text we will give only our conventions and the results for the $2M^0$ and $2\rho^0$ intermediate states (Abelian model contributions).

III. RESULTS

The differential cross section for deep-inelastic electron scattering off a ρ^0 target can be written in the laboratory frame of the ρ^0 as (see Fig. 1)

$$\frac{d^2\sigma}{d\Omega dE'}=\frac{4\alpha^2E'^2m}{|q^4|}\left[mW_2(q^2;m\nu)\cos^2(\tfrac{1}{2}\theta)+2W_1(q^2;m\nu)\sin^2(\tfrac{1}{2}\theta)\right], \quad (3.1)$$

where E' is the final electron's energy, θ is the scattering angle between the final and initial electron, and m is the mass of the ρ^0. Here q^2 is the momentum transfer squared between the initial and final electron (in traditional theories of electroproduction, q^2 is taken to be the mass of the virtual photon), and $m\nu=q\cdot p$, where p is the four-momentum of the ρ^0 target. $W_2(q^2,m\nu)$ and $W_1(q^2,m\nu)$ are the inelastic form factors. Our results were calculated in the scaling limit where $|q^2|\to\infty$, $m\nu\to\infty$, and $w=2m\nu/|q^2|$ finite. We kept all terms which contributed to the inelastic form factors W_2 and W_1 including terms of the form $\ln(\nu/m)$, but dropped terms which were down by factors of order $1/|q^2|$.[13]

We give our results for the $2\rho^0$ and $2M^0$ intermediate states for $\nu W_2(q^2,m\nu)$ and $W_1(q^2,m\nu)$. For both form factors the contribution from the $2\rho^0$ state is given in the first curly bracket and from the $2M^0$ state in the second curly bracket:

$$\begin{aligned}\nu W_2(q^2;m\nu)=\frac{f^2}{4\pi}\frac{1}{12}&\left\{\frac{1+2w(w-1)}{w^3(w-1)}\ln\frac{2m\nu w(w-1)}{m^2+\mu^2w(w-1)}\right.\\&\left.+\frac{3w^3-11w^2+12w-6}{2w^3(w-1)}+\frac{(w-1)(3-\mu^2/m^2+\frac{1}{4}\mu^4/m^4)}{w[1+(\mu^2/m^2)w(w-1)]}\right\}\\&+\frac{f^2}{4\pi}\frac{1}{12}\left\{\frac{(w-1)}{w^3}\ln\frac{2m\nu w(w-1)}{m^2(w-1)^2+\mu^2w}+\frac{w^2-6w+6}{2w^3}\right.\\&\left.+\frac{w(w-1)}{(w-1)^2+(\mu^2/m^2)w}\left[8-\frac{1}{w^2}\left(16(w-1)-8\frac{\mu^2}{m^2}(w-1)-\frac{2\mu^4}{m^4}\right)\right]\right\},\end{aligned}\quad (3.2)$$

$$W_1(q^2;m\nu)=\frac{f^2}{4\pi}\frac{1}{12}\left\{\frac{3}{4}-\frac{1}{2}\frac{w-1}{w^2}\right\}+\frac{f^2}{4\pi}\frac{1}{12}\left\{\frac{1}{4}-\frac{1}{2}\frac{w-1}{w^2}\right\}. \quad (3.3)$$

Here m is ρ^0 mass and μ is M^0 mass. We notice that there are $\ln(2m\nu/m^2)$ terms in νW_2, but that W_1 scales exactly (i.e., W_1 is only a function of w). The logarithmic terms in νW_2 do not represent significant deviations from scaling. In fact for $|q^2|$ between 1 and 10 $(\mathrm{GeV}/c)^2$, the log terms' contribution to νW_2 is about 20% for $w=3$, and decreases to about 1% for $w=10$. This deviation from scaling could almost be consistent with experimental deviations from scaling. (We stress, however, that our νW_2 and W_1 do not look like experiment but do have some of the same qualitative features as experiment.)

It is amusing to note the large-w behavior of our structure functions. In particular, the $2M^0$ contribution to νW_2 gives a constant or Pomeron-like behavior to νW_2 [for $|q^2|<50$ $(\mathrm{GeV}/c)^2$]. This is easily understood in terms of the double-ρ^0 exchange in the t channel [see Fig. 2(a)]. In this order, the ρ^0 lies on a flat trajectory with $\alpha_\rho=1$,[8] hence one expects a Pomeron-like singularity at $J=1$. It will be interesting to see how this "Pomeron" behaves in higher orders, where the ρ trajectory is expected to pick up a slope.[9]

IV. POSSIBLE GAUGE-PARTON PICTURES

We now attempt to interpret our results in terms of a partonlike picture. Here we picture a nucleon target, composed (in general) of various partonic constituents. We find that the V mesons are the only partons whose contributions to W_1 hold up in

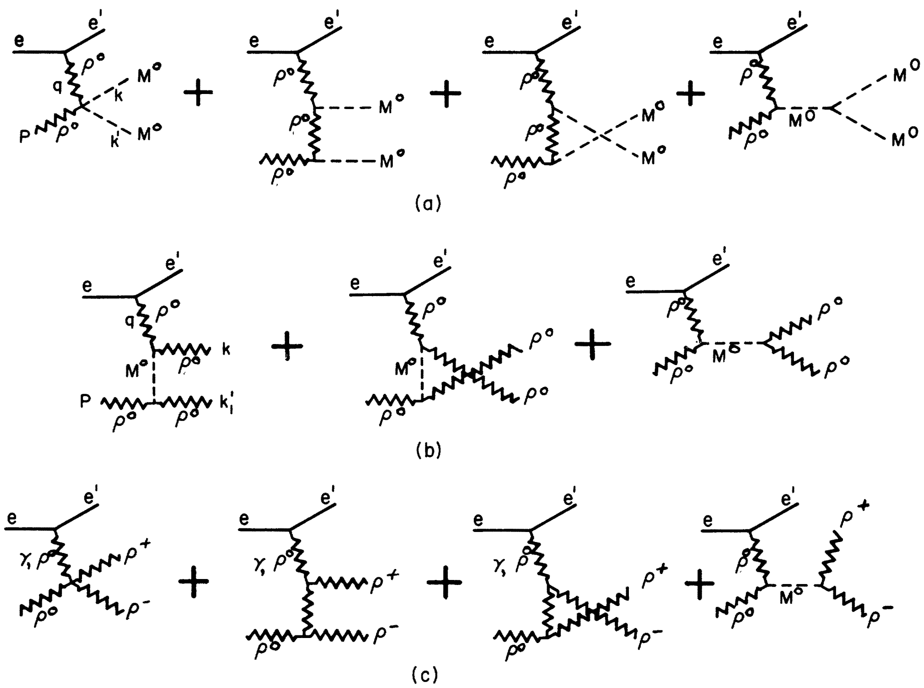

FIG. 2. (a) $2M^0$ intermediate state and kinematics; (b) $2\rho^0$ intermediate state and kinematics; (c) $\rho^+\rho^-$ intermediate state.

the scaling limit. In general, partons are not used in quite the usual way (as pictured in Fig. 3); they are emitted from the nucleon target, and interact through a "core" which has two parts: (1) a more ordinary parton kernel, involving inelastic as well as elastic parton processes, (2) a di-parton kernel involving production of $2M^0, 2\rho^0, \rho^+\rho^-, \ldots$ pairs as in Figs. 2(a)–2(c). Each of these di-parton pairs has a mass of order $|q^2|$, in contrast to ordinary partons such as in type (1). It is these di-parton pairs that give the nonvanishing W_1 in our models. The calculation in this paper is the first simple model of the di-parton kernel. (If one has in mind, a totally $[\ln(\nu/m)]$-free "di-parton" kernel, then this model is not completely satisfactory. What is needed is the analog of the usual field-theory cutoff[14]; in this case a more complicated gauge-invariant momentum-transfer cutoff appears to be needed. In this connection the ideas of Arnowitt, Friedman, and Nath[15] may be useful.) After that, the program is the following: We imagine having $W_1^{(1,2)}$ and $W_2^{(1,2)}$, where the superscripts refer to core contributions of type (1) and (2), respectively. Then, the baryon structure functions are obtained in the following way:

$$W_1(w) = \int_{1/w}^{1} dx\, \rho(x)[W_1^{(1)}(xw) + W_1^{(2)}(xw)] \quad (4.1)$$

and similarly for W_2. Here x is the fraction of the longitudinal momentum that V^μ carries as a constituent of the nucleon and $\rho(x)$ is the probability for this to occur.[16]

V. FINAL REMARKS

A first comment concerns the Callan-Gross[11] (CG) result that $\nu W_2/W_1 \to \infty$ (like a power). In fact, their formal manipulations have broken down for

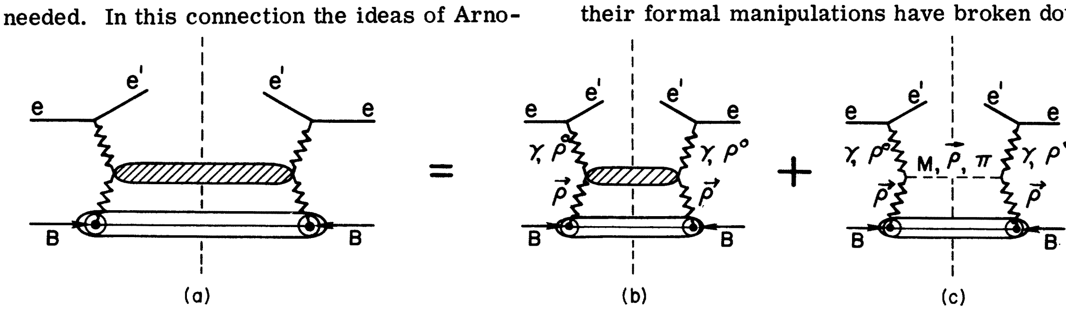

FIG. 3. (a) Core, (b) di-parton kernel, (c) ordinary parton kernel.

this case (as they did for the gluon model in perturbation theory[10]): The CG formal current-algebraic manipulations can be repeated here using the time derivative of the currents (2.5). The answer is essentially that of Ref. 4, and no transverse terms are in evidence. Thus, CG manipulations disagree with our result. In tracing back to look for the onset of error in a CG manipulation, one sees at least that there is no q_0^{-2} term in these integrals. As in the gluon calculation, this can completely destroy any resemblance of the formal answer to perturbation theory.

Our next remark concerns e^+e^- annihilation experiments. The experimental fact that this total cross section is "pointlike" ($\sigma_T \sim q^{-2}$), ruled out the old Yang-Mills theories.[17] The "pointlike" property, however, can be traced immediately to correct current dimensions; so, in our new models with current dimension three, we expect the observed $1/q^2$ behavior. Some relevant graphs contributing to the annihilation are shown in Fig. 4.

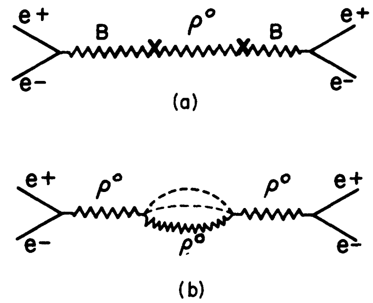

FIG. 4. (a) Old Yang-Mills contributions to e^+e^- annihilation total cross sections. (b) New gauge-theory contribution to pointlike e^+e^- annihilation.

APPENDIX

Here we show some of the details of our calculation. The sum of the diagrams in Figs. 2(a) and 2(b) corresponding to the $2M^0$ and $2\rho^0$ intermediate states, respectively, give (after averaging over the target ρ^0 polarization) a contribution equal to

$$
\begin{aligned}
W_{\mu\nu}^{(2M)} = \frac{f^2}{24(2\pi)^2} \int \frac{d^3k}{2\omega_k}\frac{d^3k'}{2\omega_{k'}} &\left[1 + \frac{3\mu^2}{s-\mu^2} + \frac{2m^2}{t-m^2}\Delta(k-q) + \frac{2m^2}{u-m^2}\Delta(p-k)\right]_{\mu\alpha} \\
&\times \Delta_{\alpha\beta}(p)\left[1 + \frac{3\mu^2}{s-\mu^2} + \frac{2m^2}{t-m^2}\Delta(k-q) + \frac{2m^2}{u-m^2}\Delta(p-k)\right]_{\beta\nu}\delta^4(p+q-k-k'),
\end{aligned}
$$

$$
\begin{aligned}
W_{\mu\nu}^{(2\rho)} = \frac{f^2m^4}{6(2\pi)^2}\int \frac{d^3k}{2\omega_k}\frac{d^3k'}{2\omega_{k'}} &\left\{ \frac{\Delta_{\mu\nu}(p)\operatorname{Tr}[\Delta(k)\Delta(k')]}{s-\mu^2} + \frac{\Delta_{\mu\nu}(k)\operatorname{Tr}[\Delta(p)\Delta(k')]}{t-\mu^2} \right. \\
&+ \frac{\Delta_{\mu\nu}(k')\operatorname{Tr}[\Delta(p)\Delta(k)]}{u-\mu^2} + \frac{[\Delta(k)\Delta(k')\Delta(p)+\Delta(p)\Delta(k')\Delta(k)]_{\mu\nu}}{(s-\mu^2)(t-\mu^2)} \\
&\left. + \frac{[\Delta(k')\Delta(k)\Delta(p)+\Delta(p)\Delta(k)\Delta(k')]_{\mu\nu}}{(s-\mu^2)(u-\mu^2)} + \frac{[\Delta(k)\Delta(p)\Delta(k')+\Delta(k')\Delta(p)\Delta(k)]_{\mu\nu}}{(t-\mu^2)(u-\mu^2)} \right\} \\
&\times \delta^4(p+q-k-k').
\end{aligned}
$$

Here we have written $\Delta_{\mu\nu}(p) = g_{\mu\nu} - p_\mu p_\nu/m^2$ etc. After the necessary simplifications, and dropping all terms proportional to q_μ or q_ν (since they give zero when dotted with the lepton current), we obtain the following expressions:

$$
\begin{aligned}
W_{\mu\nu}^{(2M)} = \frac{f^2}{4\pi}\frac{1}{48}\int \frac{dt}{2p\cdot q} &\left\{ g_{\mu\nu}\left[1 + \frac{3\mu^2}{s-\mu^2} + \frac{2m^2}{t-m^2} + \frac{2m^2}{u-m^2}\right]^2 \right. \\
&- \frac{p_\mu p_\nu}{m^2}\left[\left(\frac{3\mu^2}{s-\mu^2} + \frac{2m^2}{t-m^2} + \frac{2m^2}{u-m^2}\right)^2 + \frac{\mu^2}{u-m^2}\left(\frac{\mu^2}{u-m^2} + \frac{3\mu^2}{s-\mu^2} + \frac{2m^2}{t-m^2}\right)\right] \\
&+ \left(\frac{k_\mu p_\nu}{m^2} + \frac{p_\mu k_\nu}{m^2}\right)\left[\frac{2m^2(2m^2-\mu^2)}{(t-m^2)^2} + \frac{2m^2(2m^2-\mu^2)+\mu^4}{(u-m^2)^2} + \frac{3\mu^2(2m^2-\mu^2)}{(s-\mu^2)(t-m^2)}\right. \\
&\qquad \left. + \frac{3\mu^2(2m^2+\mu^2)}{(s-\mu^2)(u-m^2)} + \frac{6m^4+2m^2\mu^2-\mu^4+2m^2q^2}{(t-m^2)(u-m^2)}\right] \\
&+ \frac{k_\mu k_\nu}{m^2}\left[-(\mu^4+8m^4-4m^2\mu^2)\left(\frac{1}{(t-m^2)^2} + \frac{1}{(u-m^2)^2}\right) - \frac{12m^2\mu^2}{s-\mu^2}\left(\frac{1}{t-m^2} + \frac{1}{u-m^2}\right)\right. \\
&\qquad \left.\left. - \frac{2(6m^4+2m^2\mu^2-\mu^4+2m^2q^2)}{(t-m^2)(u-m^2)}\right]\right\};
\end{aligned}
$$

$$W^{(2\rho)}_{\mu\nu}=\frac{f^2}{4\pi}\frac{1}{12}\int\frac{dt}{2p\cdot q}\Bigg\{g_{\mu\nu}\left[\frac{3}{4}-m^2\delta\left(\frac{1}{s-\mu^2}+\frac{1}{t-\mu^2}+\frac{1}{u-\mu^2}\right)+\frac{2m^4(3m^2-3\mu^2+q^2)}{(s-\mu^2)(t-\mu^2)(u-\mu^2)}\right.$$
$$\left.+m^4(2+\delta^2)\left(\frac{1}{(s-\mu^2)^2}+\frac{1}{(t-\mu^2)^2}+\frac{1}{(u-\mu^2)^2}\right)\right]$$
$$-p_\mu p_\nu\left[m^2(2+\delta^2)\left(\frac{1}{(s-\mu^2)^2}+\frac{1}{(u-\mu^2)^2}\right)+\frac{m^2(4-2\delta)(q^2+3m^2-3\mu^2)}{(s-\mu^2)(t-\mu^2)(u-\mu^2)}\right.$$
$$\left.+\frac{q^2+3m^2-3\mu^2}{(s-\mu^2)(u-\mu^2)}-\frac{2m^2\delta^2}{(s-\mu^2)(t-\mu^2)}\right]$$
$$-k_\mu k_\nu\left[m^2(2+\delta^2)\left(\frac{1}{(t-\mu^2)^2}+\frac{1}{(u-\mu^2)^2}\right)+\frac{m^2(4-2\delta)(q^2+3m^2-3\mu^2)}{(s-\mu^2)(t-\mu^2)(u-\mu^2)}\right.$$
$$\left.+\frac{q^2+3m^2-3\mu^2}{(t-\mu^2)(u-\mu^2)}-\frac{2m^2\delta^2}{(s-\mu^2)(t-\mu^2)}\right]$$
$$+(p_\mu k_\nu+k_\mu p_\nu)\left[\frac{m^2(2+\delta^2)}{(u-\mu^2)^2}+\frac{m^2\delta^2}{u-\mu^2}\left(\frac{1}{t-\mu^2}+\frac{1}{s-\mu^2}\right)+\frac{m^2\delta^2}{(s-\mu^2)(t-\mu^2)}\right.$$
$$\left.-\frac{(q^2+3m^2-3\mu^2)[(u-\mu^2)^2+2m^2(2+\delta)-\frac{1}{2}(q^2+3m^2-3\mu^2)]}{(s-\mu^2)(t-\mu^2)(u-\mu^2)}\right]\Bigg\}.$$

For the $2M^0$ intermediate state we have

$$s+t+u=q^2+m^2+2\mu^2\,.$$

For the $2\rho^0$ intermediate state we have

$$s+t+u=q^2+3m^2\,,$$

where $\delta=(1-\mu^2/2m^2)$.

Before extracting W_1 and W_2 from these expressions we have added the necessary q_μ or q_ν terms (their contribution is always zero) so that $W_{\mu\nu}$ has a gauge-invariant structure. For example we have replaced $g_{\mu\nu}\to(g_{\mu\nu}-q_\mu q_\nu/q^2)$, etc.

After doing the integrals and taking the limit $q^2\to\infty$, $p\cdot q\to\infty$, we obtain the expressions of Eqs. (3.2) and (3.3).

*Research supported by the Air Force Office of Scientific Research, Office of Aerospace Research, United States Air Force, under Contract number F44620-70-C-0028.

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[13]In our calculations, the inequality

$$\frac{2M\nu w(w-1)}{10}>M^2+\mu^2w(w-1)\,,\;M^2(w-1)+\mu^2w$$

was always satisfied; this keeps us away from the threshold $w=1$.

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