Unified picture of the algebra of strengths, $SU(6)_W^{\text{currents}}$, and $SU(6)_W^{\text{strong}}$†

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A unitary transformation is introduced which carries the entire set of integrated local current densities generating the group $[U(6)\times U(6)]_{\beta W}^{\text{currents}}$ [with diagonal subgroup $SU(6)_W^{\text{currents}}$] into a newly defined $[U(6)\times U(6)]_{\beta W}^{\text{strong}}$ [with diagonal subgroup $SU(6)_W^{\text{strong}}$] which commutes with the boost in the z direction and with the free-field Hamiltonian. Strong current-hadron scattering amplitudes are constructed and analyzed in the deep Regge region in order to extract the hadron couplings to vector and tensor Regge exchanges. This analysis is based upon a canonical algebra of strong bilocal currents which is not equivalent to the bilocal algebra of the current system.

I. INTRODUCTION

Some years ago Cabibbo, Horwitz, and Ne'eman[1] pointed out that some features of high-energy hadron scattering could be understood by assuming that factorized Regge residues satisfy a $[U(3)\times U(3)]_\beta$ algebra. The original Cabibbo-Horwitz-Ne'eman (CHN) hypothesis was recently modified[2] so as to be consistent with the idea of "two component" duality, and extended to a full $[U(6)\times U(6)]_\beta$ algebra through the inclusion of the strengths associated with pseudoscalar-meson couplings and the "magnetic" couplings to vector mesons. This algebra of strengths was seen to be consistent with the results of graphical duality, such as the absence of exotic resonances in baryon-baryon and baryon-meson scattering.

In spite of the successes of the algebra of Regge residues, it has not been possible to date to establish a link with other algebraic systems acting on hadrons, i.e., the integrated chiral currents[3] $[U(6)]_W^{\text{currents}}$ and the supermultiplet symmetries such as[4] $[U(6)]_W^{\text{strong}}$. A step forward towards a unifying picture between these two algebraic systems has recently been taken by Melosh,[5] following the suggestion of Fritzsch and Gell-Mann[6] that there should be a unitary transformation connecting them. The present work is an attempt to incorporate the algebra of strengths into this unified picture.

In order to achieve this purpose, we construct[7] in Sec. II, a $[U(6)\times U(6)]$ group, which we denote by $[U(6)\times U(6)]_{\beta W}^{\text{strong}} = \mathcal{W}_B$, whose generators are invariant under longitudinal boosts; it reduces at rest to $[U(6)\times U(6)]_\beta$ and its diagonal subgroup is $[U(6)]_W^{\text{strong}}$. Hadrons are supposed to transform irreducibly under the group $\mathcal{W}_B$. This group is obtained by applying a unitary transformation to the subgroup $[U(6)\times U(6)]_{\beta W}^{\text{currents}}$ of the U(12) group generated by an algebra isomorphic to the space integrals of the local currents of the free-quark model.

In Sec. III we discuss the structure of the transformed fields and currents and the effect of the transformation on the whole U(12). In particular we show that the anticommutation relations of the new fields ("strong" fields) do not display microcausality and the new currents ("strong" currents) are not local currents. The group whose generators are the space integrals of strong currents can therefore annihilate the vacuum and the symmetry limit can be realized linearly, i.e., Coleman's theorem[8] does not apply. We show, furthermore, that the transformation leads to 72 distinct "good" operators in the infinite-momentum frame, in contrast with the ordinary U(12) where only 36 operators survive in the infinite-momentum frame.[9]

In Sec. IV we construct some amplitudes, assuming that the transformed currents play a dynamical role in hadron-hadron scattering. We then show that commutators of strong currents lead to scaling in the Bjorken region where, in a realistic model, the commutators are expected to have the same structure as in the free-quark model. The assumption that the leading singularity in the Bjorken region is also leading in the deep Regge region enables us to show that factorized Regge residues close on a $[U(6)\times U(6)]_{\beta W}^{\text{strong}}$ algebra.

It is shown in Appendix C that the algebra of transformed currents does not become equivalent to the canonical algebra of local currents even at infinite momentum. The mechanism for this inequivalence is discussed in detail.

In Sec. V we give a general discussion of our results, with some comments on the meaning of the lack of microcausality in the anticommutation relations of the transformed fields. We review briefly previous attempts to justify—on theoretical grounds—the algebra of strengths. We argue that they are either inadequate or, at best, they lead to a static $[U(6)\times U(6)]_{\beta}^{\text{currents}}$ algebra of factorized Regge residues (which coincides with the static limit of our $[U(6)\times U(6)]_{\beta W}^{\text{strong}}$). We conclude by pointing out that there is still some freedom, within our theoretical framework, for the assumption of the analytic properties of the Regge exchanges. This freedom does not change the algebraic structure of the strengths, but it raises an interesting question, namely, are short-distance effects in any way related to typically long-distance effects like Regge behavior?

II. THE CLASSIFICATION GROUP

Let us consider a subgroup $[U(6)\times U(6)]_{\beta W}^{\text{currents}}$ of the U(12) generated by the space integrals of local currents:

$$\begin{aligned} F(\lambda_a) &= \int d^3x\, q^\dagger(x)\tfrac{1}{2}\lambda_a q(x)\,,\\ F(\beta\sigma_x\lambda_a) &= \int d^3x\, q^\dagger(x)\beta\sigma_x\tfrac{1}{2}\lambda_a q(x)\,,\\ F(\beta\sigma_y\lambda_a) &= \int d^3x\, q^\dagger(x)\beta\sigma_y\tfrac{1}{2}\lambda_a q(x)\,,\\ F(\sigma_z\lambda_a) &= \int d^3x\, q^\dagger(x)\sigma_z\tfrac{1}{2}\lambda_a q(x)\,; \end{aligned} \tag{2.1}$$

$$\begin{aligned} F(\beta\lambda_a) &= \int d^3x\, q^\dagger(x)\beta\tfrac{1}{2}\lambda_a q(x)\,,\\ F(\sigma_x\lambda_a) &= \int d^3x\, q^\dagger(x)\sigma_x\tfrac{1}{2}\lambda_a q(x)\,,\\ F(\sigma_y\lambda_a) &= \int d^3x\, q^\dagger(x)\sigma_y\tfrac{1}{2}\lambda_a q(x)\,,\\ F(\beta\sigma_z\lambda_a) &= \int d^3x\, q^\dagger(x)\beta\sigma_z\tfrac{1}{2}\lambda_a q(x)\,, \end{aligned} \tag{2.2}$$

where $a=0,\ldots,8$. Although the set of operators listed in (2.1) and (2.2) is the same as the $[U(6)\times U(6)]_{\beta}$ used by Dashen and Gell-Mann[10] as a rest classification algebra, the "diagonal" subgroup (2.1) is chosen to be the $SU(6)_W^{\text{currents}}$, and the representation's structure is therefore different. The $SU(6)_W^{\text{currents}}$ commute with α_3, but not with the free-field Hamiltonian[11]

$$H_0=\int d^3x\, q^\dagger(x)(-i\vec{\alpha}\cdot\vec{\nabla}+\beta m)q(x)\,. \tag{2.3}$$

The "nondiagonal" part (2.2) does not commute with α_3 or H_0. Let us construct a unitary transformation G which transforms the entire algebra into a system representing an approximate symmetry of the hadrons, invariant with respect to Lorentz boosts along the z axis. As a first step, we use the transformation $V=e^{iS}$ introduced by Melosh,[5] where

$$S=\tfrac{1}{2}\int d^3x\, q^\dagger(x)\tan^{-1}\left(\frac{\vec{\gamma}_\perp\cdot\vec{\partial}_\perp}{m}\right)q(x)\,. \tag{2.4}$$

It will be useful to record a few properties of this transformation:

$$e^{iS}q(x)e^{-iS}=e^{-is(-i\vec{\partial}_\perp)}q(x)\,, \tag{2.5}$$

where

$$s(-i\vec{\partial}_\perp)=\tfrac{1}{2}\tan^{-1}\left[\frac{i\vec{\gamma}_\perp\cdot(-i\vec{\partial}_\perp)}{m}\right], \tag{2.6}$$

$$e^{-is(-i\vec{\partial}_\perp)}=\frac{\kappa+m+\vec{\gamma}_\perp\cdot(-i\vec{\partial}_\perp)}{[2\kappa(\kappa+m)]^{1/2}}\,, \tag{2.7}$$

and

$$e^{-2is(-i\vec{\partial}_\perp)}=\frac{1}{\kappa}[m+\vec{\gamma}_\perp\cdot(-i\vec{\partial}_\perp)]\,, \tag{2.8}$$

where $\kappa=[(\vec{\gamma}_\perp\cdot\vec{\partial}_\perp)^2+m^2]^{1/2}$.

The transformation (2.4) carries the free-field Hamiltonian (2.3) to the form

$$\begin{aligned} H_M &= V^{-1}H_0V\\ &=\int d^3x\, q^\dagger(x)(-i\alpha_3\partial_3+\beta\kappa)q(x)\,. \end{aligned} \tag{2.9}$$

We now introduce an additional unitary transformation [with SU(3) singlet behavior, and even C and P eigenvalues], which commutes with (2.1) and brings (2.2) to a form which commutes with H_M. Let $V'=e^{iS'}$, where

$$S'=\tfrac{1}{2}\int d^3x\, q^\dagger(x)\tan^{-1}\left(\frac{\gamma^3\partial_3}{\kappa}\right)q(x)\,. \tag{2.10}$$

This transformation has the properties

$$e^{iS'}q(x)e^{-iS'}=e^{-is'(-i\partial_3)}q(x)\,, \tag{2.11}$$

where

$$s'(-i\partial_3)=\tfrac{1}{2}\tan^{-1}\left[\frac{i\gamma^3(-i\partial_3)}{\kappa}\right], \tag{2.12}$$

$$e^{-is'(-i\partial_3)}=\frac{E+\kappa+\gamma^3(-i\partial_3)}{[2E(E+\kappa)]^{1/2}}\,, \tag{2.13}$$

and

$$e^{-2is'(-i\partial_3)}=\frac{1}{E}[\kappa+\gamma^3(-i\partial_3)]\,, \tag{2.14}$$

where $E=[(\vec{\gamma}\cdot\vec{\nabla})^2+m^2]^{1/2}$.

The combined transformation

$$G=VV' \tag{2.15}$$

brings H_0 to the Foldy-Wouthuysen[12] form

$$G^{-1}H_0G=\int d^3x\, q^\dagger(x)\beta E q(x)\,, \tag{2.16}$$

but it is not identical to the transformation used by Foldy and Wouthuysen. V', applied to (2.2) in the sense $V'FV'^{-1}$, brings these operators to the form

$$\begin{aligned}&\int d^3x\, q^\dagger(x)\, B_M \tfrac{1}{2}\lambda_a\, q(x)\,,\\ &\int d^3x\, q^\dagger(x) B_M \beta\sigma_x \tfrac{1}{2}\lambda_a\, q(x)\,,\\ &\int d^3x\, q^\dagger(x) B_M \beta\sigma_y \tfrac{1}{2}\lambda_a\, q(x)\,,\\ &\int d^3x\, q^\dagger(x) B_M \sigma_z \tfrac{1}{2}\lambda_a\, q(x)\,,\end{aligned} \tag{2.17}$$

where $B_M=(-i\alpha_3\partial^3+\beta\kappa)/E$. The charges of (2.1) are invariant under V'.

Since the operators (2.17) and (2.1) commute with H_M, the combined transformation

$$G=VV' \tag{2.18}$$

therefore brings (2.1) and (2.2) to the form

$$W=GFG^{-1}\,, \tag{2.19}$$

which commutes with H_0. Furthermore,

$$Gq(x)G^{-1}=e^{-is'(-i\partial_3)}e^{-is(-i\vec{\partial}_\perp)}\,q(x) \equiv g^{-1}q(x) \tag{2.20}$$

and

$$g\beta g^{-1}=\frac{(-i\vec{\alpha}\cdot\vec{\nabla}+\beta m)}{E}=B, \tag{2.21}$$

while $\beta\sigma_\perp$ and σ_z are left invariant by $e^{-is'(-i\partial_3)}$.

Hence,

$$\begin{aligned}W(\beta\lambda_a)&=GF(\beta\lambda_a)G^{-1}\\ &=\int d^3x q^\dagger(x) B\tfrac{1}{2}\lambda_a\, q(x)\,,\\ W(\sigma_x\lambda_a)&=GF(\sigma_x\lambda_a)G^{-1}\\ &=\int d^3x\, q^\dagger(x) B\, e^{is(-i\vec{\partial}_\perp)}\\ &\qquad\times\beta\sigma_x\tfrac{1}{2}\lambda_a\, e^{-is(-i\vec{\partial}_\perp)}\, q(x)\,,\\ W(\sigma_y\lambda_a)&=GF(\sigma_y\lambda_a)G^{-1}\\ &=\int d^3x\, q^\dagger(x) B\, e^{is(-i\vec{\partial}_\perp)}\\ &\qquad\times\beta\sigma_y\tfrac{1}{2}\lambda_a\, e^{-is(-i\vec{\partial}_\perp)}\, q(x)\,,\\ W(\beta\sigma_z\lambda_a)&=GF(\beta\sigma_z\lambda_a)G^{-1}\\ &=\int d^3x\, q^\dagger(x) B\, e^{is(-i\vec{\partial}_\perp)}\\ &\qquad\times\sigma_z\tfrac{1}{2}\lambda_a\, e^{-is(-i\vec{\partial}_\perp)} q(x)\,.\end{aligned} \tag{2.22}$$

The transformed $SU(6)_W$ charges $W(\lambda_a)$, $W(\beta\sigma_x\lambda_a)$, $W(\beta\sigma_y\lambda_a)$, $W(\sigma_z\lambda_a)$ have the same form as (2.22), but without the operator B (these are explicitly given by Melosh[5]).

Defining the quark fields in the usual way,

$$q(x)=\int\frac{d^3p}{(2\pi)^{3/2}}\left(\frac{m}{E}\right)^{1/2}\sum_s[b(p,s)u(p,s)e^{-ip\cdot x} + d^\dagger(p,s)v(p,s)e^{ip\cdot x}]\,, \tag{2.23}$$

where s refers to unitary spin as well as spin, $b(p,s)$ annihilates a quark, and $d^\dagger(p,s)$ creates an antiquark, one finds that

$$\begin{aligned}W(\beta\lambda_0)&=GF(\beta\lambda_0)G^{-1}\\ &=\frac{1}{\sqrt{6}}\sum_s\int d^3p[b^\dagger(p,s)b(p,s) + d^\dagger(p,s)d(p,s)]\end{aligned} \tag{2.24}$$

explicitly commutes with the boost operator M^{03}. As there are indications that this property may hold in a realistic theory, we shall assume it to hold true hereafter. All of (2.22) [along with the transformed charges of (2.11)] is therefore boost-invariant, and the group

$$\mathcal{W}_B=[U(6)\times U(6)]^{\text{strong}}_{\beta W} \tag{2.25}$$

generated by the charges (2.19) therefore provides a boost-invariant classification of hadron states.

For states for which $\vec{p}=0$ (for all quarks contained in the hadron) the group $\mathcal{W}_B$ reduces essentially to the $[U(6)\times U(6)]^{\text{strong}}_{\beta}$ used by CHN for the classification of states at rest, and only in this case can it be reduced over the static $SU(6)^{\text{strong}}$ as well. We shall assume in what follows that the physical hadron states are classified according to irreducible representations of $\mathcal{W}_B$.

III. STRUCTURE OF THE TRANSFORMED FIELDS AND STRONG CURRENTS

Carrying out the operations indicated in (2.20), one finds that

$$\begin{aligned}\hat{q}(x)&=Gq(x)G^{-1}\\ &=(E+i\beta\partial^0)\frac{E+\kappa+i\gamma^3\partial_3}{[2E(E+\kappa)]^{1/2}[2\kappa(\kappa+m)]^{1/2}}q(x)\,.\end{aligned} \tag{3.1}$$

Since $q(x)$ satisfies the Klein-Gordon equation, it follows that

$$(E-i\beta\partial^0)\hat{q}(x)=0$$

or

$$i\partial^0\hat{q}(x)=\beta E\hat{q}(x)\,, \tag{3.2}$$

i.e., the "strong fields" satisfy the Foldy-Wouthuysen equation.

In terms of the momentum space representation (2.23), the relation (3.1) is

$$\hat{q}(x)=\hat{q}_+(x)+\hat{q}_-(x)\,, \tag{3.3}$$

$$\hat{q}_+(x)=\left(\frac{1+\beta}{2}\right)\int\frac{d^3p}{(2\pi)^{3/2}}\left(\frac{m}{\kappa(\kappa+m)}\right)^{1/2}\frac{1}{(E+\kappa)^{1/2}}(E+\kappa-\gamma^3p^3)\sum_s b(p,s)u(p,s)e^{-ip\cdot x}\,, \tag{3.4}$$

$$\hat{q}_-(x)=\left(\frac{1-\beta}{2}\right)\int\frac{d^3p}{(2\pi)^{3/2}}\left(\frac{m}{\kappa(\kappa+m)}\right)^{1/2}\frac{1}{(E+\kappa)^{1/2}}(E+\kappa+\gamma^3p^3)\sum_s d^\dagger(p,s)v(p,s)e^{+ip\cdot x}\,. \tag{3.5}$$

The decomposition (3.3) has the following properties:

(a) For Γ in the Dirac×SU(3) algebra of $[\mathrm{U}(6)\times\mathrm{U}(6)]_\beta$,

$$\hat{q}(x)^\dagger\Gamma\hat{q}(x)=\hat{q}_+(x)^\dagger\Gamma\hat{q}_+(x)+\hat{q}_-(x)^\dagger\Gamma\hat{q}_-(x); \tag{3.6}$$

these densities contain no pair terms, and decompose into currents carried by quarks and antiquarks separately.

(b) For Γ_5 in the part of Dirac×SU(3) algebra of U(12) outside of $[\mathrm{U}(6)\times\mathrm{U}(6)]_\beta$,

$$\hat{q}(x)^\dagger\Gamma_5\hat{q}(x)=\hat{q}_+(x)^\dagger\Gamma_5\hat{q}_-(x)+\hat{q}_-(x)^\dagger\Gamma_5\hat{q}_+(x); \tag{3.7}$$

these densities contain only pair terms, and create and annihilate quark-antiquark pairs.

In Appendix A it is shown that

$$\{\hat{q}(x),\hat{q}(x')^\dagger\}=i(\beta E+i\partial_0)\Delta(x-x')$$

$$=i\beta\partial_0\Delta_1(x-x')-\partial_0\Delta(x-x') \tag{3.8}$$

$$\{\hat{q}_+(x),\hat{q}_+(x')^\dagger\}=(1+\beta)i\partial_0\Delta_+(x-x')\,, \tag{3.9}$$

$$\{\hat{q}_-(x),\hat{q}_-(x')^\dagger\}=-(1-\beta)i\partial_0\Delta_-(x-x')\,. \tag{3.10}$$

Although $\Delta(x-x')$ is causal (it is zero outside of the light cone), the operator E is nonlocal, and the noncausal Δ_1 function enters into the anticommutator. The noncausal nature of the anticommutation relations is due to the nonlocal spreading effect of the transformation G, and is characteristic of Foldy-Wouthuysen-type fields (we discuss this point further in Sec. V). We shall call densities such as those shown in Eqs. (3.6) and (3.7) *quasilocal* to distinguish them from densities constructed of local covariant fields. At equal times, however, we recover the canonical anticommutation relations

$$\{\hat{q}(x),\hat{q}(x')\}|_{x^0=x'^0}=\delta^3(\vec{x}-\vec{x}')\,, \tag{3.11}$$

$$\{\hat{q}_+(x),\hat{q}_+(x')^\dagger\}|_{x^0=x'^0}=\frac{1+\beta}{2}\delta^3(\vec{x}-\vec{x}')\,, \tag{3.12}$$

$$\{\hat{q}_-(x),\hat{q}_-(x')^\dagger\}|_{x^0=x'^0}=\frac{1-\beta}{2}\delta^3(\vec{x}-\vec{x}')\,. \tag{3.13}$$

From property (a), we see that $[\mathrm{U}(6)\times\mathrm{U}(6)]_{\beta W}^{\mathrm{strong}}$ current densities form a representation of charge conjugation,[13] and may be classified according to $C=(+)$ or $(-)$, with 36 operators in each class. Since the transformation G is invariant under charge conjugation (and parity), and all of the charges become "good" under this transformation, we obtain 72 densities and charges even in the limit for states with $p_z\to\infty$.[14]

The tensor properties of the strong currents under operations of the proper Lorentz group are severely restricted by the transformation G (space-time translation symmetry is preserved since G depends on t only through the fields). We shall show, in what follows, that *up to surface terms* all of the currents of $[\mathrm{U}(6)\times\mathrm{U}(6)]_{\beta W}^{\mathrm{strong}}$ transform under longitudinal boosts like the fourth component of a four vector (or third component of an axial vector), and the remainder of the transformed U(12) densities like (pseudo-) scalars (the connection with spin is, however, not so direct). Matrix elements of the strong densities between states of equal momentum (for which the surface terms do not contribute) therefore display these simple tensor properties. It is also true that commutators of the quasilocal current densities, according to Eqs. (3.8)–(3.13), also have these properties for the Dirac×SU(3) content up to surface terms, and we may therefore construct amplitudes of the type considered in deep-inelastic lepton-hadron scattering.

To prove these assertions, we remark that up to surface terms, the transformation G applied to any density yields the same result as found in the integrands of the corresponding charges. For the SU(3) currents $[(\lambda_a)$ in $\mathrm{SU}(6)_W]$, it is clear that one obtains the fourth component of a vector. The remaining $\mathrm{SU}(6)_W$ densities contain Dirac×SU(3) matrices that commute with α_3, so a Lorentz boost in the z direction induces the same mixing with α_3 as found in the boost of a third or fourth component. This property ensures the boost invariance of the space integral of these densities. As one can see from (2.22), the remaining transformed densities of $(\mathrm{U}(6)\times\mathrm{U}(6)]_{\beta W}^{\mathrm{strong}}$ are identical to those of $\mathrm{SU}(6)_W^{\mathrm{strong}}$ except for the Lorentz-invariant factor B [this factor may be inserted by commutation of any of the densities of $SU(6)_W^{strong}$ with the first of (2.22) at equal times]. We therefore conclude that all of the densities of $\mathcal{W}_B$ transform like third or fourth components of a vector under z boosts.

For the remaining densities of U(12), we remark that $\gamma^5\beta$ commutes with the transformations g defined by (2.20). Hence

$$\gamma^5\beta = g\gamma^5\beta g^{-1} = (g\gamma^5 g^{-1})(g\beta g^{-1}) = g\gamma^5 g^{-1}\cdot B,$$

or, due to the fact that $B^2 = 1$,

$$g\gamma^5 g^{-1} = \gamma^5\beta B. \tag{3.14}$$

Although B is invariant, the extra factor of β reduces these densities to (pseudo-) scalars. This completes the proof of the first part of our assertion.

For the second part, consider the commutator

$$\begin{aligned}[\hat{q}(x)^\dagger\Gamma_1\hat{q}(x), \hat{q}(x')^\dagger\Gamma_2\hat{q}(x')] &= i[\hat{q}(x)^\dagger\Gamma_1\beta\,\Gamma_2\hat{q}(x') \\ &\quad + \hat{q}(x')^\dagger\Gamma_2\beta\,\Gamma_1\hat{q}(x)]\,\partial_0\Delta_1(x-x') \\ &\quad - [\hat{q}(x)^\dagger\Gamma_1\Gamma_2\hat{q}(x') - \hat{q}(x')^\dagger\Gamma_2\Gamma_1\hat{q}(x)]\,\partial_0\Delta(x-x'),\end{aligned} \tag{3.15}$$

where we have used $\partial_0'\Delta_1(x-x') = -\partial_0\Delta_1(x-x')$ and $\partial_0'\Delta(x'-x) = \partial_0\Delta(x-x')$, and the Γ_i are any Dirac $\times SU(3)$ matrices in U(12). We again remark that the presence of the Δ_1 term reflects the nonlocality induced by the transformation G; this contribution vanishes at equal times.

It is clear that the effective Dirac $\times$ SU(3) part of the strong bilocals appearing in (3.15) are the same as those of the corresponding quasilocal densities, where commutators with the momentum operator play the role of surface terms. This completes the proof of the second part of our assertion.

For the set of currents for which Γ_1 and Γ_2 do not contain γ^5, commutators of the form (3.15) "close" on a system of $8\times9\times2\times2 = 288$ types of operators (the number of elements in the Dirac algebra without γ^5 times the number of unitary spin matrices combined through f or d, and carried by quark or antiquark fields).

To tabulate the tensor properties of the strong densities, we restrict ourselves to the Melosh representation (where V' is applied to the densities, but not V), since V preserves the Lorentz properties, with respect to z boosts, up to surface terms. In Table I we list the Dirac$\times$SU(3) matrices, as well as the $CP = ++$ operator

TABLE I. List of current density classes before and after application of the transformation V' [cf., Eq. (2.10)]. The tensors $0, 3, 0', 3', S, P$ refer to zero and three components of a four-vector, zero and three components of an axial-vector, scalar, and pseudoscalar, respectively, under z boosts and parity reflection.

J	Class	Tensor	C	P	J_M	Class	Tensor
λ_a	good	$0(V^0)$	$-$	$+$	λ_a	good	0
$\sigma_\perp\lambda_a$	bad	$S(A_\perp)$	$+$	$+$	$B_M\beta\sigma_\perp\lambda_a$	good	$3'$
$\sigma_3\lambda_a$	good	$3'(A_3)$	$+$	$+$	$\sigma_3\lambda_a$	good	$3'$
$\beta\lambda_a$	bad	$S(S)$	$+$	$+$	$B_M\lambda_a$	good	0
$\beta\sigma_\perp\lambda_a$	good	$3'(T^{13,23})$	$-$	$+$	$\beta\sigma_\perp\lambda_a$	good	$3'$
$\beta\sigma_3\lambda_a$	bad	$S(T^{12})$	$-$	$+$	$B_M\sigma_3\lambda_a$	good	$3'$
$\gamma^5\lambda_a$	good	$0'(A^0)$	$+$	$-$	$B_M\beta\gamma^5\lambda_a$	bad	P
$\gamma^5\sigma_\perp\lambda_a$	bad	$P(V_\perp)$	$-$	$-$	$\gamma^5\sigma_\perp\lambda_a$	bad	P
$\gamma^5\sigma_3\lambda_a$	good	$3(V_3)$	$-$	$-$	$B_M\beta\gamma^5\sigma_3\lambda_a$	bad	P
$\gamma^5\beta\lambda_a$	bad	$P(P)$	$+$	$-$	$\beta\gamma^5\lambda_a$	bad	P
$\gamma^5\beta\sigma_\perp\lambda_a$	good	$0'(T^{0\perp})$	$-$	$-$	$B_M\gamma^5\sigma_\perp\lambda_a$	bad	P
$\gamma^5\beta\sigma_3\lambda_a$	bad	$P(T^{03})$	$-$	$-$	$\beta\gamma^5\sigma_3\lambda_a$	bad	P

$$B_M = (-i\alpha^3\partial_3 + \beta\kappa)/E, \tag{3.16}$$

which occur in the densities between $q^\dagger(x)$ and $q(x)$ when surface terms are ignored, for the original U(12) densities (J) and for the transformed system (J_M).

Since B_M is invariant, we may classify[15] the densities according to whether or not the remaining factors commute with α_3. The first six lines of Table I contain the "good" densities J_M of $[U(6)\times SU(6)]_{\beta W}^{strong}$, and we note that these are just those of $[SU(6)]_W^{strong}$ with and without the factor B_M. Adding and subtracting corresponding operators from the first column of the first and second set of three lines, we obtain the densities J of the two subgroups of $[U(6)\times U(6)]_\beta$. These are not charge conjugation eigenstates; transformed by G they become

$$\hat{q}(x)^\dagger\frac{1\pm\beta}{2}\Gamma\hat{q}(x) = \hat{q}_\pm(x)^\dagger\Gamma\hat{q}_\pm(x). \tag{3.17}$$

The plus subscript corresponds to a "quark" current, and the minus to an "antiquark" current. They are essentially interchanged by charge conjugation; sums and differences of these two sets of 36 operators therefore have definite charge conjugation, and, as pointed out previously, we obtain 72 distinct operators even in the infinite-momentum frame.

For the "bad" densities, we remark that

$$\hat{q}(x)^\dagger \Gamma_5 \hat{q}(x) = \hat{q}_+(x)^\dagger \Gamma_5 \hat{q}_-(x) + \hat{q}_-(x)^\dagger \Gamma_5 \hat{q}_+(x),$$
$$\hat{q}(x)^\dagger \beta \Gamma_5 \hat{q}(x) = \hat{q}_+(x)^\dagger \Gamma_5 \hat{q}_-(x) - \hat{q}_-(x)^\dagger \Gamma_5 \hat{q}_+(x), \quad (3.18)$$

so that these densities also have definite charge conjugation properties and create or annihilate quark-antiquark pairs. Of the 72 distinct operators obtained by taking sums and differences of these, 36 are Hermitian conjugates of the others.

IV. THE ALGEBRA OF REGGE RESIDUES

In order to study the algebraic properties of the couplings of Regge exchanges to hadrons, we shall construct spin-averaged absorptive parts of the form

$$M_{ab} = \int e^{iq\cdot x} \langle N(p) | [\hat{J}_a(x), \hat{J}_b(0)] | N(p) \rangle d^4x, \quad (4.1)$$

where $\hat{J}_a(x), \hat{J}_b(0)$ are "strong currents," and the hadron states $|N(p)\rangle$ are in small representations of $[U(6)\times U(6)]^{\text{strong}}_{\beta W}$. Since we shall extract the algebraic properties of the commutator on the light cone and go to the deep Regge region to determine the Regge couplings, it is necessary to study the light-cone behavior of certain current commutators. Let us consider the vector-vector commutator

$$\begin{aligned}[\hat{v}_a^0(x), \hat{v}_b^0(0)] = &-\tfrac{1}{2}\{[\hat{v}_c(x,0) - \hat{v}_c(0,x)] d_{abc} \\ &+ i[\hat{v}_c(x,0) + \hat{v}_c(0,x)] f_{abc}\}\partial_0 \Delta(x) \\ &+ \tfrac{1}{2} i \{[\hat{s}_c(x,0) + \hat{s}_c(0,x)] d_{abc} \\ &+ i[\hat{s}_c(x,0) - \hat{s}_c(0,x)] f_{abc}\}\partial_0 \Delta_1(x),\end{aligned} \quad (4.2)$$

the vector-"scalar" commutator[16]

$$\begin{aligned}[\hat{v}_a^0(x), \hat{s}_b^0(0)] = &-\tfrac{1}{2}\{[\hat{s}_c(x,0) - \hat{s}_c(0,x)] d_{abc} \\ &+ i[\hat{s}_c(x,0) + \hat{s}_c(0,x)] f_{abc}\}\partial_0 \Delta(x) \\ &+ \tfrac{1}{2} i \{[\hat{v}_c(x,0) + \hat{v}_c(0,x)] d_{abc} \\ &+ i[\hat{v}_c(x,0) - \hat{v}_c(0,x)] f_{abc}\}\partial_0 \Delta_1(x),\end{aligned} \quad (4.3)$$

and the "axial-vector–pseudoscalar" commutator

$$\begin{aligned}[\hat{a}_a^0(x), \hat{p}_b(0)] = &\tfrac{1}{2} i \{[\hat{s}_c(x,0) + \hat{s}_c(0,x)] d_{abc} \\ &+ i[\hat{s}_c(x,0) - \hat{s}_c(0,x)] f_{abc}\}\partial_0 \Delta(x) \\ &- \tfrac{1}{2}\{[\hat{v}_c(x,0) - \hat{v}_c(0,x)] d_{abc} \\ &+ i[\hat{v}_c(x,0) + \hat{v}_c(0,x)] f_{abc}\}\partial_0 \Delta_1(x),\end{aligned} \quad (4.4)$$

where, in our model,

$$\begin{aligned}\hat{v}_c^0(x) &= \hat{q}(x)^\dagger \frac{\lambda_c}{2} \hat{q}(x), \\ \hat{s}_c^0(x) &= \hat{q}(x)^\dagger \beta \frac{\lambda_c}{2} \hat{q}(x), \\ \hat{a}_c^0(x) &= \hat{q}(x)^\dagger \gamma^5 \frac{\lambda_c}{2} \hat{q}(x), \\ \hat{p}_c(x) &= i\hat{q}(x)^\dagger \beta\gamma^5 \frac{\lambda_c}{2} \hat{q}(x), \\ \hat{v}_c(x,0) &= \hat{q}(x)^\dagger \frac{\lambda_c}{2} \hat{q}(0), \\ \hat{s}_c(x,0) &= \hat{q}(x)^\dagger \beta \frac{\lambda_c}{2} \hat{q}(0).\end{aligned} \quad (4.5)$$

In the Bjorken limit of (4.1), the commutators (4.2), (4.3), and (4.4) contribute to the integral only in the neighborhood of the light cone. It is shown in Appendix B that in the Bjorken limit both $\Delta(x)$ and $\Delta_1(x)$ contributions to the amplitude scale in the same way. The model expressions (4.2) and (4.3) therefore provide a representation of the general form of a light-cone expansion (with singularities characteristic of Bjorken scaling) in the scaling region, which we would expect to find in a realistic model.

Let us analyze the diagonal (in momentum) matrix elements of (4.2), (4.3), and (4.4). The symmetric and antisymmetric combinations

$$\begin{aligned}\hat{v}_a^{(\pm)}(x) &= \tfrac{1}{2}[\hat{v}_a(x,0) \pm \hat{v}_a(0,x)], \\ \hat{s}_a^{(\pm)}(x) &= \tfrac{1}{2}[\hat{s}_a(x,0) \pm \hat{s}_a(0,x)]\end{aligned} \quad (4.6)$$

are elements of two distinct tensor operators belonging to the adjoint representation of the group W_B (including charge conjugation). For the first of these,

$$\hat{v}_a^{(+)}(x) \sim W(\lambda_a), \qquad \hat{s}_a^{(+)}(x) \sim W(\beta\lambda_a), \quad (4.7)$$

and for the second,

$$\hat{v}_a^{(-)}(x) \sim W(\beta\lambda_a), \qquad \hat{s}_a^{(-)}(x) \sim W(\lambda_a), \quad (4.8)$$

where ~ means "transforms like." Applying the Wigner-Eckart theorem to these tensor operators, (we explicitly assume that, in the symmetry limit in which we are working, the singlet components have the same reduced matrix elements)

$$\begin{aligned}\langle N(p) | \hat{v}_a^{(+)}(x) | N(p)\rangle &= C_{v,a}^{NN} f^{(+)}(x\cdot p) p^0, \\ \langle N(p) | \hat{s}_a^{(+)}(x) | N(p)\rangle &= C_{s,a}^{NN} f^{(+)}(x\cdot p) p^0,\end{aligned} \quad (4.9)$$

$$\begin{aligned}\langle N(p) | \hat{v}_a^{(-)}(x) | N(p)\rangle &= iC_{s,a}^{NN} f^{(-)}(x\cdot p) p^0, \\ \langle N(p) | \hat{s}_a^{(-)}(x) | N(p)\rangle &= iC_{v,a}^{NN} f^{(-)}(x\cdot p) p^0,\end{aligned} \quad (4.10)$$

where $f^{(\pm)}(x\cdot p)$ are even and odd real functions, respectively, and the factor p^0 corresponds to the transformation property of the bilocals under z boosts (the additional x^0 term is less singular on the light cone). The coefficients $C_{s,a}^{NN}$ and $C_{v,a}^{NN}$ are Clebsch-Gordan coefficients normalized according to

$$2p^0C^{NN}_{v,a}\,\delta^3(\vec{p}-\vec{p}\,')=\langle N(p)|W(\lambda_a)|N(p)\rangle\ ,\qquad 2p^0C^{NN}_{s,a}\,\delta^3(\vec{p}-\vec{p}\,')=\langle N(p)|W(\beta\lambda_a)|N(p')\rangle\ . \tag{4.11}$$

In the Bjorken region, the diagonal (in momentum) matrix elements of (4.2), (4.3), and (4.4) can therefore be written as

$$\begin{aligned}\langle N(p)|[\hat{v}^0_a(x),\hat{v}^0_b(0)]|N(p)\rangle &= -i[f^{(-)}(x\cdot p)d_{abc}C^{NN}_{s,c} + f^{(+)}(x\cdot p)f_{abc}C^{NN}_{v,c}]p^0\partial_0\Delta(x)\\ &\quad + i[f^{(+)}(x\cdot p)d_{abc}C^{NN}_{s,c} - f^{(-)}(x\cdot p)f_{abc}C^{NN}_{v,c}]p^0\partial_0\Delta_1(x)\,,\end{aligned} \tag{4.12}$$

$$\begin{aligned}\langle N(p)|[\hat{v}^0_a(x),\hat{s}^0_b(0)]|N(p)\rangle &= -i[f^{(+)}(x\cdot p)f_{abc}C^{NN}_{s,c} + f^{(-)}(x\cdot p)d_{abc}C^{NN}_{v,c}]p^0\partial_0\Delta(x)\\ &\quad + i[f^{(+)}(x\cdot p)d_{abc}C^{NN}_{v,c} - f^{(-)}(x\cdot p)f_{abc}C^{NN}_{s,c}]p^0\partial_0\Delta_1(x)\,,\end{aligned} \tag{4.13}$$

and

$$\begin{aligned}\langle N(p)|[\hat{a}^0_a(x),\hat{p}_b(0)]|N(p)\rangle &= i[f^{(+)}(x\cdot p)d_{abc}C^{NN}_{s,c} - f^{(-)}(x\cdot p)f_{abc}C^{NN}_{v,c}]p^0\partial_0\Delta(x)\\ &\quad - i[f^{(-)}(x\cdot p)d_{abc}C^{NN}_{s,c} + f^{(+)}(x\cdot p)f_{abc}C^{NN}_{v,c}]p^0\partial_0\Delta_1(x)\,.\end{aligned} \tag{4.14}$$

As shown in Appendix B, the $i\partial_0\Delta$ and $\partial_0\Delta_1$ contributions scale in the same way; using Eqs. (B7) and (B8), we obtain

$$(M^{VV}_{ab})_{\text{spin zero}} \underset{\text{Bj}}{\rightarrow} \pi[F^{(+)}(1/\omega)+iF^{(-)}(1/\omega)] \times(d_{abc}C^{NN}_{s,c}+if_{abc}C^{NN}_{v,c})\,, \tag{4.15}$$

$$(M^{VS}_{ab})_{\text{spin zero}} \underset{\text{Bj}}{\rightarrow} \pi[F^{(+)}(1/\omega)+iF^{(-)}(1/\omega)] \times(d_{abc}C^{NN}_{v,c}+if_{abc}C^{NN}_{s,c})\,, \tag{4.16}$$

$$(M^{AP}_{ab})_{\text{spin zero}} \underset{\text{Bj}}{\rightarrow} -\pi i[F^{(+)}(1/\omega)-iF^{(-)}(1/\omega)] \times(d_{abc}C^{NN}_{s,c}-if_{abc}C^{NN}_{v,c})\,, \tag{4.17}$$

where

$$f^{(\pm)}(x\cdot p)=\int e^{i\xi(x\cdot p)}F^{(\pm)}(\xi)\,d\xi. \tag{4.18}$$

In the large-ω (deep Regge) limit, at least part of the absorptive amplitudes calculated above in the Bjorken region can be expected to show Regge behavior.[17] The asymptotic form of these amplitudes is determined primarily by the long-distance behavior of $f^{(\pm)}(x\cdot p)$, i.e., large x_- on the null plane $x_+=0$. At small x_-, $f^{(+)}(x\cdot p)$ takes on the value of a universal charge; $f^{(-)}(x\cdot p)$, however, vanishes at small distances. The shape of the function $f^{(+)}(x\cdot p)$ could be very different for meson and baryon states, and its universal value at $x=0$ could have no bearing on the universality of Regge couplings. The scale of $f^{(-)}(x\cdot p)$, moreover, cannot be determined by its value at $x=0$. We have shown elsewhere,[18] however, that the observed universality of Regge couplings may follow from the canonical algebra of strong bilocals. In what follows, we restrict ourselves primarily to a discussion of couplings within multiplets.

It is possible, for example, that $F^{(-)}(1/\omega)$ does not take a factorized Regge asymptotic form; this possibility would permit a diffractive part in all of the three amplitudes. Picking out the negative charge conjugation exchange from Eq. (4.15) and the positive from Eq. (4.17), one obtains in this case the result given in a preliminary communication.[7] In Appendix C, it is shown that the relation between the $p_z\to\infty$ limit of the algebra of ordinary local currents and the corresponding limit of the algebra of strong currents is not that of a direct unitary equivalence, since V' does not have a well-defined operator limit; we did not find, therefore, the complete structure of the bilocal coupling operators in this earlier work. Our algebraic conclusions remain correct, however, since they correspond to a special case of the present analysis.

Another possibility is that $F^{(+)}(1/\omega)+iF^{(-)}(1/\omega)$ provides a factorized Regge coupling, but that $F^{(+)}(1/\omega)-iF^{(-)}(1/\omega)$ is diffractive; in fact, an amplitude of the type Eq. (4.17) might be expected to have a strong diffractive component. It is also possible, of course, that both $F^{(+)}(1/\omega)\pm F^{(-)}(1/\omega)$ take on factorized Regge form.

Let us assume that $F^{(+)}(1/\omega)+iF^{(-)}(1/\omega)$ takes on the factorized Regge asymptotic form

$$F^{(+)}(1/\omega)+iF^{(-)}(1/\omega)=\sum_n a_n\omega^n \rightarrow \beta\omega^\alpha\,, \tag{4.19}$$

where[19]

$$\beta=\beta_+^{\text{str. curr.}}\beta^{\text{hadr.}}\ . \tag{4.20}$$

It follows from Eqs. (4.15) and (4.16) that the couplings of vector and vector-scalar strong currents to Regge exchanges are given by

$$\beta_+^{\text{str. curr.}}f_{abc}\,,\quad C=(-)\qquad \beta_+^{\text{str. curr.}}d_{abc}\,,\quad C=(+) \tag{4.21}$$

and the hadron couplings (including the singlet) by

$$\beta^{\text{hadr.}}C^{NN}_{v,c}\,,\quad C=(-)\qquad \beta^{\text{hadr.}}C^{NN}_{s,c}\,,\quad C=(+)\,. \tag{4.22}$$

This result implies exchange degeneracy for hadron-hadron amplitudes.

If $F^{(+)}$ factorizes and $F^{(-)}$ does not, we would reach the same conclusions for the hadron Regge couplings.

The connection with exchanged spin is obtained directly by charge conjugation, which determines the symmetry of the amplitude under u-channel crossing, and therefore the signature factor in each case.[20] From this and Eq. (4.11), it follows that the W charges,

$$\begin{aligned} \langle N(p)\,|\,W(\lambda_a)\,|\,N(p)\rangle &= 2p^0\delta^3(\vec{p}-\vec{p}')\gamma^{NN}_{v,a}\,, \\ \langle N(p)\,|\,W(\beta\lambda_a)\,|\,N(p)\rangle &= 2p^0\delta^3(\vec{p}-\vec{p}')\gamma^{NN}_{s,a}\,, \end{aligned} \tag{4.23}$$

are proportional to the couplings of hadrons to vector and tensor trajectories, respectively, as originally conjectured in CHN.

V. DISCUSSION

We have shown that the one-particle coupling of hadrons to Regge exchanges in the forward direction is given by the algebra of $[\mathrm{U}(6)\times\mathrm{U}(6)]^{\mathrm{strong}}_{\beta W}$, an algebra constructed around $\mathrm{SU}(6)^{\mathrm{strong}}_W$ and acting on small ("constituent") representations. If the strong bilocal operators are sensitive only to the representation structure of the constituent quarks of the hadron states, yielding as reduced matrix elements functions with similar asymptotic behavior ($\xi\to 0$), we would find a coupling scheme which is universal for mesons and baryons in addition to the symmetry that we have derived for multiplets. This result, in fact, can be shown to follow from the canonical algebra of strong bilocals.[18] The Fourier transforms of the matrix elements of the bilocals $\hat{v}^{(+)}_a(x)$ and $\hat{s}^{(+)}_a(x)$ as $\xi\to\infty$ correspond to universal charges; universal Regge couplings therefore suggest a dual relation between large- and small-ξ behavior.

If $F^{(+)}(\xi)+iF^{(-)}(\xi)$ factorizes [as assumed in Eq. (4.20)], the elastic amplitude Eq. (4.15) in the deep Regge region would be completely described by simple linear couplings; there would seem to be, in this case, no room for a diffractive part.[21] On the other hand, for example, if $F^{(+)}(\xi)$ factorizes, but $F^{(-)}(\xi)$ does not, the behavior of $F^{(-)}(\xi)$ would be characteristic of cut contributions, which would not be expected to have linear universal couplings.[22]

It is shown in Appendix C that the unitary operator V' does not have a well-defined limit, but its action on the fields *is* asymptotically well defined. Since the resulting anticommutation relations for the fields defined by Eq. (3.1) are not asymptotically the same as those of the local free-quark fields, the limiting transformation is not canonical, and the algebras of local currents and quasilocal strong currents at infinite momentum are therefore not equivalent. The extent of a residual similarity of the algebras in the deep Regge region is described in this appendix in the framework of the free-field model.

The anticommutation relations (3.8)–(3.10) of the strong fields do not exhibit microcausality (even though the equal-time anticommutation relations are the usual ones). It was not our object to construct a consistent relativistic field theory with these operators, but only to use them as a calculational aid in discussing the structure of a theory with strong currents. However, it is interesting to recall that the variable $\vec{x}$ appearing as an argument in a covariant wave function does not correspond in a simple way to the position of a particle. The position operator as shown, for example, by Newton and Wigner,[23] is highly nonlocal in such a representation. In the Foldy-Wouthuysen representation[12] [closely related to (3.1)], on the other hand, the position operator has a simple local representation $[i(\partial/\partial\vec{p})]$. The coordinate $\vec{x}$ appearing in the Foldy-Wouthuysen wave function does correspond to the position of the particle in a way analogous to that of the Schrödinger wave function in the nonrelativistic theory. The restriction of interaction to the point described by a position eigenvalue would be unrealistic from the viewpoint of a local covariant field theory, and it is therefore not surprising that the fields of a second quantized Foldy-Wouthuysen–type description do not satisfy local causal anticommutation relations. Fields transformed by the Melosh operator V have similar properties (in the transverse directions). It seems that this loss of microcausality is necessary in order to consistently discuss any model for an approximate symmetry such as $\mathrm{SU}(6)^{\mathrm{strong}}_W$ (Ref. 8).

Finally, we wish to discuss some of the previous attempts to relate the pattern of observed high-energy Regge couplings to the algebra of currents. Testa[24] and Cabibbo and Testa[25] used light-cone techniques in the current system; their amplitudes were also related to the matrix elements of bilocal operators, and could therefore be assumed to have Regge asymptotic behavior for $\omega\to\infty$. The couplings were assumed to be approximately determined by the first (local) term in the expansion of the bilocals, i.e., the matrix elements of charges that are the integrals of local currents. This approximation is not valid since Regge asymptotic behavior must depend strongly on long distance ($x_-\to\infty$) properties of the bilocal operator, and not just on its short distance (local) limit. A similar result would have been obtained from an application of the Wigner-Eckart theorem, assuming the states to be classified according to a group generated by charges that are the integrals of local currents. Aside from the difficulties raised by Coleman's theorem,[8] these couplings (at least in the case of Ref. 25) would refer to a classification scheme applicable to hadrons in which all quarks "contained" would have to be at rest. This configuration would not be consistent with the type of state required by Melosh[5] in his proposed resolution of the G_A/G_V problem.

In our treatment, the physical particle states are classified according to a group generated by integrals of densities that are not local. Regge asymptotic behavior is extracted from the transform of a quasibilocal operator including its long distance ($x_- \to \infty$) properties, and the application of the Wigner-Eckart theorem leads to the result that the couplings are given by the matrix elements of charges that are not the integrals of local currents. Two examples of distinct phenomenological consequences are as follows:

(1) If, as concluded by Cabibbo and Testa,[25] it is the local currents that generate the $[U(3)\times U(3)]_\beta$ symmetry of the Regge couplings, then the Regge region of deep-inelastic neutrino scattering should display the tensor property of the couplings. In our view, the local currents do not transform irreducibly under the classification group, and hence the ratio of isovector-odd charge conjugation to isovector-even charge conjugation Regge coupling strengths in deep-inelastic neutrino scattering, for example, would not be expected to be consistent with the ratio of Clebsch-Gordan coefficients of $[U(3)\times U(3)]_\beta$.

(2) Cabibbo and Testa[25] pointed out that the identification of the matrix elements of the local densities u_0 and u_8 with the couplings to tensor trajectories implies that they are of the same order of magnitude. This leads, in some models,[26] to disagreement with extrapolated values of the σ term in πN scattering. There is no reason for these matrix elements to have the same order of magnitude in our theory, since these local densities do not provide the couplings to tensor trajectories. Moreover, the identification of these densities with the symmetry-generating charges would lead to the absence of a D-type contribution to the baryon masses in these models in disagreement with the mass spectrum.

Estimates of these effects can be calculated in the framework of our model; this will be done elsewhere.

Kislinger and Young[27] extracted expected asymptotic behavior from the matrix elements of bilocal operators and discussed the transformation properties of the remaining Regge residues under the Dashen–Gell-Mann form-factor algebra.[28] Their approach did not require a pole-dominance argument [they referred, however, only to local SU(3) currents]. It was possible to show in this framework that the commutator of vector exchange residues does not generate exotic couplings, but the structure of the rest of the algebra and a connection with $[U(6)\times U(6)]_\beta$ was not accessible. An effective completion of their program was carried out for the current system by Reddy.[29]

APPENDIX A: ANTICOMMUTATION RELATIONS OF THE "STRONG FIELDS"

From Eqs. (3.4) and (3.5),

$$\{\hat{q}_+(x), \hat{q}_+(x')^\dagger\} = \left(\frac{1+\beta}{2}\right)\int \frac{d^3p}{(2\pi)^3}\frac{m}{\kappa(\kappa+m)}\frac{1}{E+\kappa}(E+\kappa-\gamma^3p^3)\sum_s u(p,s)u(p,s)^\dagger(E+\kappa+\gamma^3p^3)\left(\frac{1+\beta}{2}\right)e^{-ip\cdot(x-x')} \tag{A1}$$

and

$$\{\hat{q}_-(x), \hat{q}_-(x')^\dagger\} = \left(\frac{1-\beta}{2}\right)\int \frac{d^3p}{(2\pi)^3}\frac{m}{\kappa(\kappa+m)}\frac{1}{E+\kappa}(E+\kappa+\gamma^3p^3)\sum_s v(p,s)v(p,s)^\dagger(E+\kappa-\gamma^3p^3)\left(\frac{1-\beta}{2}\right)e^{ip\cdot(x-x')}. \tag{A2}$$

Using the well-known relations

$$\sum_s u(p,s)u(p,s)^\dagger = [(\gamma^\mu p_\mu + m)/2m]\beta\,, \qquad \sum_s v(p,s)v(p,s)^\dagger = -[(m-\gamma^\mu p_\mu)/2m]\beta\,, \tag{A3}$$

the Dirac operators entering (A1) and (A2) can be easily evaluated between the projection operators $(1\pm\beta)/2$, since odd operators cannot contribute. Dropping all odd operators, one obtains

$$\left(\frac{1+\beta}{2}\right)(E+\kappa-\gamma^3p^3)\frac{\beta E-\gamma^3p^3-\vec{\gamma}_\perp\cdot\vec{p}_\perp+m}{2m}\beta(E+\kappa+\gamma^3p^3)\left(\frac{1+\beta}{2}\right) = \frac{1+\beta}{2}\frac{\kappa(\kappa+m)(E+\kappa)}{m}$$

and (A4)

$$\left(\frac{1-\beta}{2}\right)(E+\kappa+\gamma^3p^3)\frac{\beta E-\gamma^3p^3-\vec{\gamma}_\perp\cdot\vec{p}_\perp-m}{2m}\beta(E+\kappa-\gamma^3p^3)\left(\frac{1-\beta}{2}\right) = \frac{1-\beta}{2}\frac{\kappa(\kappa+m)(E+\kappa)}{m},$$

from which it follows that

$$\{\hat{q}_+(x), \hat{q}_+(x')^\dagger\} = [(1+\beta)/2] \int [d^3p/(2\pi)^3] e^{-ip\cdot(x-x')}, \tag{A5}$$

$$\{\hat{q}_-(x), \hat{q}_-(x')^\dagger\} = [(1-\beta)/2] \int [d^3p/(2\pi)^3] e^{ip\cdot(x-x')}. \tag{A6}$$

Defining, as usual,

$$\Delta(x-x') = -[i/(2\pi)^3] \int (d^3p/2p^0) \times (e^{-ip\cdot(x-x')} - e^{ip\cdot(x-x')}),$$

$$i\Delta(x-x') = \Delta_+(x-x') - \Delta_-(x-x'), \tag{A7}$$

$$\Delta_1(x-x') = \Delta_+(x-x') + \Delta_-(x-x'),$$

we see that

$$\begin{aligned} i[\beta(-\nabla^2+m^2)^{1/2}+i\partial_0]\Delta(x-x') &= \frac{1}{(2\pi)^3}\int \frac{d^3p}{2p^0}\{[\beta(\vec{p}^{\,2}+m^2)^{1/2}+p^0]e^{-ip\cdot(x-x')} - [\beta(\vec{p}^{\,2}+m^2)^{1/2}-p^0]e^{ip\cdot(x-x')}\} \\ &= \frac{1}{(2\pi)^3}\int \frac{d^3p}{2}[(1+\beta)e^{-ip\cdot(x-x')} + (1-\beta)e^{ip\cdot(x-x')}] \\ &= i\beta\partial_0\Delta_1(x-x') - \partial_0\Delta(x-x'). \end{aligned} \tag{A8}$$

Adding (A5) and (A6), one obtains, with (A8), the anticommutation relations (3.8). Since

$$i\partial_0\Delta_\pm(x-x') = \pm[1/(2\pi)^3]\int (d^3p/2)e^{\mp ip\cdot(x-x')},$$

Eqs. (3.9) and (3.10) follow.

APPENDIX B: SCALING OF THE LIGHT-CONE BILOCALS

Consider the typical integrals

$$\int d^4x\, e^{iq\cdot x} p_0 f(x\cdot p)\partial_0\Delta(x), \tag{B1}$$

$$\int d^4x\, e^{iq\cdot x} p_0 f(x\cdot p)\partial_0\Delta_1(x). \tag{B2}$$

Representing $f(x\cdot p)$ as

$$f(x\cdot p) = \int d\xi\, e^{i\xi(x\cdot p)}F(\xi), \tag{B3}$$

(B1) becomes

$$\begin{aligned} &-2\pi p^0 \int d\xi\, F(\xi)(q^0+\xi p^0)\int d^3k\, \delta^3(q+\xi p-k)\epsilon(q^0+\xi p^0)\delta((q^0+\xi p^0)^2-(k^0)^2) \\ &\qquad = -2\pi p^0\int d\xi\, F(\xi)(q^0+\xi p^0)\delta(q^2+\xi^2p^2+2\nu\xi-m^2)\epsilon(q^0+\xi p^0), \end{aligned} \tag{B4}$$

where $\nu = p\cdot q$. Going to the Bjorken limit $[\nu\to\infty, -q^2\to\infty, 2\nu/(-q^2)=\omega=\text{const.}]$, we then obtain

$$-2\pi p^0\frac{1}{2\nu}F\left(\frac{1}{\omega}\right)\left(q^0+\frac{1}{\omega}p^0\right). \tag{B5}$$

The factor $p^0[q^0+(1/\omega)p^0]$ represents the tensor properties of the amplitude; choosing a timelike vector $n^\mu = (1, 0, 0, 0)$, we may write

$$\begin{aligned} p^0\left(q^0+\frac{1}{\omega}p^0\right) &= n^\mu p_\mu n^\nu\left(q_\nu+\frac{1}{\omega}p_\nu\right) \\ &= n^\mu n^\nu\left\{\frac{1}{2}\left[p_\mu\left(q_\nu+\frac{1}{\omega}p_\nu\right)+\left(q_\mu+\frac{1}{\omega}p_\mu\right)p_\nu\right] - g_{\mu\nu}\left(\nu+\frac{1}{\omega}p^2\right)\right\} + g^{00}\left(\nu+\frac{1}{\omega}m^2\right). \end{aligned} \tag{B6}$$

It is the second term of (B6) that corresponds to the spin-zero part of the amplitude, and we therefore find that the spin-zero part of (B1) is

$$-\pi F(1/\omega) \tag{B7}$$

in the Bjorken limit.

Similarly, substituting (B3) into (B2), we find that it yields

$$-2\pi i p^0\int d\xi\, F(\xi)(q^0+\xi p^0) \times\int d^3k\, \delta^3(q+\xi p-k)\delta((q^0+\xi p^0)^2-(k^0)^2) \underset{\mathrm{Bj}}{\to} -\pi i F\left(\frac{1}{\omega}\right). \tag{B8}$$

APPENDIX C: RELATION BETWEEN THE LIGHT-CONE ALGEBRAS OF LOCAL CURRENTS AND QUASILOCAL STRONG CURRENTS AT INFINITE MOMENTUM

At finite momenta, the light-cone algebras of local currents and quasilocal strong currents are not expected to be equivalent since they are not unitarily related. The strong current $\hat{v}_a(x)$ is related to a corresponding local current by

$$\hat{v}_a(x) = G(x_0)v_a(x)G^{-1}(x_0)\,. \tag{C1}$$

The commutator of two such quasilocal currents *at unequal times* cannot be expressed as a unitary transformation acting on the commutator of the corresponding two local currents, since the operator $G^{-1}(x_0)G(y_0)$ does not reduce to unity for $x_0 \neq y_0$. In the infinite-momentum frame ($p_z \to \infty$), if $G(x_0)$ were to approach a limiting form, the x_0 dependence would disappear and we would expect to find the two algebras to be isomorphic. Unlike the Melosh transformation V, V' contains "bad" operators, and the transformation G does not have a well-defined limit as an operator. It is this property which is responsible for the transformation of certain bad operators in the U(12) system of local currents to "good" operators in the strong system. The mechanism for this phenomenon is most easily discussed on the level of the free-quark fields, since the canonical light-cone algebra is constructed on the basis of their anticommutation relations.

For the covariant quarks, the anticommutation relation

$$\{q(x), q(y)^\dagger\} = i(i\gamma^\mu\partial_\mu + m)\gamma^0\Delta(x-y) \tag{C2}$$

becomes asymptotically

$$\{q'(x'), q'(y')^\dagger\} \sim -\frac{(1+\alpha_3)}{(1-v^2)^{1/2}}\partial_-'\Delta(x'-y') \tag{C3}$$

$$= \frac{(1+\alpha_3)}{(1-v^2)^{1/2}} i\partial_-'[\Delta_+(x'-y') - \Delta_-(x'-y')]\,, \tag{C4}$$

where we have defined

$$q'(x') \equiv U(\Lambda)q(x)U(\Lambda)^{-1} = \mathcal{S}q(\Lambda^{-1}x) \tag{C5}$$

and $x' = \Lambda^{-1}x$ for $p_z \to \infty$. The strong quark anticommutation relation, however, becomes asymptotically

$$\{\hat{q}'(x'), \hat{q}'(y')^\dagger\} \sim \frac{i\partial_-'}{(1-v^2)^{1/2}}[i\Delta(x'-y') + \beta\Delta_1(x'-y')] \tag{C6}$$

$$= \frac{i\partial_-'}{(1-v^2)^{1/2}}[(1+\beta)\Delta_+(x'-y') - (1-\beta)\Delta_-(x'-y')]\,, \tag{C7}$$

where

$$\hat{q}'(x') = U\hat{q}(x)U^{-1} \tag{C8}$$

can be calculated directly by applying the transformation Eq. (3.1) or Eq. (2.20) [the invariance of the Dirac equation, used in deriving Eq. (3.1), preserves the equivalence of the two forms], with derivatives taken with respect to x^μ, to $q'(x')$. In the $p_z \to \infty$ limit, the transformation Eq. (C8) has the asymptotic form

$$\hat{q}'(x') \sim \frac{1}{\sqrt{2}}(1+\gamma^3\sigma')\frac{\kappa + m - i\vec{\gamma}^\perp\cdot\vec{\partial}_\perp}{[2\kappa(\kappa+m)]^{1/2}}q'(x')\,, \tag{C9}$$

where $\sigma' = i\partial_-'/|i\partial_-'|$ is defined in terms of the Fourier representation, and we note that

$$\frac{1}{\sqrt{2}}(1+\gamma^3\sigma') = e^{(\pi/4)\gamma^3\sigma'}\,. \tag{C10}$$

The relation between Eqs. (C4) and (C7) can be easily understood in view of the asymptotic form Eq. (C9). Calculating the anticommutator between $\hat{q}'(x')$ and $\hat{q}'(y')^\dagger$ using Eqs. (C4) and (C9), we find

$$\{\hat{q}'(x'), \hat{q}'(y')^\dagger\} = \frac{1}{\sqrt{2}}(1+\gamma^3\sigma')\frac{\kappa+m-i\vec{\gamma}^\perp\cdot\vec{\partial}_\perp}{[2\kappa(\kappa+m)]^{1/2}}\frac{(1+\alpha_3)}{(1-v^2)^{1/2}} \times\frac{\kappa+m+i\vec{\gamma}^\perp\cdot\vec{\partial}_\perp}{[2\kappa(\kappa+m)]^{1/2}}\frac{1}{\sqrt{2}}(1-\gamma^3\sigma') \times i\partial_-'[\Delta_+(x'-y') - \Delta_+(x'-y')]\,, \tag{C11}$$

where all derivatives refer to x'^μ ($\vec{x}_\perp' = \vec{x}_\perp$). Since $\vec{\gamma}^\perp$ commutes with α_3, the operators induced by the Melosh transformation cancel to unity (it is therefore possible for the Melosh transformation to have a well-defined asymptotic limit at $p_z \to \infty$), but

$$\tfrac{1}{2}(1+\gamma^3\sigma')(1+\alpha_3)(1-\gamma^3\sigma') = (1+\beta\sigma')\,. \tag{C12}$$

Since $\Delta_+(x')$ contains $e^{-ik^+x'^-}$ and $\Delta_-(x')$ contains $e^{+ik^+x'^-}$, this result brings Eq. (C11) to the form of Eq. (C7).

We have therefore verified that the asymptotic forms of the anticommutation relations of the local fields $q(x)$ and the strong fields at $p_z \to \infty$ are not the same. Equation (C8), however, implies that in any finite frame

$$\hat{q}'(x') = (UGU^{-1})q'(x')(UG^{-1}U^{-1})\,, \tag{C13}$$

so that if UGU^{-1} were to exist as a unitary transformation at $p_z \to \infty$, the anticommutation relations would necessarily be the same. From this contradiction it follows that G (in particular, V') has no well-defined operator limit, and the algebra of local currents cannot therefore be strictly equivalent to the algebra of quasilocal strong currents.

The mechanism for the survival of certain bad operators in the current system can also be easily understood with the help of Eq. (C9). For example (up to surface terms),

$$\tfrac{1}{2}(1-\gamma^3\sigma')\beta(1+\gamma^3\sigma')=\alpha_3\sigma' \qquad \text{(C14)}$$

occurs asymptotically in the G transform of $q^\dagger(x)\beta q(y)$; $\alpha_3\sigma'$ commutes with the remaining operators induced by the Melosh transformation and is "good" in the current system (this result agrees with the infinite-momentum limit of the form of the operator B).

In the following, we show that:

(a) It is the residual pair terms of S' (which retain time dependence) that provide the asymptotic results discussed above in the infinite-momentum limit; the one-particle terms vanish, and the pair terms do not have a well-defined operator limit.

(b) The part of the transformation G already proposed by Melosh, i.e., the operator S, does have a well-defined limit. The pair terms vanish, and the nontrivial transformation induced by this operator at infinite momentum is entirely due to the one-particle terms.

(c) There is a formal residual similarity between the two algebras in the deep Regge region within the context of the free-field model; the correspondence is obtained by discarding pair terms, negative frequency parts in the wave functions of $j(x,0)$, and positive-frequency parts in the wave functions of $j(0,x)$ in the system of local currents.

In terms of free-field annihilation-creation operators, the transformation (2.10) may be written as

$$S'=\tfrac{1}{2}\int d^3k\left(\frac{m}{k^0}\right)[b^\dagger(k,s)b(k,s')u^\dagger(k,s)i\gamma^3u(k,s')+d^\dagger(k,s')d(k,s)v^\dagger(k,s)i\gamma^3v(k,s')$$
$$+e^{2ik^0x^0}b^\dagger(k,s)d^\dagger(k_{(-)},s')u^\dagger(k,s)i\gamma^3v(k_{(-)},s')$$
$$-e^{-2ik^0x^0}d(k,s)b(k_{(-)},s')v^\dagger(k,s)i\gamma^3u(k_{(-)},s')]\tan^{-1}\left(\frac{k^3}{\kappa}\right), \qquad \text{(C15)}$$

where we use the notation $k_{(-)}=(k^0,-\vec{k})$. With the help of the transformation law

$$Ub(k,s)U^{-1}=\left[\frac{k'^0}{(\Lambda k')^0}\right]^{1/2}b(k',s), \qquad \text{(C16)}$$

where $k'=\Lambda^{-1}k$, we find

$$US'U^{-1}=\tfrac{1}{2}\int d^3k'\left(\frac{m}{k'^0}\right)\Bigg\{\frac{k'^0}{(\Lambda k')^0}[b^\dagger(k',s)b(k',s')u^\dagger(k,s)i\gamma^3u(k,s')+d^\dagger(k',s')d(k',s)v^\dagger(k,s)i\gamma^3v(k,s')]$$
$$+\frac{(k'^0k'^0_{(-)})^{1/2}}{[(\Lambda k')^0(\Lambda k'_{(-)})^0]^{1/2}}[b^\dagger(k',s)d^\dagger(k'_{(-)},s')u^\dagger(k,s)i\gamma^3v(k_{(-)},s)e^{2ik^0x^0}$$
$$-d(k',s)b(k'_{(-)},s')v^\dagger(k,s)i\gamma^3u(k_{(-)},s')e^{-2ik^0x^0}]\Bigg\}\tan^{-1}\left(\frac{k^3}{\kappa}\right), \qquad \text{(C17)}$$

and note that

$$\tfrac{1}{2}(k'+k'_{(-)})=\frac{k^0}{(1-v^2)^{1/2}}(1,0,0,-v) \qquad \text{(C18)}$$

and

$$(\Lambda k'_{(-)})^0=k^0=(\Lambda k')^0. \qquad \text{(C19)}$$

Using

$$u^\dagger(k,s)i\gamma^3u(k,s')=\frac{i}{2m}[\sigma^3,\vec{\sigma}_\perp\cdot\vec{k}^\perp]_{ss'}$$
$$=-v^\dagger(k,s)i\gamma^3v(k,s'), \qquad \text{(C20)}$$

we see that the one-particle terms of (C17) can be written as

$$(US'U^{-1})^{(1)}=\tfrac{1}{2}\int\frac{d^3k'}{(\Lambda k')^0}[b^\dagger(k',s)b(k',s')$$
$$-d^\dagger(k',s')d(k',s)]$$
$$\times\frac{i}{2m}[\sigma^3,\vec{\sigma}_\perp\cdot\vec{k}^\perp]_{ss'}\tan^{-1}\left(\frac{(\Lambda k')^3}{\kappa'}\right). \qquad \text{(C21)}$$

For $v\to1$,

$$(\Lambda k')^0=\frac{k'^0+vk'^3}{(1-v^2)^{1/2}}\sim\frac{k'^+}{(1-v^2)^{1/2}};$$

$$(\Lambda k')^3\sim k'^+/(1-v^2)^{1/2}$$

also, so that the inverse tangent function may be replaced by $\frac{1}{2}\pi$, and the whole operator goes to zero as $O((1-v^2)^{1/2})$ (provided that we have convergence of the integral).

For the two-body terms, Eqs. (C18) and (C19) permit us to write the kinematical coefficient as

$$\frac{(k'^0 k'^0_{(-)})^{1/2}}{[(\Lambda k')^0(\Lambda k'_{(-)})^0]^{1/2}} = \frac{[2k'^0(\Lambda k')^0/(1-v^2)^{1/2} - k'^{02}]^{1/2}}{(\Lambda k')^0} \to \left(\frac{2k'^0}{k'^+}\right)^{1/2} \tag{C22}$$

as $v \to 1$. The exponentials are

$$e^{\pm 2ik^0x^0} = e^{\pm 2i(\Lambda k')^0 x^0} \sim \exp[\pm 2ik'^+x^0/(1-v^2)^{1/2}]. \tag{C23}$$

Finally, using

$$u^\dagger(k,s)i\gamma^3 v(k_{(-)},s') = i\frac{k^0+m}{2m}\left[\sigma^3 + \frac{\vec{\sigma}\cdot\vec{k}\,\sigma^3\,\vec{\sigma}\cdot\vec{k}}{(k^0+m)^2}\right]_{ss'} = -v^\dagger(k,s)i\gamma^3 u(k_{(-)},s'), \tag{C24}$$

we obtain

$$(US'U^{-1})^{(2)} \sim \frac{i}{\sqrt{2}}\tfrac{1}{2}\pi\int d^3k'\left(\frac{k'^+}{k'^0}\right)^{1/2}\{b^\dagger(k',s)d^\dagger(k'_{(-)},s')\exp[2ik'^+x^0/(1-v^2)^{1/2}] + d(k',s)b(k'_{(-)},s')\exp[-2ik'^+x^0/(1-v^2)^{1/2}]\}\frac{(\sigma_3)_{ss'}}{(1-v^2)^{1/2}}, \tag{C25}$$

where the limit of Eq. (C18) defining $k'_{(-)}$, i.e.,

$$k'_{(-)} = \frac{2(\Lambda k')^0}{(1-v^2)^{1/2}}(1,0,0,-v) - k' \tag{C26}$$

must be taken with caution (cross terms preserve the norm). The contribution (C25) may be easily undetected if the limit is taken in terms of the fields $q(x)$ in Eq. (2.10), and a change of variables is made in coordinate space.[30] The factor $(1-v^2)^{1/2}$ in the denominator prevents us from applying the Riemann-Lebesgue lemma to the exponential factors uniformly in v, and an interchange in the order of the integrations and limits is unjustified. The operator limit of Eq. (C25) is therefore not well defined.

To complete our study of the infinite-momentum limit of S', we now show that the limiting value of the commutator $[S', q(x)]$ is due to the two-body part of S'. Extracting only the two-body part of S' from Eq. (C15) and carrying out the required spin sums, we find

$$[S'^{(2)}, q(x)] = \tfrac{1}{2}\int\frac{d^3k}{(2\pi)^{3/2}}\left(\frac{m}{k^0}\right)^{3/2}\left\{e^{ik\cdot x}d(k,s)\frac{\gamma^0k^0+\gamma^jk^j+m}{2m}\gamma^0 i\gamma^3 v(k,s) - e^{-ik\cdot x}b(k,s)\frac{\gamma^0k^0+\gamma^jk^j-m}{2m}\gamma^0 i\gamma^3 u(k,s)\right\}\tan^{-1}\left(\frac{k^3}{\kappa}\right). \tag{C27}$$

Under an infinite Lorentz boost,

$$[(US'U^{-1})^{(2)}, Uq(x)U^{-1}] \sim -(\tfrac{1}{2})(\tfrac{1}{2}\pi)\int\frac{d^3k'}{(2\pi)^{3/2}}\left(\frac{m}{k'^0}\right)^{1/2}\frac{1}{k'^+/(1-v^2)^{1/2}} \times\left\{b(k',s)\left(\frac{(\gamma^0+\gamma^3)k'^+}{2(1-v^2)^{1/2}} + \frac{\vec{\gamma}^\perp\cdot\vec{k}^\perp+m}{2}\right)\gamma^0 i\gamma^3 u(\Lambda k',s)e^{-i\Lambda k'\cdot x} - d^\dagger(k',s)\left(\frac{(\gamma^0+\gamma^3)k'^+}{2(1-v^2)^{1/2}} + \frac{\vec{\gamma}^\perp\cdot\vec{k}^\perp+m}{2}\right)\gamma^0 i\gamma^3 v(\Lambda k',s)e^{i\Lambda k'\cdot x}\right\}. \tag{C28}$$

The mass and transverse pieces do not contribute due to the factor $(1-v^2)^{1/2}$ in front, and

$$\frac{(\gamma^0+\gamma^3)\gamma^0}{2} = \frac{1-\alpha^3}{2}.$$

Passing γ^3 through this factor, we obtain the projection operator $(1+\alpha^3)/2$ acting on $u(\Lambda k',s)$ and $v(\Lambda k',s)$; in the limit $v\to 1$, it takes on the eigenvalue unity on these wave functions. We therefore obtain

$$[(US'U^{-1})^{(2)}, Uq(x)U^{-1}] \sim -i\tfrac{1}{4}\pi\gamma^3\sigma' q'(x'), \tag{C29}$$

as required by Eqs. (C8), (C9), and (C10). The one-particle contribution to this commutator contains the factors

$$(\gamma^\mu k_\mu \pm m)\gamma^0 \sim \frac{(\gamma^0-\gamma^3)\gamma^0}{2(1-v^2)^{1/2}} = \frac{1+\alpha^3}{2(1-v^2)^{1/2}};$$

passing γ^3 through to the left leaves the projection operator $(1-\alpha^3)/2$ to act on the wave functions $u(k,s)$, $v(k,s)$. The one-body terms in S' therefore do not contribute to the transformation $q \to \hat{q}$ in the infinite-momentum limit.

In contrast to this situation, it is just the one-particle terms of the Melosh operator S which contribute to the transformation $q\to\hat{q}$ in the infinite-momentum limit, and only these terms survive in the limiting form of the operator. In terms of annihilation-creation operators, the transformation (2.4) may be written as

$$S=\tfrac{1}{2}\int d^3k\left(\frac{m}{k^0}\right)\Big\{b^\dagger(k,s)b(k,s')u^\dagger(k,s)\frac{i\vec{\gamma}^\perp\cdot\vec{k}^\perp}{|\vec{k}^\perp|}u(k,s') - d(k,s)d^\dagger(k,s')v^\dagger(k,s)\frac{i\vec{\gamma}^\perp\cdot\vec{k}^\perp}{|\vec{k}^\perp|}v(k,s')$$
$$+b^\dagger(k,s)d^\dagger(k_{(-)},s')u^\dagger(k,s)\frac{i\vec{\gamma}^\perp\cdot\vec{k}^\perp}{|\vec{k}^\perp|}v(k_{(-)},s')e^{2ik^0x^0}$$
$$-d(k,s)b(k_{(-)},s')v^\dagger(k,s)\frac{i\vec{\gamma}^\perp\cdot\vec{k}^\perp}{|\vec{k}^\perp|}u(k_{(-)},s')e^{-2ik^0x^0}\Big\}\tan^{-1}\left(\frac{|\vec{k}^\perp|}{m}\right). \quad \text{(C30)}$$

The asymptotic form of the first of the two-body terms is

$$\tfrac{1}{2}m\int\frac{d^3k'}{k'^0}\left(\frac{2k'^0}{k'^+}\right)^{1/2}\exp[2ik'^+x^0/(1-v^2)^{1/2}]\,b^\dagger(k',s)d^\dagger(k'_{(-)},s')u^\dagger(\Lambda k')\frac{i\vec{\gamma}^\perp\cdot\vec{k}^\perp}{|\vec{k}'^\perp|}v(\Lambda k'_{(-)})\tan^{-1}\left(\frac{|\vec{k}'^\perp|}{m}\right). \quad \text{(C31)}$$

With the help of the asymptotic result $(\Lambda\vec{k}'^\perp=\vec{k}'^\perp=\vec{k}^\perp)$

$$u^\dagger(k,s)\vec{\gamma}^\perp\cdot\vec{k}^\perp v(k_{(-)},s')=\frac{k^0+m}{2m}\left(\vec{\sigma}^\perp\cdot\vec{k}^\perp+\frac{\vec{\sigma}\cdot\vec{k}(\vec{\sigma}^\perp\cdot\vec{k}^\perp)\vec{\sigma}\cdot\vec{k}}{(k^0+m)^2}\right)_{ss'},$$
$$\sim\frac{k'^+}{(1-v^2)^{1/2}}[\vec{\sigma}^\perp\cdot\vec{k}'^\perp+\sigma_3(\vec{\sigma}^\perp\cdot\vec{k}'^\perp)\sigma_3+O((1-v^2)^{1/2})]_{ss'}=O(1), \quad \text{(C32)}$$

we see that the expression (C31) is well conditioned, and the Riemann-Lebesgue lemma can be uniformly applied (a similar result holds for the second of the two-body terms).

The asymptotic form of the first of the one-body terms of (C30) is

$$(1-v^2)^{1/2}\tfrac{1}{2}m\int\frac{d^3k'}{k'^+}\,b^\dagger(k',s)b(k',s')u^\dagger(\Lambda k',s)\frac{i\vec{\gamma}^\perp\cdot\vec{k}'^\perp}{|\vec{k}'^\perp|}u(\Lambda k',s')\tan^{-1}\left(\frac{|\vec{k}'^\perp|}{m}\right). \quad \text{(C33)}$$

Using

$$u^\dagger(k,s)\vec{\gamma}^\perp\cdot\vec{k}^\perp u(k,s')=\frac{1}{2m}[\vec{\sigma}^\perp\cdot\vec{k}^\perp,\vec{\sigma}\cdot\vec{k}]_{ss'},$$
$$=\frac{k^3}{2m}[\vec{\sigma}^\perp\cdot\vec{k}^\perp,\sigma_3]_{ss'}\sim\frac{k'^+}{2m(1-v^2)^{1/2}}[\vec{\sigma}^\perp\cdot\vec{k}'^\perp,\sigma_3]_{ss'},$$

we see that the factor $(1-v^2)^{1/2}$ cancels, and (with a similar result for the second term) the infinite-momentum limit of the one-body terms of (C30) is well defined.[5]

It is of interest in this simple example to see explicitly how the contribution of the two-body terms to the transformed quark field vanishes in the infinite-momentum limit. Carrying out the necessary spin sums, the commutator of S with $q(x)$ at equal time is found to be

$$[S,q(x)]=\int\frac{d^3k}{(2\pi)^{3/2}}\left(\frac{m}{k^0}\right)^{3/2}\left\{-\left(\frac{\gamma^0k^0-\gamma^jk^j+m}{2m}\right)\gamma^0 i\frac{\vec{\gamma}^\perp\cdot\vec{\mathrm{k}}^\perp}{|\vec{\mathrm{k}}^\perp|}u(k,s)b(k,s)e^{-ik\cdot x}\right.$$
$$+\frac{\gamma^0k^0-\gamma^jk^j-m}{2m}\gamma^0\frac{i\vec{\gamma}^\perp\cdot\vec{\mathrm{k}}^\perp}{|\vec{\mathrm{k}}^\perp|}v(k,s)d^\dagger(k,s)e^{ik\cdot x}$$
$$+\frac{\gamma^0k^0+\gamma^jk^j+m}{2m}\gamma^0\frac{i\vec{\gamma}^\perp\cdot\vec{\mathrm{k}}^\perp}{|\vec{\mathrm{k}}^\perp|}v(k,s)d^\dagger(k,s)e^{ik\cdot x}$$
$$\left.-\frac{\gamma^0k^0+\gamma^jk^j-m}{2m}\gamma^0\frac{i\vec{\gamma}^\perp\cdot\vec{\mathrm{k}}^\perp}{|\vec{\mathrm{k}}^\perp|}u(k,s)b(k,s)e^{-ik\cdot x}\right\}, \qquad \text{(C34)}$$

where the last two terms are due to the two-body parts of S. The sum of all terms yields, of course, the usual result. However, in the $p_z\to\infty$ limit, the one-body terms in Eq. (C34) take on an asymptotic form [all factors of $(1-v^2)^{1/2}$ cancel] proportional to

$$\frac{\gamma^0-\gamma^3}{2}\gamma^0 i\vec{\gamma}^\perp\cdot\vec{\mathrm{k}}^\perp u(\Lambda k',s)=\frac{1+\alpha^3}{2}i\vec{\gamma}^\perp\cdot\vec{\mathrm{k}}^\perp u(\Lambda k',s)$$
$$\sim i\vec{\gamma}^\perp\cdot\vec{\mathrm{k}}^\perp u(\Lambda k',s),$$
$$\frac{\gamma^0-\gamma^3}{2}\gamma^0 i\vec{\gamma}^\perp\cdot\vec{\mathrm{k}}^\perp v(\Lambda k',s)=\frac{1+\alpha^3}{2}i\vec{\gamma}^\perp\cdot\vec{\mathrm{k}}^\perp v(\Lambda k',s)$$
$$\sim i\vec{\gamma}^\perp\cdot\vec{\mathrm{k}}^\perp v(\Lambda k',s),$$

and the two-body terms

$$\frac{\gamma^0+\gamma^3}{2}\gamma^0 i\vec{\gamma}^\perp\cdot\vec{\mathrm{k}}^\perp u(\Lambda k',s)=\frac{1-\alpha^3}{2}i\vec{\gamma}^\perp\cdot\vec{\mathrm{k}}^\perp u(\Lambda k',s)$$
$$\to 0,$$
$$\frac{\gamma^0+\gamma^3}{2}\gamma^0 i\vec{\gamma}^\perp\cdot\vec{\mathrm{k}}^\perp v(\Lambda k',s)=\frac{1-\alpha^3}{2}i\vec{\gamma}^\perp\cdot\vec{\mathrm{k}}^\perp v(\Lambda k',s)$$
$$\to 0.$$

We have shown, so far, that the algebra of strong currents is not equivalent to the algebra of local currents, and studied in some detail the mechanism responsible for this inequivalence. In the remaining paragraphs of this appendix we shall discuss the residual similarity between the two algebras in the deep Regge region, within the context of the free-field model.

Let us consider the asymptotic forms of the commutation relations among local currents

$$Uv_a^0(x)U^{-1}=v_a'^0(x')$$
$$\sim\frac{1}{(1-v^2)^{1/2}}[v_a^0(x')+v_a^3(x')], \qquad \text{(C35)}$$

and the strong currents

$$U\hat{v}_a^0(x)U^{-1}=\hat{v}_a'^0(x'), \qquad \text{(C36)}$$

where $x'=\Lambda^{-1}x$. Using the relation (C3), we obtain

$$[v_a'^0(x'),v_b'^0(0)]$$
$$\sim-\frac{1}{(1-v^2)^{1/2}}\partial_-'\Delta(x')\left\{(d_{abc}+if_{abc})q'^\dagger(x')\tfrac{1}{2}\lambda_c q'(0)\right.$$
$$\left.-(d_{abc}-if_{abc})q'^\dagger(0)\tfrac{1}{2}\lambda_c q'(x')\right\}, \qquad \text{(C37)}$$

and using the relation (C7) in the form

$$\{\hat{q}'(x'),\hat{q}'(y')^\dagger\}\sim-\frac{1+\beta\sigma'}{(1-v^2)^{1/2}}\partial_-'\Delta(x'-y'), \qquad \text{(C38)}$$

we find

$$[\hat{v}_a'^0(x'),\hat{v}_b'^0(0)]$$
$$\sim-\frac{1}{(1-v^2)^{1/2}}\left\{(d_{abc}+if_{abc})\hat{q}'^\dagger(x')\tfrac{1}{2}\lambda_c\frac{(1+\beta\sigma_\Delta')}{2}\hat{q}'(0)\right.$$
$$\left.-(d_{abc}-if_{abc})\hat{q}'^\dagger(0)\tfrac{1}{2}\lambda_c\frac{(1-\beta\sigma_\Delta')}{2}\hat{q}'(x')\right\}$$
$$\times\partial_-'\Delta(x'), \qquad \text{(C39)}$$

where σ_Δ' acts on $\Delta(x')$ and not on the field operators. We have shown in Appendix B that the Δ_1 singularity produces a scaling contribution which is just i times the scaling contribution of the Δ singularity. This means that Δ_+ scales, but Δ_- provides only a vanishing contribution [cf. Eqs. (A7)] in the kinematical region of interest ($q^0>0$, $\xi>0$). In their application to the asymptotic calculation of scaling amplitudes, we may write Eqs. (C37) and (C39) as

$$[v_a'^0(x'),v_b'^0(0)]$$
$$\sim+\frac{i\partial_-'\Delta_+(x')}{(1-v^2)^{1/2}}\left\{(d_{abc}+if_{abc})q'^\dagger(x')\tfrac{1}{2}\lambda_c q'(0)\right.$$
$$\left.-(d_{abc}-if_{abc})q'^\dagger(0)\tfrac{1}{2}\lambda_c q'(x')\right\} \qquad \text{(C40)}$$

and

$$[\hat{v}_a'^0(x'), \hat{v}_b'^0(0)] \sim + \frac{i\partial'_-\Delta_+(x')}{(1-v^2)^{1/2}} \left\{ (d_{abc} + if_{abc})\hat{q}'^\dagger(x')\tfrac{1}{2}\lambda_c\left(\frac{1+\beta}{2}\right)\hat{q}'(0) - (d_{abc} - if_{abc})\hat{q}'^\dagger(0)\tfrac{1}{2}\lambda_c\left(\frac{1-\beta}{2}\right)\hat{q}'(x')\right\}. \qquad \text{(C41)}$$

Up to surface terms (we are, in effect, considering only diagonal in momentum matrix elements), we may use the transformation (C9) and the relation

$$\tfrac{1}{2}(1-\gamma^3\sigma')\beta(1+\gamma^3\sigma') = \alpha^3\sigma'$$

to obtain

$$[\hat{v}_a'^0(x), \hat{v}_b'^0(0)] \sim \frac{i\partial'_-\Delta_+(x)}{(1-v^2)^{1/2}} \left\{ (d_{abc} + if_{abc})q'^\dagger(x')\tfrac{1}{2}\lambda_c\frac{1+\sigma'}{2}q'(0) - (d_{abc} - if_{abc})q'^\dagger(0)\tfrac{1}{2}\lambda_c\frac{1-\sigma'}{2}q'(x')\right\}, \qquad \text{(C42)}$$

where σ' acts on the quark fields. This result is identical to that of Eq. (C40) for the local currents except for the selection of positive-frequency components of the quark fields in $v_c'^0(x', 0)$ and negative-frequency components in $v_c'^0(0, x')$ (this combination maintains definite charge conjugation for the coefficients of d_{abc} and f_{abc}) and the exclusion of pair terms.

The similarity between the two algebras that we have found is based upon an explicit use of the free-field model, and therefore (except for general transformation properties) could be directly applicable only in the zeroth order of perturbation theory.

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[7]L. Gomberoff, L. P. Horwitz, and Y. Ne'eman, Phys. Lett. 45B, 131 (1973).

[8]S. Coleman, Phys. Lett. 19, 144 (1965).

[9]H. Fritzsch and M. Gell-Mann, in *Proceedings of the XVI International Conference on High Energy Physics, Chicago-Batavia, Ill., 1972,* edited by J. D. Jackson and A. Roberts (NAL, Batavia, Ill., 1973), Vol. 2, p. 135.

[10]R. Dashen and M. Gell-Mann, Phys. Rev. Lett. 17, 340 (1966).

[11]We shall work with the free-quark field as a model for the physical hadron world.

[12]L. L. Foldy and S. A. Wouthuysen, Phys. Rev. 78, 29 (1950). The Foldy-Wouthuysen transformation would provide us with a good boost-invariant $[U(3)\times U(3)]_\beta^{\text{strong}}$, and the work of Sec. IV could be carried through in this context. The resulting classification scheme would not, however, be as useful for other applications; we wish to have, in particular, a boost-invariant U(6) subgroup.

[13]We define charge conjugation to include the phase η_c defined by Y. Dothan, Nuovo Cimento 30, 399 (1963). See also, Y. Ne'eman, *Algebraic Theory* (Benjamin, New York, 1967), pp. 36 and 124.

[14]In the current U(12), V^0 and V^3 go, at infinite momentum, to operators which are identical because parity is not defined in this limit. In view of this result it is clear that the $p_z \to \infty$ limit of the algebra of strong currents is not equivalent, in a simple way, to the $p_z \to \infty$ limit of the algebra of ordinary local currents. This point will be discussed further in Sec. IV and in Appendix C.

[15]Our definition does not exactly coincide with that of Ref. 9; our "good" is a combination of their "good" and "terrible."

[16]The commutator (4.2) permits us to reach the singlet-even charge conjugation couplings, and the commutator (4.3) the singlet-odd charge conjugation couplings.

[17]As pointed out by Fritzsch and Gell-Mann (Ref. 9), "good-good" commutators of their ordinary currents vanish (except locally) in the $p_z \to \infty$ limit. The commutators of the "strong currents" of Eq. (3.6) do not have this property.

[18]Vijayanarayana T. N. Reddy, L. Gomberoff, L. P. Horwitz, and Y. Ne'eman (unpublished). In this work we have followed a procedure first applied to the current system by V. T. N. Reddy (Ref. 29).

[19]If the amplitude given by Eq. (4.17) were to take on a factorized Regge asymptotic form also, we would obtain a "bad" strong current coupling to Regge exchange $\beta_-^{\text{str.curr.}}$ which is, in general, not equal to $\beta_+^{\text{str.curr.}}$. The bad strong currents and the good strong currents form two distinct, unrelated, tensors under $\mathcal{W}_B$.

[20]Both even and odd terms can contribute to even and odd charge conjugation parts. According to Eqs. (4.9), (4.10), and (4.18), we may have signatures $1+(-1)^n$ and $1+(-1)^{n+1}$ for the even-charge conjugation couplings $s^{(+)}$ and $v^{(-)}$, and $1-(-1)^n$, $1-(-1)^{n+1}$ for the odd-charge conjugation couplings $s^{(-)}$ and $v^{(+)}$.

[21]E. D. Bloom and F. J. Gilman [Phys. Rev. Lett. 25, 1140 (1970)] have remarked that diffraction scattering does not appear to dominate on the light cone ($\omega \leq 3$).

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[23]T. D. Newton and E. P. Wigner, Rev. Mod. Phys. 21, 400 (1949).

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[25]N. Cabibbo and M. Testa, Phys. Lett. 42B, 369 (1972).

[26]S. L. Glashow and S. Weinberg, Phys. Rev. Lett. 20, 224 (1968); M. Gell-Mann, R. J. Oakes, and B. Renner, Phys. Rev. 175, 2195 (1968).

[27]M. Kislinger and K. Young (unpublished).

[28]R. F. Dashen and M. Gell-Mann, Phys. Rev. Lett. 17, 340 (1966).

[29]V. T. N. Reddy, University of Texas (Austin) report, 1973.

[30]In the usual treatment of the infinite-momentum frame as simply a change of variables, some care must therefore be taken to assure that "bad" operators such as S' are effective in the sense, for example, of Eq. (C29).