Regge behavior and daughter degeneracies in fourth-order spinor-vector scattering*

Samuel Prum
Department of Physics and Astronomy, Tel-Aviv University, Ramat-Aviv, Israel†
and Department of Physics, University of California at Los Angeles, Los Angeles, California 90024
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For the equal-mass scattering of spin-$\frac{1}{2}$ nucleons and neutral vector mesons, we discuss the Regge behavior of the amplitude and consider the questions posed by the existence of degenerate daughter trajectories within the framework of fourth-order perturbation theory. A formalism for the identification of daughter contributions is developed and the algebraic solution to the equations constraining the residues of the daughters is presented. By applying these ideas to the results of a previous calculation, we find that, while to fourth order sufficient information exists to resolve the degeneracy and to test for its breaking, the daughter contributions cannot be clearly identified due to the presence of new singularities having the form $t^{-1}\ln^2 t$ to fourth order. The possible origin of these terms is discussed and is tentatively ascribed to the existence of fixed poles of order three at the negative half-integer values of J.

I. INTRODUCTION

In recent years there has been a continued interest in understanding the Regge ideas from the point of view of perturbation theory. This interest stems from the added insight that can be obtained by looking at models, such as perturbation theory, when attempting to guess the general analytic properties of scattering amplitudes. Characteristic of this type of approach has been the success achieved in representing a Regge trajectory as a complete set of ladder diagrams.

A question of particular concern regarding perturbative expansions for interactions of particles with spin has been whether the concept of an "elementary particle" in renormalized Lagrangian field theory can be fitted within the framework of an analytic scattering theory. While examining this question several years ago, Gell-Mann, Goldberger, Low, Marx, and Zachariasen in various combinations[1-3] discovered one example, that of a spin-$\frac{1}{2}$ fermion interacting with a massive vector boson through a conserved current, for which this ambiguity is resolved. Studying the leading asymptotic behavior for the first few orders of the perturbation expansion, GGLMZ found that the fermion lies on a Regge trajectory $\alpha(s)$ passing through $l\equiv J-\frac{1}{2}=0$ when $s=m^2$, and that the critical requirements for the existence of such a result are the occurrence of a "nonsense" channel and the presence of the factoring property[4-6] in the Born approximation. This analysis has been extended through sixth order in the coupling constant,[7,8] with some work also having been done on the nth-order case,[9] and it has been shown that the set of two-body Feynman graphs associated with this process have a leading behavior which is consistent with the existence of two poles: one of trajectory $\alpha(s)$ and even signature, the other of trajectory $-\alpha(s)$ and odd signature. The suggestion has also been made that the entire amplitude is analytic.[10,11]

A natural continuation to the study of the fermion trajectory is to examine how the kinematic constraints among helicity amplitudes at $s=0$ are satisfied in this spinor-vector scattering model. Abers, Cassandro, Teplitz, and Muzinich[12] have used second-order perturbation theory to study this question and to test the conspiracy hypothesis. They have found that there exist infinite sequences of daughter trajectories passing through each negative half-integer value of the angular momentum, all of which conspire in a complicated way with the fermion trajectory to satisfy the kinematic constraints. In fact, there appear to be at least three sequences of these daughters, since a single sequence, while satisfying the kinematic constraints, does not have a factorizable residue matrix in Born approximation. The exact nature of this daughter degeneracy, however, can only be ascertained through the study of higher-order terms in the perturbation expansion.

It is our purpose in this paper to discuss the solution to the question of degenerate daughter trajectories within the framework of fourth-order perturbation theory and to apply these considerations to a calculation of the asymptotic nonpolynomial terms of the fourth-order scattering amplitude for large momentum transfer t and $s=0$.[13] Our analysis shows that under the hypothesis of three degenerate daughters, the fourth order contains sufficient information to resolve the degeneracy, and simple conditions exist that test for its breaking. When the Regge picture is applied to the interpretation of the fourth-order calculation, one observes that in addition to the contributions due to trajectories, there are terms which can be tentatively ascribed to the existence of fixed poles of order three at all negative half-integer values of J. These additional singularities complicate the isolation and analysis of the daughter contributions, so that when these complications are neglected, one finds that the daughter trajectories at $J=-\frac{1}{2}$ separate but that none of the members of the triplet lie one unit of angular momentum below the mother trajectory. Furthermore, these new singularities exhibit a more dominant leading behavior than the daughter contributions (i.e., $t^{-1}\ln^2 t$ versus $t^{-1}\ln t$).

Our plan for this paper is as follows: In Sec. II we show how information on a Regge trajectory and its residue appear in perturbation theory. For this purpose we extend the Reggeization formalism of GGLMZ to include signature, and derive expressions for the contributions of the mother and daughter trajectories as power series in the coupling constant $g^2=e^2/4\pi$.

Under the assumption of three degenerate daughter trajectories at each value of J, we show in Sec. III that the data available from second- and fourth-order calculations constrain the factorizable residues and the trajectory slopes at each J into systems of 12 equations in 12 unknowns. A unique solution to this system of equations is found by reducing the problem to a series of eigenvalue equations.

The application of this formalism to a calculation[13] of the fourth-order elastic scattering of a fermion and a neutral vector boson is considered in the following sections. Section IV is concerned with constructing the parity-conserving signatured amplitudes. The features of these amplitudes are discussed in Sec. V, where an attempt is made to understand the origin of the terms proportional to $t^{-n}\ln^2 t$, for $n\geq 1$; to fourth order, these terms cannot derive from a Regge trajectory. The most plausible explanation we find is the existence of fixed poles of order three in the left-half J plane. In Sec. VI we consider the daughter contributions and show how these can be extracted from the amplitude. Then, by combining the fourth-order information with the results of the second-order calculation,[3,12] we solve for the residues and trajectory slopes of the daughters at $J=-\frac{1}{2}$. Our results are summarized and discussed in Sec. VII.

II. REGGE-POLE CONTRIBUTIONS IN PERTURBATION THEORY

When studying the Regge properties of a scattering amplitude by means of perturbation theory, it is helpful to keep in mind the general form of the contribution to the amplitude, for each order of perturbation theory, coming from a single scalar Regge pole. Such a contribution is given asymptotically by

$$T(s,t)\sim\beta(s)t^{\alpha(s)}=\beta(s)\exp[\alpha(s)\ln t], \tag{2.1}$$

where $\beta(s)$ is the residue corresponding to the pole $\alpha(s)$. In a theory in which the interaction is characterized by a collection of coupling strengths, which for simplicity we call g^2, such as Lagrangian field theory, one can write the trajectory and the residue as power series in g^2:

$$\alpha(s)=\alpha_0(s)+g^2\alpha_1(s)+g^4\alpha_2(s)+\cdots, \tag{2.2}$$

$$\beta(s)=g^2\beta_1(s)+g^4\beta_2(s)+\cdots. \tag{2.3}$$

An expression for the asymptotic form of the amplitude to each order in g^2 can then be derived. It has the form

$$T(s,t)\sim K(s,t)t^{\alpha_0(s)}, \tag{2.4}$$

where $K(s,t)$ is given by

$$K(s,t)=g^2\beta_1+g^4(\beta_2+\beta_1\alpha_1\ln t)+g^6[\beta_3+(\beta_2\alpha_1+\beta_1\alpha_2)\ln t+\tfrac{1}{2}\beta_1(\alpha_1\ln t)^2]+\cdots \tag{2.5}$$

for each trajectory present in the theory. Thus, the contributions of several poles correspond to polynomials in t^{-1} multiplying powers of $\ln t$, i.e., terms of the form $t^{-n}(\ln t)^m$ for n and m non-negative integers; each trajectory being associated with a power of t^{-1}. In principle then, a given order of perturbation theory yields the value of the residue to that order in g^2 and of the trajectory to one order less in g^2.

The generalization of these ideas to the scattering of particles with spin involves more complicated expressions for the asymptotic form of the scattering amplitude, with the residues becoming matrices. However, the principle involved in identifying the residue and trajectory contributions remains the same. Several authors[1-3,14-19] have discussed the Reggeization procedure for the case involving particles with spin. We follow the method introduced by GGLMZ, based on the helicity formalism of Jacob and Wick,[20] and define, for a two-body scattering process of the type $a+b\to c+d$, parity-conserving helicity amplitudes that are free of kinematic singularities. These are given by the rule

$$T^{(\pm)}_{\lambda_c,\lambda_d;\lambda_a,\lambda_b}(s,t)=(1+z)^{-|\nu+\mu|/2}(1-z)^{-|\nu-\mu|/2}T_{\lambda_c,\lambda_d;\lambda_a,\lambda_b}\pm(-1)^{\mu+M}\eta_c\eta_d(-1)^{S_c+S_d-\nu}\times(1-z)^{-|\nu+\mu|/2}(1+z)^{-|\nu-\mu|/2}T_{-\lambda_c,-\lambda_d;\lambda_a,\lambda_b}. \tag{2.6}$$

Here λ denotes helicity, S spin, and η intrinsic parity; $\mu=\lambda_a-\lambda_b$, $\nu=\lambda_c-\lambda_d$, $M=\max(|\mu|, |\nu|)$, z is the cosine of the scattering angle θ, and v is $\frac{1}{2}$ for half-integral J and zero for integral J. The parentheses around the parity signs are used so that they may be distinguished from the signature.

A Regge formalism using these amplitudes can then be developed in analogy to GGLMZ, and one can extend their results to parity-conserving signatured amplitudes $T^{(\pm)\pm}_{\lambda_c,\lambda_d;\lambda_a,\lambda_b}$, which satisfy a partial-wave expansion of the form

$$T^{(\pm)\pm}_{\lambda_c,\lambda_d;\lambda_a,\lambda_b}(s,t)=\sum_J(2J+1)[t^{(\pm)\pm}_{\lambda_c,\lambda_d;\lambda_a,\lambda_b}(s)E^{J(+)}_{\mu\nu}(z)+t^{(\mp)\mp}_{\lambda_c,\lambda_d;\lambda_a,\lambda_b}(s)E^{J(-)}_{\mu\nu}(z)], \quad (2.7)$$

where $t^{(\pm)\pm}_{\lambda_c,\lambda_d;\lambda_a,\lambda_b}(s)$ are parity-conserving signatured partial-wave amplitudes, and the functions $E^{J(\pm)}_{\mu\nu}(z)$ are linear combinations of derivatives of Legendre polynomials and are tabulated by GGLMZ. In terms of the right-hand- and left-hand-cut contributions to the scattering amplitude, these parity-conserving signatured amplitudes are given by

$$\tfrac{1}{2}T^{(\pm)\pm}_{\{\lambda_i\}}(s,z)=T^{(\pm)}_{R\{\lambda_i\}}(s,z)\pm(-1)^{M-v-1}T^{(\pm)}_{L\{\lambda_i\}}(s,-z), \quad (2.8)$$

where T_R and T_L denote the respective right- and left-cut contributions, and the symbol $\{\lambda_i\}=\lambda_c, \lambda_d; \lambda_a, \lambda_b$.

By carrying out the Reggeization process for this partial-wave expansion one finds that the contribution of a trajectory $J=\alpha(s)$ to the amplitude is given by

$$T^{(\pm)\pm}_{\{\lambda_i\}}(s,z)\sim-\pi[(2\alpha+1)/\sin\pi\alpha]\times[\beta^{J(\pm)\pm}_{\{\lambda_i\}}(s)E^{J(+)}_{\mu\nu}(-z)+\beta^{J(\mp)\mp}_{\{\lambda_i\}}(s)E^{J(-)}_{\mu\nu}(-z)]. \quad (2.9)$$

Since we are interested in studying the elastic scattering of electrons and massive photons, we define $l=J-\frac{1}{2}$, and set $S_a=S_c=1$, $S_b=S_d=\frac{1}{2}$, $\eta_a=\eta_c=-1$, $\eta_b=\eta_d=+1$. To avoid duplicating amplitudes we restrict the indices λ_b and λ_d to the value $+\frac{1}{2}$ and suppress them entirely, thus writing simply $T^{(\pm)\pm}_{\lambda_a,\lambda_c}$, with λ_a and λ_c equal to -1, 0, and $+1$.

The power-series (asymptotic) expansions for the functions $E^{J(\pm)}_{\mu\nu}(z)$ that are relevant to our analysis are given in Appendix A. By making use of these expansions, we are able to give the first few leading terms of the asymptotic amplitude contributed by a trajectory $l=\alpha(s)$ having a residue $\beta(s)$. We find

$$T^{(\pm)\pm}_{\kappa,\nu}\sim F_\alpha\epsilon_{\nu\kappa}(-z)^\alpha\left[\beta^{\alpha(\pm)\pm}_{\kappa,\nu}+\frac{\alpha}{2\alpha+1}\beta^{\alpha(\mp)\mp}_{\kappa,\nu}(-z)^{-1}+O((-z)^{-2})\right], \quad (2.10a)$$

$$T^{(\pm)\pm}_{-1,\nu}\sim\alpha F_\alpha(-z)^{\alpha-1}\left[\beta^{\alpha(\pm)\pm}_{-1,\nu}+\frac{\alpha-1}{2\alpha+1}\beta^{\alpha(\mp)\mp}_{-1,\nu}(-z)^{-1}+O((-z)^{-2})\right], \quad (2.10b)$$

$$T^{(\pm)\pm}_{-1,-1}\sim\alpha^2F_\alpha(-z)^{\alpha-1}\left[\beta^{\alpha(\pm)\pm}_{-1,-1}+\frac{3(\alpha-1)}{2\alpha+1}\beta^{\alpha(\mp)\mp}_{-1,-1}(-z)^{-1}+O((-z)^{-2})\right], \quad (2.10c)$$

where

$$F_\alpha=-2^{\alpha+3/2}\pi(\alpha+1)\Gamma(\alpha+\tfrac{3}{2})[\sqrt{\pi}\,\sin\pi\alpha\,\Gamma(\alpha+2)]^{-1}.$$

The Greek indices κ and ν have been used to denote the sense values 0 and 1, and $\epsilon_{\kappa\nu}=1$ unless $\kappa=0$, $\nu=1$, in which case $\epsilon_{\kappa\nu}=-1$.

The analogy of these expressions to the simpler forms of Eq. (2.1) is quite evident, and the contributions of a trajectory to each order in the perturbation expansion, though more complicated, can be similarly identified. For example, in the case of the electron trajectory which we denote by $\mu(s)$ with residue $\xi_{ij}(s)$, this contribution is of the form

$$T^{(\pm)\pm}_{\kappa,\nu}\sim-g^2\sqrt{2}\,\epsilon_{\nu\kappa}(\mu_1^\pm)^{-1}(\xi_2^{(\pm)\pm})_{\kappa,\nu}F^\pm(g^2), \quad (2.11a)$$

$$T^{(\pm)\pm}_{-1,\nu}\sim-g^2\sqrt{2}(-z)^{-1}[(\xi_1^{(\pm)\pm})_{-1,\nu}F^\pm(g^2)-(-z)^{-1}(\xi_1^{(\mp)\mp})_{-1,\nu}F^\mp(g^2)], \quad (2.11b)$$

$$T^{(\pm)\pm}_{-1,-1}\sim-g^2\sqrt{2}(-z)^{-1}[\mu_1^\pm(\xi_0^{(\pm)\pm})_{-1,-1}F^\pm(g^2)-3(-z)^{-1}\mu_1^\pm(\xi_0^{(\mp)\mp})_{-1,-1}\times F^\mp(g^2)], \quad (2.11c)$$

where

$$F^\pm(g^2)\equiv1+g^2[\mu_1^\pm\ln(-z)+f(z^{-1})]+O(g^4),$$

and $f(z^{-1})$ is a polynomial in z^{-1}. Here we have made use of the fact that $\mu(s)$ is of the form $\mu=0+g^2\mu_1+g^4\mu_2+\cdots$ and that its residue $\xi(s)$ ($\xi=\xi_0+g^2\xi_1+g^4\xi_2+\cdots$) is sense-choosing, so that at $s=0$, ξ_0 is nonzero for nonsense-nonsense amplitudes, and ξ_1 vanishes for the sense-sense amplitudes.

For a daughter trajectory at $l=-1$, its contribution to the amplitude is given by the following expressions:

$$T^{(\pm)\pm}_{\kappa,\nu} \sim g^2\sqrt{2}\,\epsilon_{\nu\kappa}(-z)^{-1}M^{(\pm)\pm}_{\kappa,\nu}\,, \tag{2.12a}$$

$$T^{(\pm)\pm}_{-1,\nu} \sim -g^2\sqrt{2}\,(-z)^{-2}M^{(\pm)\pm}_{-1,\nu}\,, \tag{2.12b}$$

$$T^{(\pm)\pm}_{-1,-1} \sim g^2\sqrt{2}\,(-z)^{-2}M^{(\pm)\pm}_{-1,-1}\,, \tag{2.12c}$$

$$M^{(\pm)\pm}_{i,j} \equiv (\gamma_1^{(\pm)\pm})_{i,j} + g^2[\,\delta_1^{\pm}(\gamma_1^{(\pm)\pm})_{i,j}\ln(-z) + f(z^{-1})] + O(g^4)\,,$$

where we have labeled the daughter trajectory by $\delta(s)$ ($\delta = -1 + g^2\delta_1 + \cdots$), and its residue by $\gamma = g^2\gamma_1 + g^4\gamma_2 + \cdots$.

An examination of Eq. (2.11) shows that in fourth-order perturbation theory the contribution of the electron trajectory to the amplitude is proportional to a polynomial in $(-z)^{-1}$ times $\ln(-z)$ plus a second polynomial in $(-z)^{-1}$. The leading term of this contribution is of the form $(-z)^0\ln(-z)$ for the sense-sense values, and of the form $(-z)^{-1}\ln(-z)$ for the nonsense values. Similarly, Eq. (2.12) shows that a daughter at $l=-1$ gives a leading-power contribution of the form $(-z)^{-1}\ln(-z)$ to sense-sense values of the amplitude, and of the form $(-z)^{-2}\ln(-z)$ to nonsense values of the amplitude. For the purpose of determining the fourth-order contributions due to the various trajectories, it is not necessary to go through the task of a partial-wave inversion of the amplitude, since one can simply make use of equations such as Eqs. (2.11) and (2.12) to examine the coefficients of terms proportional to $(-z)^{-n}\ln(-z)$, where n is a non-negative integer, and identify the respective contributions. This procedure is illustrated in Sec. VI.

III. CONSTRAINTS ON DEGENERATE DAUGHTER TRAJECTORIES IN SPINOR-VECTOR SCATTERING

Let us examine what constraints the fourth-order analysis places on the daughter trajectories and its residues. The work of ACTM using second-order perturbation theory has indicated that three families of degenerate daughter trajectories exist. For each daughter there are four unknowns to be determined: a factorizable residue matrix $(\gamma_1^{(k)})_{i,j}$ involving three parameters, and a trajectory "slope" $\delta_1^{(k)}$, where the index k labels the daughter trajectories. Thus, there are a total of twelve unknowns.

At each negative integral value of l, second-order perturbation theory provides six equations of constraint, which have the form

$$Q = \sum_{k=1}^{3}\gamma_1^{(k)}. \tag{3.1}$$

For each value of l, Q is a known nonfactorizing symmetric matrix containing the second-order contributions due to the residues of the daughter trajectories.

In Sec. II we have seen that the information that can be obtained from fourth order is of two types: The $\ln(-z)$-dependent terms constrain the residues and the trajectory slopes, while the purely polynomial terms depend only on the residue slopes $\gamma_2^{(k)}$. For the purpose of studying the daughter degeneracies only the $\ln(-z)$-dependent terms are of immediate interest since only they provide the additional constraints that are needed. These are given by means of the following equation:

$$R = \sum_{k=1}^{3}\delta_1^{(k)}\gamma_1^{(k)}, \tag{3.2}$$

where for each negative integer l, R is a known symmetric matrix determined from the fourth-order calculation. Together, Eqs. (3.1) and (3.2) form a system of twelve equations in twelve unknowns.

Consider the solution to this system of equations: First of all, note that a trivial test exists to determine whether the degeneracy is broken; for if the degeneracy remained, all of the trajectory slopes would be equal, in which case R and Q would be related by a multiplicative factor. Thus, the noncommutativity of Q and R provides an immediate test for the breaking of the degeneracy. Proceeding with the solution we express each residue $\gamma_1^{(k)}$ by means of the factorization property as the outer product of a vector with itself, i.e.,

$$\gamma_1^{(k)} = \vec{V}_k\otimes\vec{V}_k\,, \tag{3.3}$$

where the $\vec{V}_k$, $k=1$, 2, and 3, are linearly independent (but not orthogonal), since Q is of rank three. The matrices Q and R can be expanded in terms of their corresponding eigenvalues and eigenvectors, both of which are real since Q and R are symmetric. Thus we can write

$$Q = \sigma_1\vec{x}_1\otimes\vec{x}_1 + \sigma_2\vec{x}_2\otimes\vec{x}_2 + \sigma_3\vec{x}_3\otimes\vec{x}_3\,, \tag{3.4}$$

$$R = \rho_1\vec{y}_1\otimes\vec{y}_1 + \rho_2\vec{y}_2\otimes\vec{y}_2 + \rho_3\vec{y}_3\otimes\vec{y}_3 \tag{3.5}$$

where the σ_k are the eigenvalues of Q with corresponding orthonormal eigenvectors $\vec{x}_k$, and the ρ_k are the eigenvalues of R with corresponding orthonormal eigenvectors $\vec{y}_k$.

Both sets of eigenvectors are linearly independent. Thus they can be used to represent the vectors $\vec{V}_k$ by means of two unknown real matrices A and B, which are defined by the following expressions:

$$\vec{V}_k = \sum_{n=1}^{3}A_{kn}\vec{x}_n = \sum_{n=1}^{3}B_{kn}\vec{y}_n\,. \tag{3.6}$$

These matrices have the property that $(B^{-1}A)_{mn} = \vec{y}_m\cdot\vec{x}_n$, so that only one of them is really unknown. Defining $L = B^{-1}A$ and rewriting for system of twelve equations in terms of A we find

$$A^T A = \sigma\,, \tag{3.7}$$

$$A^T \delta_1 A = L^T \rho L\,, \tag{3.8}$$

where $\sigma \equiv \mathrm{diag}(\sigma_1, \sigma_2, \sigma_3)$, $\delta_1 \equiv \mathrm{diag}(\delta_1^{(1)}, \delta_1^{(2)}, \delta_1^{(3)})$, and $\rho \equiv \mathrm{diag}(\rho_1, \rho_2, \rho_3)$. The matrix L is known from the eigenvectors of Q and R and has the property of being unitary.

Considering first Eq. (3.7), we see that σ is required to be positive definite if a solution is to exist. Later on we will see that this is in fact the case. A particular solution to this equation is given by $A_P = \mathrm{diag}(\sqrt{\sigma_1}, \sqrt{\sigma_2}, \sqrt{\sigma_3})$, and the most general solution is $A = UA_P$, for an arbitrary unitary matrix U. Equation (3.8) can then be used to determine the conditions on U. One finds a relation of the form

$$\delta_1 = UMU^{-1}\,, \tag{3.9}$$

with M being given by

$$M = A_P{}^{-1} L^T \rho L A_P{}^{-1}\,, \tag{3.10}$$

all of which are known quantities. We see, therefore, that U is required to be the similarity transformation that diagonalizes M, and the trajectory slopes are the eigenvalues of M. This solution is unique if one requires U to be orthonormal.

This determines $A = UA_P$ and gives us the solution for the trajectory slopes $\delta_1^{(k)}$. One can then work back and construct the residues $\gamma_1^{(k)}$ by means of Eq. (3.6). Our unknowns are therefore expressible in terms of the eigenvalues and the eigenvectors of Q and R, and the solution of the twelve equations reduces to the task of solving a series of eigenvalue problems.

The fourth-order calculation, we find, can provide an answer to the question of degenerate daughter trajectories in spinor-vector scattering. In order to apply these ideas, it is necessary to calculate the asymptotic fourth-order amplitude at fixed s, and for large $|t|$; from this one can in principle extract the matrices Q and R. In the following sections we turn to the evaluation of Q and R and to the analysis of the fourth-order "data."

IV. THE ASYMPTOTIC BEHAVIOR OF THE FOURTH-ORDER PROCESS

The evaluation of asymptotic next-to-leading-order terms in Feynman integrals with spin numerators is a topic in itself for which very few applicable techniques exist. This is due mainly to the fact that spin introduces essential complications into the theory, thus making even low-order perturbative calculations a difficult task. In our case, we are interested in studying the fourth-order contribution to the amplitude describing the scattering of electrons and massive photons for large t and for large u. We restrict ourselves to considerations of the amplitude for $s = 0$, where the kinematic constraints hold, and in addition, we consider only the case of equal masses. This equal-mass restriction is made in order to simplify the kinematic constraints, which would otherwise have to be satisfied at $s = 0$ and $s = (m_1 - m_2)^2$, where m_1 and m_2 are the masses of the particles. Although no proof exists, it is anticipated that the equal-mass case results are at least qualitatively the $m_1 \to m_2$ limit of the more general case. Even in this simplified form, the calculation is rather extensive. The technical aspects are discussed in a companion paper,[13] and here we will be concerned only with those features of the interaction that are relevant to the Regge behavior of the theory.

In fourth order, the set of graphs that exhibits the Regge behavior consists of boxes, crossed-boxes, and second-order corrections. These diagrams are shown in Fig. 1. The amplitude represented by these graphs contains both t-channel and u-channel discontinuities, so that one must calculate the contributions due to each cut and then construct signatured amplitudes by using the prescription given in Eq. (2.8).

As we have seen in our previous discussion, for the purpose of studying the properties of daughters, only the nonpolynomial terms of the high-energy expansion of the amplitude are needed. This is a fortunate simplification in our case, for one can then set aside questions of renormalization which only have a bearing on the polynomial terms. Following the conventions defined in ACTM, with the spinor as the "number two" particle in the Jacob

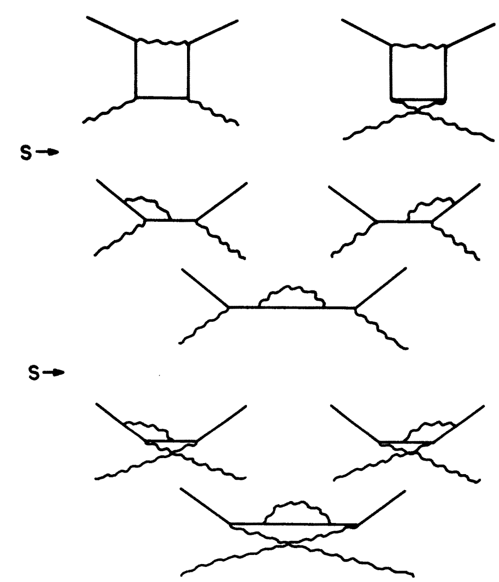

FIG. 1. Fourth-order diagrams for spinor-vector scattering.

and Wick formalism,[20] we find the following expressions for the nonpolynomial part of the helicity-dependent scattering amplitude:

(a) The right-hand-cut contributions for the six independent amplitudes are given by

$$\hat{T}_{1,1/2;\,1,1/2}=(-3\zeta+22\zeta^2-60\zeta^3+92\zeta^4)M_t+(1-28\zeta+148\zeta^2-196\zeta^3)N_t\,, \quad (4.1a)$$

$$\hat{T}_{1,1/2;\,0,-1/2}=\sqrt{2}(-2\zeta+14\zeta^2-38\zeta^3+44\zeta^4)M_t+\sqrt{2}(1-16\zeta+58\zeta^2-52\zeta^3)N_t\,, \quad (4.1b)$$

$$\hat{T}_{0,1/2;\,0,1/2}=(-2\zeta+28\zeta^2-112\zeta^3+154\zeta^4)M_t+(2-24\zeta+112\zeta^2-168\zeta^3)N_t\,, \quad (4.1c)$$

$$\hat{T}_{1,-1/2;\,0,1/2}=\sqrt{2}(6\zeta^2-26\zeta^3+20\zeta^4)M_t+\sqrt{2}(1-4\zeta+6\zeta^2-12\zeta^3)N_t\,, \quad (4.1d)$$

$$\hat{T}_{1,1/2;\,-1,1/2}=(-\zeta+14\zeta^2-64\zeta^3+84\zeta^4)M_t+(1-8\zeta+16\zeta^2-12\zeta^3)N_t\,, \quad (4.1e)$$

$$\hat{T}_{-1,1/2;\,-1,1/2}=(\zeta+6\zeta^2-36\zeta^3+44\zeta^4)M_t+(1-4\zeta+44\zeta^2-116\zeta^3)N_t\,, \quad (4.1f)$$

with $\zeta\equiv m^2/t$, $g^2\equiv e^2/4\pi$, and

$$T_{R\{\lambda_i\}}=ig^4(t-4m^2)^{1/2}[8m\pi^2(1-4m^2/t)^{3/2}]^{-1}\times\hat{T}_{\{\lambda_i\}}\,, \quad (4.2a)$$

$$M_t\equiv[(1-2m^2/t)(1-4m^2/t)]^{-1}\times\{\ln[(t-3m^2)/m^2]\}^2\,, \quad (4.2b)$$

$$N_t\equiv[1-3m^2/t]^{-1}\ln(4m^2/t)\,. \quad (4.2c)$$

(b) The left-hand-cut contributions are of the form

$$\hat{T}_{1,1/2;\,1,1/2}=(\xi-\xi^2-22\xi^3+22\xi^4)M_u+(5\xi-31\xi^2-25\xi^3+132\xi^4)N_u\,, \quad (4.3a)$$

$$\hat{T}_{1,1/2;\,0,-1/2}=\sqrt{2}(-\xi+4\xi^2)M_u+\sqrt{2}(4\xi-25\xi^2+54\xi^3-42\xi^4)N_u\,, \quad (4.3b)$$

$$\hat{T}_{0,1/2;\,0,1/2}=(2\xi-6\xi^2-4\xi^3+4\xi^4)M_u+(18\xi-62\xi^2+62\xi^3)N_u\,, \quad (4.3c)$$

$$\hat{T}_{1,-1/2;\,0,1/2}=\sqrt{2}(-\xi+6\xi^2-12\xi^3+4\xi^4)M_u+\sqrt{2}(2\xi-13\xi^2-4\xi^3+42\xi^4)N_u\,, \quad (4.3d)$$

$$\hat{T}_{1,1/2;\,-1,1/2}=(\xi-3\xi^2+6\xi^3-10\xi^4)M_u+(11\xi-19\xi^2+27\xi^3-48\xi^4)N_u\,, \quad (4.3e)$$

$$\hat{T}_{-1,1/2;\,-1,1/2}=(\xi-5\xi^2+2\xi^3+6\xi^4)M_u+(9\xi-55\xi^2+91\xi^3-36\xi^4)N_u\,, \quad (4.3f)$$

with $\xi\equiv m^2/u$, and

$$T_{L\{\lambda_i\}}=-g^4\sqrt{u}\,[8m\pi^2(1-4m^2/u)^{3/2}]^{-1}\hat{T}_{\{\lambda_i\}}\,, \quad (4.4a)$$

$$M_u\equiv[(1-m^2/u)(1-2m^2/u)]^{-1}\times\{\ln[(u-3m^2)/m^2]\}^2\,, \quad (4.4b)$$

$$N_u\equiv(1-4m^2/u)[(1-2m^2/u)(1-m^2/u)]^{-1}\times\ln(4m^2/u)\,. \quad (4.4c)$$

From these expressions one constructs parity-conserving amplitudes that are free of kinematic singularities, in accordance with Eq. (2.6). Expanding in an asymptotic series in t, and forming signatured amplitudes, one obtains the following expressions for the right-signatured amplitudes $A^{(\pm)+}_{i,j}\equiv[g^4/(4\sqrt{2}\,\pi^2)]\hat{A}^{(\pm)+}_{i,j}$:

$$\hat{A}^{(\pm)+}_{1,1}\sim[2+24/z+1/z^2-25/(4z^3)+\cdots]\ln(-z)+[2/z-1/z^2+19/(2z^3)-15/(2z^4)+\cdots][\ln(-1-2z)]^2, \quad (4.5a)$$

$$\hat{A}^{(\pm)+}_{0,1}\sim(\pm)\sqrt{2}[2+11/z-3/z^2+67/(4z^3)+\cdots]\ln(-z)+(\pm)\sqrt{2}[3/z-9/(2z^2)+29/(4z^3)-73/(8z^4)+\cdots][\ln(-1-2z)]^2, \quad (4.5b)$$

$$\hat{A}^{(\pm)+}_{0,0}\sim[4+24/z+6/z^2-1/(2z^3)+\cdots]\ln(-z)+[8/z^2-6/z^3+25/(2z^4)+\cdots][\ln(-1-2z)]^2, \quad (4.5c)$$

$$\hat{A}^{(\pm)+}_{-1,0}\sim(\pm)\sqrt{2}[-2/z+1/z^2-6/z^3+37/(4z^4)+\cdots]\ln(-z)+(\pm)\sqrt{2}[1/z^2-9/(2z^3)+21/(4z^4)-73/(8z^5)+\cdots][\ln(-1-2z)]^2, \quad (4.5d)$$

$$\hat{A}^{(\pm)+}_{-1,1}\sim[-2/z-12/z^2+23/z^3+153/(4z^4)+\cdots]\ln(-z)+[-6/z^3+3/z^4-51/(4z^5)+\cdots][\ln(-1-2z)]^2, \quad (4.5e)$$

$$\hat{A}^{(\pm)+}_{-1,-1}\sim[2/z+2/z^2+53/z^3-161/(4z^4)+\cdots]\ln(-z)+[-2/z^2+9/z^3-49/(2z^4)+49/z^5+\cdots][\ln(-1-2z)]^2. \quad (4.5f)$$

Similarly, the wrong-signatured amplitudes with $\hat{A}^{(\pm)-}$ defined above are found to be

$$\hat{A}^{(\pm)-}_{1,1} \sim [2+14/z-10/z^2+99/(4z^3)+\cdots]\ln(-z)+[4/z-7/z^2+15/(2z^3)-15/(2z^4)+\cdots][\ln(-1-2z)]^2, \tag{4.6a}$$

$$\hat{A}^{(\pm)-}_{0,1} \sim (\pm)\sqrt{2}\,[2+3/z-12/z^2+19/(4z^3)+\cdots]\ln(-z) + (\pm)\sqrt{2}\,[1/z-3/(2z^2)+23/(4z^3)-63/(8z^4)+\cdots][\ln(-1-2z)]^2, \tag{4.6b}$$

$$\hat{A}^{(\pm)-}_{0,0} \sim [4-12/z+16/z^2-29/(2z^3)+\cdots]\ln(-z)+[4/z+11/(2z^4)+\cdots][\ln(-1-2z)]^2, \tag{4.6c}$$

$$\hat{A}^{(\pm)-}_{-1,0} \sim (\pm)\sqrt{2}\,[-2/z+5/z^2-5/z^3-7/(4z^4)+\cdots]\ln(-z) + (\pm)\sqrt{2}\,[-1/z^2-3/(2z^3)-1/(4z^4)-23/(8z^5)+\cdots][\ln(-1-2z)]^2, \tag{4.6d}$$

$$\hat{A}^{(\pm)-}_{-1,1} \sim [-2/z+10/z^2-24/z^3+123/z^4+\cdots]\ln(-z)+[-2/z^2-10/z^4+27/(4z^5)+\cdots][\ln(-1-2z)]^2, \tag{4.6e}$$

$$\hat{A}^{(\pm)-}_{-1,-1} \sim [2/z-16/z^2+16/z^3-361/(4z^4)+\cdots]\ln(-z)+[9/z^3-49/(2z^4)+48/z^5+\cdots][\ln(-1-2z)]^2. \tag{4.6f}$$

These are the data that we need for the study of the daughters.

V. FEATURES OF THE AMPLITUDE

Consider our expressions for the nonpolynomial part of the fourth-order amplitude. The leading terms are proportional to $(-z)^0\ln(-z)$ for the sense-sense amplitudes, and to $(-z)^{-1}\ln(-z)$ for the nonsense amplitudes. These are the terms that were calculated by GGLMZ and were used to show the existence of the electron trajectory. The coefficients of these terms satisfy the factorization condition and our results find exact agreement with the values obtained by GGLMZ. These leading terms come only from the right-hand (t-channel) cut, thus conforming to the behavior expected for this process; namely, there are two mother trajectories [one with value $+\alpha(s)$ and even signature, the other with value $-\alpha(s)$ and odd signature], which together correspond to a leading behavior that derives alternately from t-channel cuts in even orders of the coupling constant g^2 and u-channel cuts in odd orders of g^2.

The next-to-leading-order terms in the amplitude are of the form $(-z)^{-m}\ln(-z)$ and $(-z)^{-n}\times[\ln(-z)]^2$, for m and n positive integers. Those terms linear in $\ln(-z)$ contain the contributions of the various trajectories in the theory; however, the presence of a contribution proportional to $[\ln(-z)]^2$ is a rather surprising result. In Sec. II we have shown that, to fourth order, such a contribution cannot have a moving trajectory as its source. Therefore, it suggests the existence of additional singularities in the complex angular-momentum plane, to the left of the electron trajectory. Before studying the daughter contributions, we consider the origin of these new terms.

There are several types of singularities in J that could conceivably give rise to this effect, such as cuts, fixed poles, and essential singularities. Unfortunately, in attempting to understand the origin of the $[\ln(-z)]^2$ terms, it is very difficult to discriminate amongst the various types of singularities, since the way they enter into the amplitude is not well understood and very much model-dependent. Most of the discussion in the literature regarding these singularities is limited to scalar models of the interaction, such as scalar ladder diagrams, and it is not clear that these results can be extended to the spin-dependent case, where shifting effects in J are possible[21] and *essential* complications exist.[13] For example, a scalar planar box diagram cannot give rise to terms proportional to $[\ln(-z)]^2$, whereas the spinor-vector planar box does. Nonetheless, we make use of these results to at least explore the qualitative effect that these singularities have, with the understanding that our results will be rather speculative in nature.

A possible argument that justifies the $[\ln(-z)]^2$ contributions through the existence of a cut is suggested if one views the box and crossed-box diagrams of Fig. 1 as depicting in some sense the exchange of two Regge poles—i.e., two electrons. The analytic properties associated with such an exchange have been studied for the scalar interaction by several authors,[22,23] who find that the iteration of two Regge poles produces a branch cut. This cut, with a branch point at $J=2\alpha(\frac{1}{4}s)-1$, is present whenever there is a third double-spectral function ρ_{tu}. Such a picture seems to suggest that a cut must exist, but, since spin can have a translational effect,[21] the position of the branch point might be shifted away from $J=0$. On the other hand, although the way in which a cut affects the asymptotic behavior of the scattering amplitude is not well understood, a crude model which assumes a partial-wave dependence of the form $[l-\alpha_c(s)]^p$, for some $p>0$ near the cut,[24] indicates that one might expect a contribution of the form $t^{\alpha_c(s)}/(\ln t)^{p+1}$ to the amplitude. Thus, if this is indeed the case, one could rule out a cut as the source of the $[\ln(-z)]^2$ terms.

A second type of singularity connected with the existence of ρ_{tu} is the singularity that was first noted by Gribov and Pomeranchuk[25] in connection with the Froissart-Gribov projection. This essential singularity is found by examining the discontinuities that are present in $Q_l(z)$ for negative integer l; however, such a singularity manifests itself only in the wrong-signature amplitudes, which is contrary to the behavior we observe.

The most plausible explanation is that offered by the existence of fixed poles at all negative half-integer values of J. The contribution $T^{FP}(s,t)$ that a fixed pole at $l=\omega$ with residue $g^4\lambda+O(g^6)$ would make to the scattering amplitude can be deduced from the Sommerfeld-Watson integral, provided that one can draw a contour around this pole. If we assume the pole to be of order n, we have as the contribution of the leading term

$$T^{FP}(s,t) \underset{t\to\infty}{\sim} \tfrac{1}{2}ig^4\lambda \times \oint dl(2l+1)P_l(-z)/[(l-\omega)^n \sin\pi l]$$

$$\sim g^4\lambda t^{\omega}(-\ln t)^{n-1}/(n-1)!\,. \tag{5.1}$$

Thus, a fixed pole of order 3 could give rise to a $[\ln(-z)]^2$ behavior in the amplitude which would justify the observed terms. Unfortunately, very little is known about the properties of these fixed poles and one is unable to draw any further implications. The only conclusion that one can form is that if these poles exist, they occur at all negative half-integer values of J, beginning with $J=-\frac{1}{2}$, since the leading coefficient of $[\ln(-z)]^2$ is z^{-1}.

VI. ISOLATION OF DAUGHTER CONTRIBUTIONS

Let us direct our attention to the terms linear in $\ln(-z)$; it is from these terms that we wish to extract information on the daughter trajectories of the theory. We have shown in Sec. II how the various trajectories exhibit themselves in the perturbation expansion, and how information on the fourth-order part of the residue matrix of a trajectory at $l=\alpha$ is contained in the coefficient of the term $(-z)^\alpha\ln(-z)$.

Since any trajectory above $l=\alpha$ can also contribute to $(-z)^\alpha\ln(-z)$, it is necessary, in order to identify the contribution due to the trajectory at a given $l=\alpha$, to determine what additional trajectories exist above it; one must then subtract out their contributions. Thus, for example, in the case of the daughter trajectory at $l=-1$ it is necessary to remove the mother trajectory and its wrong-signature twin. Once this is done, the matrix R that is necessary for the analysis discussed in Sec. III is determined.

In a similar way, one can determine R for the daughter trajectories at other negative integer values of l. However, for the purpose of studying the degeneracy question, the analysis of the daughter trajectory at $l=-1$ is easiest and sufficient.

The unexpected possible existence of fixed poles at the negative-integral values of l creates additional difficulties. This is evident from the fact that a fixed pole at $l=-1$, having an asymptotic behavior of the form $(-z)^{-1}[\ln(-z/z_0)]^2$ for some fixed z_0, could contribute to the terms linear in $\ln(-z)$ if z_0 was other than 1; i.e., if the scale of the asymptotic behavior was not really known. The true values of R would then be inaccessible. This concealment of the daughter contributions is not restricted to fixed poles, but could also come about if a cut or an essential singularity were present, since the closing path of the contour of integration in the Sommerfeld-Watson integral would have to be distorted. For the moment, let us disregard these possibilities and detail the subtraction procedure for isolating the contribution of the daughter at $l=-1$. We will then apply this procedure to the coefficients of the terms linear in $\ln(-z)$, neglecting the $[\ln(-z)]^2$ contributions.

The Regge asymptotic forms for the contributions of the two mother trajectories and the daughter at $l=-1$ have been given in Eqs. (2.11) and (2.12). It is the sum of these contributions that make up the scattering amplitude up to the second-leading-order term in z. To second order in the perturbation, this means terms of the form $(-z)^{-1}$ for the sense-sense amplitudes and of the form $(-z)^{-2}$ for the nonsense amplitudes, while to fourth order in the coupling constant they are terms proportional to $(-z)^{-1}\ln(-z)$ and $(-z)^{-2}\times\ln(-z)$, respectively. If one identifies the coefficients $C_{ij}^{(\pm)\pm}$ of these second-leading-order terms in z with an expansion of the form

$$C_{ij}^{(\pm)\pm}=g^2(C_1^{(\pm)\pm})_{ij}+g^4(C_2^{(\pm)\pm})_{ij}+\cdots\,, \tag{6.1}$$

one can write the following expression relating the mother and daughter contributions:

$$(C_1^{(\pm)\pm})_{\kappa,\nu}=\sqrt{2}\,(Q^{(\pm)\pm})_{\kappa,\nu}\epsilon_{\nu\kappa}\,, \tag{6.2a}$$

$$(C_1^{(\pm)\pm})_{-1,\nu}=\sqrt{2}\,[(\xi_1^{(\mp)\mp})_{-1,\nu}-(Q^{(\pm)\pm})_{-1,\nu}]\,, \tag{6.2b}$$

$$(C_1^{(\pm)\pm})_{-1,-1}=\sqrt{2}\,[3\mu_1^{\mp}(\xi_0^{(\mp)\mp})_{-1,-1}+(Q^{(\pm)\pm})_{-1,-1}]\,, \tag{6.2c}$$

$$(C_2^{(\pm)\pm})_{\kappa,\nu}=\sqrt{2}\,(R^{(\pm)\pm})_{\kappa,\nu}\epsilon_{\nu\kappa}\,, \tag{6.3a}$$

$$(C_2^{(\pm)\pm})_{-1,\nu}=\sqrt{2}\,[\mu_1^{\mp}(\xi_1^{(\mp)\mp})_{-1,\nu}-(R^{(\pm)\pm})_{-1,\nu}]\,, \tag{6.3b}$$

$$(C_2^{(\pm)\pm})_{-1,-1}=\sqrt{2}\,[3(\mu_1^{\mp})^2(\xi_0^{(\mp)\mp})_{-1,-1}+(R^{(\pm)\pm})_{-1,-1}]\,. \tag{6.3c}$$

The matrices Q and R have been defined previously, and they are the quantities that we need in order to carry out the analysis described in Sec. III. Q, which is the sum of the residues of the daughter trajectories, is given in the work of ACTM, who calculated the second-order amplitude and carried out the partial-wave inversion.[26] However, for the purposes of illustration and corroboration we reevaluate Q in this paper using our subtraction method.

One can identify the matrix coefficients $C_1^{(\pm)\pm}$ by expanding the second-order amplitude of ACTM in a power series in $(-z)^{-1}$. Also, from the leading term in this expansion, one can determine through the use of Eq. (2.11) the values for the mother trajectory and its residue which are needed in order to apply Eqs. (6.2) and (6.3). This procedure yields the following results:

$$C_1^{(\pm)\pm} = \pm(2\sqrt{2}\,\pi)^{-1}\begin{bmatrix} 9 & (\pm)\sqrt{2} & 1 \\ (\pm)\sqrt{2} & 2 & (\pm)\sqrt{2} \\ 1 & (\pm)\sqrt{2} & 1 \end{bmatrix}, \tag{6.4}$$

$$(\xi_1^{(\pm)\pm})_{-1,\nu} = \pm(4\pi)^{-1}(2, (\pm)\sqrt{2}) \tag{6.5}$$

$$\mu_1^{\mp}(\xi_0^{(\mp)\mp})_{-1,-1} = \mp(2\pi)^{-1}, \tag{6.6}$$

$$\mu_1^{\pm} = \mp(2\pi)^{-1}. \tag{6.7}$$

Inserting these values into the equations for Q we find

$$Q^{(\pm)\pm} = \pm(4\pi)^{-1}\begin{bmatrix} 9 & (\pm)\sqrt{2} & 1 \\ (\pm)\sqrt{2} & 2 & (\pm)\sqrt{2} \\ 1 & (\pm)\sqrt{2} & 7 \end{bmatrix}, \tag{6.8}$$

which is in complete agreement with ACTM.

To determine the matrix R we obtain the coefficients $C_2^{(\pm)\pm}$ from Eqs. (4.5) and (4.6). In so doing we disregard any contributions to the $\ln(-z)$ terms that come from writing

$$[\ln(-1-2z)]^2 \simeq [\ln(-z)]^2 + 2\ln 2[1-(2z)^{-1}]\ln(-z) + f(z^{-1}),$$

where $f(z^{-1})$ is a polynomial in z^{-1}. This is equivalent to the assumption that the fixed-pole contributions are distinguishable and purely $[\ln(1-2z)]^2$ times a polynomial in z^{-1}. We then have the following expressions:

$$C_2^{(\pm)+} = (4\sqrt{2}\,\pi^2)^{-1}\begin{bmatrix} -24 & (\mp)11\sqrt{2} & -12 \\ (\mp)11\sqrt{2} & -24 & (\pm)\sqrt{2} \\ -12 & (\pm)\sqrt{2} & 2 \end{bmatrix}, \tag{6.9a}$$

$$C_2^{(\pm)-} = (4\sqrt{2}\,\pi^2)^{-1}\begin{bmatrix} -14 & (\mp)3\sqrt{2} & 10 \\ (\mp)3\sqrt{2} & 12 & (\pm)5\sqrt{2} \\ 10 & (\pm)5\sqrt{2} & -16 \end{bmatrix}. \tag{6.9b}$$

From these coefficients, the values for the matrix R follow; they are given by

$$R^{(\pm)+} = (8\pi^2)^{-1}\begin{bmatrix} -24 & (\mp)11\sqrt{2} & 10 \\ (\mp)11\sqrt{2} & -24 & (\mp)3\sqrt{2} \\ 10 & (\mp)3\sqrt{2} & -4 \end{bmatrix}, \tag{6.10a}$$

$$R^{(\pm)-} = (8\pi^2)^{-1}\begin{bmatrix} -14 & (\mp)3\sqrt{2} & -12 \\ (\mp)3\sqrt{2} & 12 & (\mp)7\sqrt{2} \\ -12 & (\mp)7\sqrt{2} & -22 \end{bmatrix}. \tag{6.10b}$$

An examination of Q and R for each signature shows that these matrices do not commute, which implies that the degeneracy of the daughter trajectories is removed by the fourth-order interaction. An explicit numerical solution can also be computed by using the technique outlined in Sec. III. The matrix $Q^{(\pm)\pm}$ can be rewritten by means of a similarity transformation as

$$\sigma^{\pm} = (4\pi)^{-1}\,\mathrm{diag}(1.4567, 6.6845, 9.8588),$$

with eigenvectors given by

$$\vec{x}_1^{(\pm)\pm} = (0.1518, (\mp)0.9640, 0.2185),$$

$$\vec{x}_2^{(\pm)\pm} = (-0.4578, (\pm)0.1274, 0.8799),$$

$$\vec{x}_3^{(\pm)\pm} = (0.8760, (\pm)0.2336, 0.4220).$$

Similarly, for $R^{(\pm)\pm}$ we find

$$\lambda^{+} = (8\pi^2)^{-1}\,\mathrm{diag}(-40.0608, -16.2618, 4.3226),$$

$$\lambda^{-} = (8\pi^2)^{-1}\,\mathrm{diag}(-33.1094, -5.6010, 14.7104),$$

with their eigenvectors given by

$$\vec{y}_1^{(\pm)+} = (0.7289, (\pm)0.6736, -0.1222),$$

$$\vec{y}_2^{(\pm)+} = (-0.4800, (\pm)0.6307, 0.6097),$$

$$\vec{y}_3^{(\pm)+} = (0.4882, (\mp)0.3854, 0.7830),$$

$$\vec{y}_1^{(\pm)-} = (0.5536, (\pm)0.2279, 0.8010),$$

$$\vec{y}_2^{(\pm)-} = (-0.8319, (\pm)0.1059, 0.5448),$$

$$\vec{y}_3^{(\pm)-} = (0.03931, (\mp)0.9678, 0.2482).$$

From these expressions one can construct the matrix M and solve for the trajectory slopes $\delta_1^{(k)}$ and for the factoring residues. These are found to be given by the following quantities:

$$\delta_1^{+} = (2\pi)^{-1}\mathrm{diag}(-13.179, -2.989, 0.7520),$$

$$\delta_1^{-} = (2\pi)^{-1}\mathrm{diag}(-3.758, -0.7611, 9.935),$$

$\vec{V}_1^{(\pm)+} = (4\pi)^{-1}(-0.9989, (\mp)1.346, 0.2029)$,

$\vec{V}_2^{(\pm)+} = (4\pi)^{-1}(-2.143, (\pm)0.2561, 1.505)$,

$\vec{V}_3^{(\pm)+} = (4\pi)^{-1}(1.869, (\pm)0.3400, 2.155)$,

$\vec{V}_1^{(\pm)-} = (4\pi)^{-1}(1.546, (\pm)0.7544, 2.411)$,

$\vec{V}_2^{(\pm)-} = (4\pi)^{-1}(-2.571, (\mp)0.08462, 1.091)$,

$\vec{V}_3^{(\pm)-} = (4\pi)^{-1}(-0.02623, (\mp)1.193, 0.2639)$.

The residues can then be reconstructed by taking the outer product of each $\vec{V}_k$ with itself.

The values for the trajectory slopes $\delta_1^{(k)}$, which are obtained under the assumption of total decoupling of $\ln(-z)$ and $[\ln(-1-2z)]^2$ terms, can be seen to be incorrect. The reason for this is that the Lorentz-pole analysis of ACTM, which is independent of the degenerate nature of the daughter trajectories, still applies and requires that at least one of the trajectories be exactly one unit below the mother trajectory at $s=0$. This is not verified by the calculations and one concludes that the additional singularities encountered undermine the ability to extract the daughter behavior.

VII. DISCUSSION

We have studied the interaction of electrons and neutral vector mesons within the framework of fourth-order perturbation theory in order to make use of such a laboratory of Feynman graphs so as to learn more about the nature of the degenerate daughter trajectories that are associated with the electron. In our analysis, we have shown how one can extract the daughter contributions from an amplitude which is assumed to be purely Regge-like, and how one can use this information in fourth order to test for the breaking of the degeneracy of the daughter trajectories. The application of these ideas to the calculated data has exhibited a significant new phenomenon; namely, the existence of additional singularities in J to the left of the electron trajectory. These singularities by their nature dominate the next-to-leading-order behavior in the amplitude and complicate the separation and identification of the daughter trajectories. Thus we find that a simple-minded approach to the separation of the daughter contributions fails to give consistent results.

Although these additional singularities seem to be justified by ascribing them to fixed poles of order three at each negative half-integer J, our identification is to a large extent heuristic and a careful study of the properties of cuts, fixed poles, and essential singularities is needed in order to properly deal with this question. In particular, one would like to know the exact form of the contributions of these singularities to the perturbation expansion in order to enable one to extract the daughter terms. Also, since these fixed poles constitute the next-to-leading-order terms in the amplitude, understanding the role that they play in the interaction is essential to developing added insight on the kinds of corrections needed in Regge models.

For these reasons, it would be useful to turn once again to our laboratory of Feynman graphs and calculate the nonleading behavior of the sixth-order terms. Such a calculation would be able to elucidate some of the questions we have posed. For example, the $u^{-1}(\ln u)^2$ terms would now embody the daughter trajectories as well as the fixed poles, thereby giving additional constraints helpful in identifying the daughter contributions. It would also be interesting to see whether this calculation corroborates the explanation of our findings in terms of fixed poles. The elucidation of these questions, however, is not an easy task; and the complicated behavior that we have exhibited points out in some sense the limitations of the perturbative approach.

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APPENDIX A

We list below the asymptotic series expansions for the functions $E_{\mu\lambda}^{J}(z)$ of GGLMZ that are relevant to the (spin-$\frac{1}{2}$)-(spin 1) scattering process:

$$E_{1/2,1/2}^{J+}(z) = K_l z^l \left\{1 + \sum_{n=1}^{\infty} \frac{(-1)^n}{n!} \frac{l(l-1)\cdots[l-(2n-1)]}{(l+\frac{1}{2})\cdots[l-\frac{1}{2}(2n-3)]}(2z)^{-2n}\right\},$$

$$E_{1/2,1/2}^{J-}(z) = -K_l[l/(2l+1)]z^{l-1}\left\{1 + \sum_{n=1}^{\infty} \frac{(-1)^n}{n!} \frac{l(l-1)\cdots[l-2n]}{(l-\frac{1}{2})\cdots[l-\frac{1}{2}(2n-1)]}(2z)^{-2n}\right\},$$

$$E_{1/2,3/2}^{J+}(z) = -E_{3/2,1/2}^{J+}(z) = K_l\{l/[l(l+2)]^{1/2}\}z^{l-1}\left\{1 + \sum_{n=1}^{\infty} \frac{(-1)^n}{n!} \frac{(l-1)\cdots(l-2n)}{(l+\frac{1}{2})\cdots[l-\frac{1}{2}(2n-3)]}(2z)^{-2n}\right\},$$

$$E^{J-}_{1/2,3/2}(z) = -E^{J-}_{3/2,1/2}(z) = -K_l\{l(l-1)/(2l+1)[l(l+2)]^{1/2}\}z^{l-2}\left\{1+\sum_{n=1}^{\infty}\frac{(-1)^n}{n!}\frac{(l-2)\cdots[l-(2n+1)]}{(l-\frac{1}{2})\cdots[l-\frac{1}{2}(2n-1)]}(2z)^{-2n}\right\},$$

$$E^{J+}_{3/2,3/2}(z) = K_l[1/(l+2)]z^{l-1}\left\{l+\sum_{n=1}^{\infty}\frac{(-1)^n}{n!}\frac{(l-1)\cdots(l-2n)}{(l+\frac{1}{2})\cdots[l-\frac{1}{2}(2n-3)]}(l-4n)(2z)^{-2n}\right\},$$

$$E^{J-}_{3/2,3/2}(z) = -K_l[(l-1)/(l+2)(2l+1)]z^{l-2}\left\{3l+\sum_{n=1}^{\infty}\frac{(-1)^n}{n!}\frac{(l-2)\cdots[l-(2n+1)]}{(l-\frac{1}{2})\cdots[l-\frac{1}{2}(2n-1)]}(3l-4n)(2z)^{-2n}\right\},$$

where

$$J = l+\tfrac{1}{2}, \quad K_l = [\sqrt{2}\,2^l\Gamma(l+\tfrac{3}{2})]/\sqrt{\pi}\,\Gamma(l+2).$$

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†Present address: Aerojet Electro Systems Company, Azusa, California 91702.

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