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Stationary states of a spin-1 particle in a homogeneous magnetic field

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We present a simple and complete determination of the energy spectrum and eigenfunctions of a relativistic spin-1 particle with arbitrary magnetic moment in a homogeneous magnetic field. The particle is described by a four-vector field satisfying the usual second-order equation including anomalous-magnetic-moment interaction. In the light of our results and those pertaining to the case when the external field is a Coulomb field, we discuss briefly the question of consistency of the vector theory at the basic c-number level.

I. INTRODUCTION

The main objective of this paper is to present an elegant solution of the stationary-state problem for a relativistic spin-1 particle with arbitrary magnetic moment in a homogeneous and time-independent magnetic field. The particle is described by a c-number vector field ϕ_μ obeying the familiar second-order wave equation[1] as generalized by Corben and Schwinger[2] to accommodate arbitrary magnetic moments. This problem has been tackled recently in a series of papers by Tsai and collaborators,[3,4] who have obtained the energy spectrum by an algebraic method which is extremely tedious in practice (though simple in principle) and does not give any indication about the nature of the stationary-state wave functions. Our approach to the problem is also algebraic, but unlike the authors of Refs. 3 and 4 we do not rely on the algebra to decouple the different components of the field. The observation that the components can be effectively decoupled right at the outset enables us to solve the problem in a transparent way with the aid of no more than the algebra of the creation and annihilation operators of a harmonic oscillator. That the problem of a charged particle in a homogeneous magnetic field can be related to the harmonic-oscillator problem has been known for many years, but this seems to be the first time that this fact has been effectively exploited in the case of particles with spin.

The problem considered here is of considerable intrinsic interest. Further, as part of the general problem of charged vector mesons in interaction with electromagnetic fields, it is of special importance because of its exact solubility and perspicuity. In this context it complements the work of Corben and Schwinger[2] on vector particles in electric fields of the Coulomb form. At a time when vector particles are being assigned a central role in elementary-particle interactions, as in the recent unified theories of weak and electromagnetic interactions, it is pertinent to raise the question whether at least some of the difficulties appearing in the perturbation theory of the interacting charged vector field might not be due to a fundamental malaise in the basic formulation of the theory itself. We discuss this point briefly in Sec. IV. We shall comment there also on the related problem of the acausality of propagation of the c-number vector field in the presence of anomalous-magnetic-moment coupling.

We present the formal solution of the stationary-state problem in Sec. II, and in Sec. III we bring out explicitly the nature of the eigenfunctions. Section IV is devoted to discussion of the results and other related points.

II. SOLUTION OF THE EIGENVALUE PROBLEM

We start from the second-order equation for the vector field with anomalous-magnetic-moment coupling, in the usual form:

$$(m^2+\pi^\nu\pi_\nu)\phi_\mu-\pi^\nu\pi_\mu\phi_\nu+ie\kappa F_{\mu\nu}\phi^\nu=0\,, \tag{1}$$

where $\pi_\mu=p_\mu-eA_\mu=-i(\partial/\partial x^\mu)-eA_\mu$. This equation implies the subsidiary condition

$$\pi^\nu\phi_\nu=\frac{ie}{m^2}(1-\kappa)\pi_\mu F^{\mu\nu}\phi_\nu\,, \tag{2}$$

and on feeding this back into Eq. (1), one gets the true equation of motion

$$(m^2+\pi^\nu\pi_\nu)\phi_\mu-\frac{ie}{m^2}(1-\kappa)\pi_\mu\pi_\nu F^{\nu\lambda}\phi_\lambda+ie(1+\kappa)F_{\mu\nu}\phi^\nu=0\,. \tag{3}$$

As is well known, Eqs. (3) and (2) together are completely equivalent to Eq. (1).

We are interested in the particular case of a constant homogeneous magnetic field H in the z direction. Thus we take

$$A_1=-\tfrac{1}{2}yH,\quad A_2=\tfrac{1}{2}xH,\quad A_3=0\,;\qquad F_{12}=-F_{21}=H\,, \tag{4}$$

with all other components of $F_{\mu\nu}$ vanishing. It is then easy to verify that the equations which follow from (3) for the spatial components ϕ_i $(i=1,2,3)$ can be expressed in the form

$$E^2\phi_\pm=(m^2+\vec{\pi}^2)\phi_\pm+\frac{eH}{2m^2}(1-\kappa)\pi_\pm(\pi_+\phi_- -\pi_-\phi_+) \pm eH(1-\kappa)\phi_\pm, \quad (5a)$$

$$E^2\phi_3=(m^2+\vec{\pi}^2)\phi_3+\frac{eH}{2m^2}(1-\kappa)\pi_3(\pi_+\phi_- -\pi_-\phi_+), \quad (5b)$$

where

$$\pi_\pm=\pi_1\pm i\pi_2, \quad \phi_\pm=\phi_1\pm i\phi_2. \quad (6)$$

Since we are seeking stationary solutions characterized by the time dependence e^{-iEt}, we have replaced $(\pi^0)^2\equiv(p^0)^2$ in the above by E^2.

We observe now that the operators π_+ and π_- obey an algebra equivalent to that of a simple harmonic oscillator. In fact, with

$$a=(2eH)^{-1/2}\pi_+, \quad a^\dagger=(2eH)^{-1/2}\pi_-, \quad (7)$$

we have[5]

$$[a,a^\dagger]=1, \quad [a,\pi_3]=[a^\dagger,\pi_3]=0. \quad (8)$$

Since π_3 commutes with everything, it will be replaced by its eigenvalue p_3, which we shall write also as $(2eH)^{-1/2}a_3$ wherever this helps to simplify the notation. (Unlike a and $a^\dagger$, a_3 is just a number, $-\infty<a_3<\infty$.) Now, noting that in view of (7)

$$\begin{aligned}\pi_1{}^2+\pi_2{}^2&=\tfrac{1}{2}(\pi_+\pi_-+\pi_-\pi_+)\\&=eH(aa^\dagger+a^\dagger a)\\&=eH(2N+1),\end{aligned}$$

where

$$N\equiv a^\dagger a \quad (9)$$

is the number operator, we rewrite (5) as

$$(\omega^2-X_+)\phi_+=(1-\kappa)\xi^2a^2\phi_-, \quad (10a)$$

$$(\omega^2-X_-)\phi_-=-(1-\kappa)\xi^2(a^\dagger)^2\phi_+, \quad (10b)$$

$$[\omega^2-(2N+1)\xi]\phi_3=(1-\kappa)\xi^2a_3(a\phi_- -a^\dagger\phi_+), \quad (10c)$$

wherein the following abbreviations have been used:

$$\xi=(eH/m^2), \quad \omega^2=(E^2-m^2-p_3{}^2)/m^2, \quad (11)$$

and the operators X_+, X_- are defined by

$$X_\pm=[(2N+1)\xi-\tfrac{1}{2}(1-\kappa)\xi^2] \pm[(1+\kappa)\xi-\tfrac{1}{2}(2N+1)(1-\kappa)\xi^2]. \quad (12)$$

The equations (10) for ϕ_+, ϕ_-, and ϕ_3 decouple completely in the special case $\kappa=1$, and we immediately obtain the possible values of ω^2 as

$$\omega^2=(2n-1)\xi, \quad (13a)$$

the independent solutions for any n being

$$\begin{aligned}&\phi_+=|n-2\rangle, \quad \phi_-=0, \quad \phi_3=0 \quad (n=2,3,\dots),\\&\phi_+=0, \quad \phi_-=0, \quad \phi_3=|n-1\rangle \quad (n=1,2,\dots),\\&\phi_+=0, \quad \phi_-=|n\rangle, \quad \phi_3=0 \quad (n=0,1,2\dots).\end{aligned} \quad (13b)$$

The states $|n\rangle$ are the "number eigenstates" formally defined by

$$N|n\rangle=n|n\rangle,$$
$$a|0\rangle=0, \quad |n\rangle=(n!)^{-1/2}(a^\dagger)^n|0\rangle. \quad (14)$$

For any $\kappa\neq1$, one family of solutions is immediately apparent on inspection:

$$\begin{aligned}&\phi_+=\phi_-=0, \quad \phi_3=|n\rangle,\\&\omega^2=(2n+1)\xi, \quad n=0,1,2,\dots.\end{aligned} \quad (15)$$

To obtain the remaining solutions we must solve the coupled equations (10a) and (10b) first. By eliminating ϕ_- or ϕ_+ from these two equations one obtains, for any $\kappa\neq1$,

$$\{(\omega^2-X_+)(\omega^2-X_-)+(N^2+3N+2)(1-\kappa)^2\xi^4 -2[(1-\kappa)\xi^2+2\xi](\omega^2-X_+)\}\phi_+=0, \quad (16a)$$

$$\{(\omega^2-X_+)(\omega^2-X_-)+(N^2-N)(1-\kappa)^2\xi^4 -2[(1-\kappa)\xi^2-2\xi](\omega^2-X_-)\}\phi_-=0. \quad (16b)$$

Thus ϕ_+, ϕ_- are eigenstates of the number operator N. Equations (10) permit only the following possibilities:

$$\phi_-=|n+1\rangle, \quad \phi_+=c_n|n-1\rangle, \quad \phi_3=c_n'|n\rangle, \quad n=1,2,3,\dots \quad (17)$$

$$\phi_-=|1\rangle, \quad \phi_+=0, \quad \phi_3=c_0'|0\rangle, \quad (18a)$$

$$\phi_-=|0\rangle, \quad \phi_+=0, \quad \phi_3=0, \quad (18b)$$

where c_n, c_n' are constants.

Substitution of Eq. (17) has the effect of replacing N in (16a) by $(n-1)$ and in (16b) by $(n+1)$. With these replacements, the curly-bracketed expressions in the two equations become identical, and either of them can then be solved, with the result

$$\omega^2=[(2n+1)\xi+\tfrac{1}{2}(1-\kappa)\xi^2] +\epsilon(1-\kappa)\xi[1+(2n+1)\xi+\tfrac{1}{4}\xi^2]^{1/2}, \quad \epsilon=\pm1. \quad (19)$$

The constants c_n can now be determined by introducing (17) and (19) in either Eq. (10a) or (10b):

$$\begin{aligned}c_{n\epsilon}&=[n(n+1)]^{1/2}\xi \times\{1+(n+\tfrac{1}{2})\xi+\epsilon[1+(2n+1)\xi+\tfrac{1}{4}\xi^2]^{1/2}\}^{-1}\\&=[n(n+1)]^{-1/2}\xi^{-1} \times\{1+(n+\tfrac{1}{2})\xi-\epsilon[1+(2n+1)\xi+\tfrac{1}{4}\xi^2]^{1/2}\}.\end{aligned} \quad (20a)$$

Then c_n' is obtainable from (11c):

$$c_{n\epsilon}' = a_3\xi[(n+1)^{1/2} - c_{n\epsilon}n^{1/2}] \times\{\tfrac{1}{2}\xi + \epsilon[1+(2n+1)\xi + \tfrac{1}{4}\xi^2]^{1/2}\}^{-1} . \quad (20b)$$

It may be noted that these are indpendent of κ.

In the case of the two special solutions (18), Eqs. (10a) and (10b) degenerate into the single equation $(\omega^2 - X_-)|n+1\rangle = 0$, $n+1=1, 0$. The energies associated with (18a) and (18b) are therefore determined by $\omega^2 = (X_-)_{N=n+1}$, with $n=0$ and -1, respectively, i.e.,

$$\omega^2 = (X_-)_{N=1} = (2-\kappa)\xi + (1-\kappa)\xi^2 \quad (n=0) , \quad (21a)$$

$$\omega^2 = (X_-)_{N=0} = -\kappa\xi \quad (n=-1) . \quad (21b)$$

The first of these is seen to be a special case of Eq. (19) taken with $\epsilon = +1$. The value of c_0' in (18a) also then turns out to be the special case of (20b) with $n=0$ and $\epsilon=+1$. Equation (21b) too is a special case of (19) with $n=-1$, but it belongs to $\epsilon = +1$ if $(1-\frac{1}{2}\xi)>0$ and to $\epsilon=-1$ if $(1-\frac{1}{2}\xi)<0$. This is because the square root in (19), which by definition is positive, is $|1-\frac{1}{2}\xi|$ for $n=-1$.

III. THE EIGENFUNCTIONS AND THE ENERGY SPECTRUM

It is important to recognize at this point that what we have been calling the "number eigenstate" $|n\rangle$ is not really a single state at all. In fact, an infinity of states lurk behind this symbol. One can see from general considerations that this should be so. The "harmonic oscillator" operators a and $a^\dagger$ defined in the preceding section involve *two* degrees of freedom of the particle, pertaining to its motion projected on to the x-y plane. Clearly the states of a simple (one-dimensional) harmonic oscillator cannot adequately cover this two-dimensional motion, for which one needs a doubly infinite set of states. We shall now see by solving for the "vacuum state" $|0\rangle$ that it is really a superposition of an infinite number of states $|0;m\rangle$.

By definition, $a|0\rangle = 0$. Recalling that $a=(2eH)^{-1/2} \times(\pi_1 + i\pi_2)$ and substituting $\pi_1 = -i(\partial/\partial x) + \frac{1}{2}eHy$ and $\pi_2 = -i(\partial/\partial y) - \frac{1}{2}eHx$, we rewrite this defining equation as

$$\left[-i\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \tfrac{1}{2}eH(y-ix)\right]\psi_0(x,y) = 0 . \quad (22)$$

We have written $\psi_0(x,y)$ for $|0\rangle$ in the coordinate representation, suppressing the variable z which plays no role here. A change of the independent variables to

$$\xi = x+iy \quad \text{and} \quad \eta = x-iy \quad (23)$$

reduces Eq. (22) to

$$\left(\frac{\partial}{\partial\eta} + \tfrac{1}{4}eH\xi\right)\psi_0 = 0 , \quad (24)$$

with the general solution

$$\psi_0 = e^{-eH\xi\eta/4}f(\xi) \quad (25a)$$

$$= e^{-eH\rho^2/4}f(\rho e^{i\phi}) , \quad (25b)$$

where ρ and ϕ are polar coordinates in the x-y plane and $f(\xi)$ is an arbitrary function of ξ. We expect that $f(\xi)$ can be expanded in powers of ξ. The requirement of finiteness of the wave function at the origin demands that only non-negative powers of $\xi \equiv \rho e^{i\phi}$ be present. Thus ψ_0 is an arbitrary linear combination (suitably restricted to ensure acceptable asymptotic behavior as $\rho\to\infty$) of the functions

$$\psi_{0k} = b_{0k}e^{-eH\xi\eta/4}\xi^k = b_{0k}e^{-eH\rho^2/4}\rho^k e^{ik\phi} , \quad (26)$$

where the b_{0k} are constants. It is this infinite set of functions which goes under the symbol $|0\rangle$ of the preceding section. By repeated application on the ψ_{0k} with $a^\dagger$, which may be written as $(\partial/\partial\xi) - \frac{1}{4}eH\eta$ apart from constant factors, we determine the set of functions covered by the symbol $|n\rangle$ to be

$$\psi_{nk} = \frac{(a^\dagger)^n}{(n!)^{1/2}}\psi_{nk} = b_{nk}\left(\frac{\partial}{\partial\xi} - \tfrac{1}{4}eH\eta\right)^n e^{-eH\xi\eta/4}\xi^k = b_{nk}e^{-eH\xi\eta/4}\left(\frac{\partial}{\partial\xi} - \tfrac{1}{2}eH\eta\right)^n \xi^k ,$$
$$n, k = 0, 1, 2, \ldots . \quad (27)$$

Since $\xi = \rho e^{i\phi}$ and $\eta = \rho e^{-i\phi}$, it follows that $\psi_{nk} \propto e^{i(k-n)\phi}$ and hence that it is an eigenfunction of $L_z = (xp_y - yp_x)$ belonging to the eigenvalue[6] $m = (k-n)$. It is therefore advantageous to use m instead of k for labeling the states and we shall use the notation

$$|n;m\rangle \to \psi_{n,n+m}, \quad n=0,1,2,\ldots, \quad m=-n,-n+1,\ldots ,$$
$$a|n;m\rangle = n^{1/2}|n-1, m+1\rangle , \quad (28)$$
$$a^\dagger|n;m\rangle = (n+1)^{1/2}|n+1;m-1\rangle .$$

The changes in m and n are coupled since the action of a or $a^\dagger$ does not change $k\equiv n+m$.

With this elaboration of the meaning of the "number eigenstates" we return to a consideration of the energy spectrum. When the anomalous magnetic moment strength κ has any value other than unity, the spectrum consists of three distinct branches. One of these is given by (15), which gives, in view of the definition (11) of ω^2,

$$(E/m)^2 = 1 + (p_3/m)^2 + (2n+1)\xi\,, \qquad \phi_+ = \phi_- = 0, \quad \phi_3 = |n; m\rangle, \quad n = 0, 1, 2, \ldots . \tag{29}$$

[The range of m is always that given in (28).] The other two branches are given by

$$(E/m)^2 = 1 + (p_3/m)^2 + (2n+1)\xi + \tfrac{1}{2}(1-\kappa)\xi^2 + (1-\kappa)\xi[1 + (2n+1)\xi + \tfrac{1}{4}\xi^2]^{1/2}, \quad n = 0, 1, 2, \ldots , \tag{30a}$$

which includes (19) with $\epsilon = +1$ as well as (21a);

$$(E/m)^2 = 1 + (p_3/m)^2 + (2n+1)\xi + \tfrac{1}{2}(1-\kappa)\xi^2 - (1-\kappa)\xi[1 + (2n+1)\xi + \tfrac{1}{4}\xi^2]^{1/2}, \quad n = 1, 2, 3, \ldots , \tag{30b}$$

which corresponds to (19) with $\epsilon = -1$; and

$$(E/m)^2 = 1 + (p_3/m)^2 - \kappa\xi\,, \tag{30c}$$

obtained from (21b), which is a special case ($n = -1$) of (30a) or (30b) according as $(1 - \frac{1}{2}\xi) > 0$ or <0. The eigenfunctions belonging to these levels are

$$\phi_- = |n+1; m-1\rangle\,, \quad \phi_+ = c_{n\epsilon}\,|n-1; m+1\rangle\,, \quad \phi_3 = c'_{n\epsilon}|n; m\rangle\,, \tag{31}$$

where $\epsilon = +1$ and $\epsilon = -1$ go with (30a) and (30b), respectively, and the $c_{n\epsilon}$ and $c'_{n\epsilon}$ are given by Eqs. (20). The existence of three branches is associated with the three independent spin orientations of the particle. The eigenstates of the z component of spin, S_z, are characterized by

$$\phi_+ = \phi_- = 0 \quad (S_3 = 0)\,, \quad \phi_+ = \phi_3 = 0 \quad (S_3 = +1)\,, \quad \phi_- = \phi_3 = 0 \quad (S_3 = -1)\,. \tag{32}$$

In view of this, the branch described by (29) is seen to have $S_3 = 0$, but the states (31) belonging to the other two branches are *not* eigenstates of S_z, which reflects the coupling of spin and orbital motions. A curious effect of the spin-orbit coupling is that the positive ϵ branch, which includes (30a) and (30c) for any $\xi < 2$, has an extra value of n ($n = -1$) compared with the spinless case, while the negative ϵ branch has one value too few, $n = 0$ being absent in (30b). This imbalance is removed if ξ increases beyond 2.

In the special case $\kappa = 1$, the spin and orbital degrees of freedom are decoupled, as is evident from (13), and all three branches are degenerate (except for the lowest two levels).

IV. DISCUSSION

The derivation given above, besides being simple, goes beyond the work of Refs. 3 and 4 in determining all the eigenfunctions. It has been noted already by Tsai and Yildiz[3] that the spectrum of E^2 (which they obtained for the case $p_3 = 0$) is positive-definite only in the case of minimal coupling ($\kappa = 0$). For any $\kappa \neq 0$, the spectrum includes negative values (corresponding to *imaginary* values of E, which have to be considered unphysical) if ξ is large enough. If $\kappa > 0$, for instance, the expression (30c) becomes negative if the magnetic field is large enough that $m^2\kappa\xi \equiv e\kappa H > (m^2 + p_3^2)$. On the other hand, since $(E/m)^2$ of Eq. (30b) becomes $\sim(2n+1)\kappa\xi$ for large ξ, it is negative for $\kappa < 0$. It may be instructive to view this phenomenon from a somewhat different point of view, and say that as far as motion along the z axis is concerned the particle in the presence of the magnetic field behaves as if it had an effective mass $m_{\rm eff}^2 = E^2 - p_3^2$. This effective mass becomes imaginary in one or the other of the modes if $\kappa \neq 0$ and H is large enough, and in such a "tachyonic" mode real energies are possible only if $p_3^2 > |m_{\rm eff}^2|$. The occurrence of tachyonic modes implies acausal propagation of the field, as has been pointed out by us recently,[7] though this acausality does not show up in the nature of the characteristic surfaces[8] associated with Eq. (3).

The inconsistency of the theory with $\kappa \neq 0$ in the presence of large external magnetic fields[9] H such that $\kappa e H \gtrsim m^2$, which is manifested through the appearance of tachyonic modes (imaginary energies), cannot be explained away in terms of "quantum effects" which may be expected to take place at such enormous field strengths [and are not taken into account into Eq. (3)], for such an explanation would raise the question as to why a similar inconsistency does not arise with minimal coupling. In fact, in the case of spin-$\frac{1}{2}$ particles, even with an anomalous magnetic moment κ the energy does not become imaginary, however strong the magnetic field may be.[3,10] (However, the ground-state energy does become zero at $\kappa e H = m^2$, which could give rise to strange effects.[11])

Implicit in all the above considerations is the supposition that κ is a fixed quantity, independent of H. If κ arises from radiative effects, there is no reason why it should stay constant. In fact, consideration of the radiative corrections to the electron propagator in an external magnetic field H shows that though the ground-state electron energy in the presence of H initially decreases linearly with H as if the electron had[12] $\kappa = (\alpha/2\pi)$, yet as H increases to large values the energy reaches a positive minimum value and then starts increasing.[13] However, it is not at all clear that radiative effects can erode a preexisting large anomalous moment (as in the proton) sufficiently to prevent the energy from touching zero.

In the case of vector particles, too, it is possible to arrange that E^2 never becomes negative, by making κ a suitable function of H. In fact one can readily see from Eqs. (30) what the limits on κ should be (as a function of H) for this purpose. For positive κ, the most stringent condition comes from Eq. (30c), which requires that $\kappa\xi < 1$. For negative κ it is the lowest of the levels (30b), that with $n=1$, which sets the limit. One finds that the condition is

$$|\kappa| < \tfrac{1}{2}\{[1+(4/\xi^2)(1+3\xi)]^{1/2}-1\} \sim 3/\xi$$

for large ξ. Thus one should have

$$-3\xi^{-1} < \kappa < \xi^{-1}$$

at large field strengths, if imaginary values of E are to be avoided. It is conceivable that these limits might be honored if κ were solely due to radiative effects (though this question is more difficult here than in the spin-$\frac{1}{2}$ case because of the large-momentum behavior of the vector-meson propagator). But it does not seem plausible that a preexisting anomalous moment (such as that of intermediate vector bosons of the Weinberg theory,[14] $\kappa=1$) can be reduced, by radiative corrections, to the extent required by the above inequality.

Apart from this problem, the vector theory faces serious difficulties when external *electric* fields are present. In fact it was observed by Corben and Schwinger[2] many years ago that the regular solutions for the vector field equation including interaction with a static Coulomb field do not form a complete set. With just minimal coupling ($\kappa=0$), admissible solutions exist only for *equal* values of the total and oribital angular momentum quantum numbers j and l. The wave functions for $j=l+1$ and $j=l-1$ become singular at the origin. The situation is considerably improved when an anomalous magnetic moment of one unit is assumed, but two states ($j=0$, $l=1$ and $j=1$, $l=0$) are still wanting.[15]

Thus even in the simplest (exactly soluble) situations the theory of the charged vector field turns out to be inconsistent. Then a question to be seriously pondered is to what extent the use of standard perturbation-theoretic procedures based on the usual concepts of the vector field are really meaningful in problems involving charged spin-1 particles, for example, in theories involving vector mesons as particles mediating weak or other interactions.

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[5]Here we have taken e to be positive. If e is negative, one has $[\pi_-,\pi_+]=2|e|H$, so that the following identification can be made: $a=(2|e|H)^{-1/2}\pi_-$, $a^\dagger=(2|e|H)^{-1/2}\pi_+$. The effect of this redefinition is to interchange the roles of ϕ_+ and ϕ_- in the subsequent development.

[6]There should be no confusion between this quantum number m and the mass m. Which is meant will be evident from the context.

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[9]This is reminiscent of the case of a Dirac particle in the Coulomb field due to a static charge Ze, where the wave functions become singular at the origin when Z exceeds 137. In the present case, however, the eigenfunctions continue to be regular even when E^2 goes negative.

[10]The method of the present paper can be used with advantage in the case of spin $\frac{1}{2}$ as well as higher spins. These will be considered in detail elsewhere.

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[15]Recently Hung Cheng and Tai Tsun Wu [Phys. Rev. D 5, 2978 (1972)] have considered the scattering of charged vector particles by a Yukawa potential. They conclude that "the Corben-Schwinger problem possesses a finite unique solution if and only if the anomalous magnetic moment is greater than unity."