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Gauge model for chiral-symmetry breaking and muon-electron mass ratio*

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We present a unified gauge model of weak and electromagnetic interactions, which has the following features: (a) Chiral $U(2)_L \times U(2)_R$ is an exact "natural" symmetry of strong interactions in zeroth order and therefore its violations (which arise in order g^4) are computable, and adjusting parameters suitably we can get $(\sqrt{2}\epsilon_0+\epsilon_8)/(\sqrt{2}\epsilon_0-2\epsilon_8)\sim\alpha$; (b) violations of isospin symmetry as observed in, for example, $\Delta I\neq 0$ mass differences are, then, computable and of order α; (c) this model can be extended to include leptons in such a way that m_e/m_μ is computable and is of order g^4. We also speculate on the possibility of spontaneous *CP* violation arising as a higher-order effect.

I. INTRODUCTION

It has recently been suggested[1] that gauge theories[2] may provide an elegant framework for understanding the approximate symmetries of nature. This comes about because of the interplay between the constraints of gauge invariance and renormalizability in the Lagrangian, which imposes restrictions on the nature of the fermion (and/or Higgs boson) mass matrix to zeroth order in the gauge couplings. The restrictions may arise either due to the choice of representations[1] of the gauge group to which the fermions and Higgs bosons are assigned, or due to constraints on the vacuum expectation values[3] of Higgs fields arising under such circumstances. In either case, since in general gauge couplings violate the symmetries of the zeroth-order mass matrix, the radiative corrections will modify the zeroth-order mass relations and their contributions will have to be finite[1,4] in order to be consistent with renormalizability. Attempts have been made to make use of this feature of gauge theories to understand the p-n mass difference,[4,5] μ-e mass ratio,[3] Cabibbo angle,[6] etc. In this paper, we will discuss the possibility of understanding the origin of approximate chiral $U(2)_L\times U(2)_R$ invariance of strong interactions and the μ-e mass ratio in terms of a unified gauge model of leptons and hadrons exploiting the principles mentioned above.

We will present a gauge model which realizes the required zeroth-order symmetry (i.e., $m_{\mathfrak{N}} = m_{\mathcal{P}} = 0$ and $m_e = 0$, $\mathfrak{N}$ and $\mathcal{P}$ stand for neutron and proton quark) to yield calculable breaking of chiral $U(2)_L\times U(2)_R$ as well as electron mass in higher orders in gauge coupling. The model outlined in Sec. II requires only four SU(4) quarks ($\mathcal{P}, \mathfrak{N}, \lambda, \mathcal{P}'$) of Glashow, Iliopoulos, and Maiani[7] and four observed leptons (ν_e, e^-, μ^-, and ν_μ). In zeroth order (and in order g^2 as well), only λ and $\mathcal{P}'$ quarks as well as μ^- and ν_μ are massive and the remaining fermions are massless. However, to order g^4 in gauge couplings, $\mathcal{P}$ quark, $\mathfrak{N}$ quark, and the electron and ν_e acquire mass. The masses of the $\mathcal{P}$ and the $\mathfrak{N}$ quarks (which are now computable) break chiral $U(2)_L\times U(2)_R$ symmetry of strong interactions and in conventional notation we get $(\sqrt{2}\,\epsilon_0+\epsilon_8)/(\sqrt{2}\,\epsilon_0-2\epsilon_8)\sim g_L{}^4$, where we arrange vacuum expectation values so that the magnitude of this ratio is of order α. In such a model, chiral $U(2)_L\times U(2)_R$ symmetry of strong interactions must be spontaneously broken, with pions (and a light mass η) being the corresponding Goldstone bosons. As is well known,[8] pion mass, in such a philosophy, is proportional to $\sqrt{2}\,\epsilon_0+\epsilon_8$ and therefore, in our model, will arise as a purely weak and electromagnetic effect. This approach is in general agreement with the philosophy of chiral-symmetry breaking advocated by Gell-Mann, Oakes, and Renner.[8] Note that all our above statements are independent of how one chooses to introduce strong interactions into the gauge models, as long as renormalizability is not destroyed in so doing.

The electron mass also arises in fourth order in the gauge couplings and is proportional to the muon mass. Since the masses of $\mathfrak{N}$ and e^- arise in order g^4, their apparent magnitude is smaller than one would like; but in this model, one has a large number of free parameters and one can arrange their magnitudes to get a larger magnitude for their values.

To construct the above model, we use the gauge group $SU(2)_L\times SU(2)_R\times SU(2)_A\times U(1)$, with electric charge given by $Q=I_{3L}+\frac{1}{2}Y$, where Y is the U(1) quantum number. There is a basic asymmetry between the left- and right-handed gauge groups. The role of the gauge group $SU(2)_A$ is merely to ensure the computability conditions. Not surprisingly, a number of Higgs multiplets are required in this model. In an appendix, we present a slight generalization of this model involving six quarks and six leptons, where $U(3)_L\times U(3)_R$ is a "natural" zeroth-order symmetry of strong interactions, and therefore chiral-symmetry breaking arising in higher orders is computable. In a second appendix, we speculate on a possible spontaneous generation of *CP* violtaion in higher orders in such models.

II. CONSTRUCTION OF THE MODEL

We work with the gauge group $SU(2)_L\times SU(2)_R \times SU(2)_A\times U(1)$. We denote its representations by (x, y, z, Y) and as mentioned earlier, the electric charge is given by $Q=I_{3L}+\frac{1}{2}Y$, where I_{3L} is one of the $SU(2)_L$ generators and Y is the U(1) generator. We have four quarks and four leptons in our model. We denote them by $(\mathcal{P}_0, \mathfrak{N}_0, \lambda_0, \mathcal{P}_0')$ and $(\nu_0, e_0^-, \mu_0^-, \nu_0')$. The subscript zero is used to denote the fact that the mass matrix is not diagonal in these fields. The physical quarks and leptons which are eigenstates of the mass matrix are linear combinations of the above fields and will be denoted by $(\mathcal{P}, \mathfrak{N}, \lambda, \mathcal{P}')$ and $(\nu_e, e^-, \mu^-, \nu_\mu)$. The fermions and Higgs bosons are assigned to the following representation of the gauge group.

Quarks		Representation content
$\psi_{1L}=\begin{pmatrix}\mathcal{P}_0\\ \mathfrak{N}_0\end{pmatrix}$,	$\psi_{2L}=\begin{pmatrix}\mathcal{P}_0'\\ \lambda_0\end{pmatrix}$	$(\frac{1}{2}, 0, 0, 1)$
$\psi_{1R}=\begin{pmatrix}\lambda_0\\ \mathfrak{N}_0\end{pmatrix}_R$		$(0, \frac{1}{2}, 0, 0)$
$\psi_{2R}=\begin{pmatrix}\mathcal{P}_0\\ \mathcal{P}_0'\end{pmatrix}_R$		$(0, \frac{1}{2}, 0, 2)$

(1)

Leptons		Representation content
$\psi_{3L}=\begin{pmatrix}\nu_0\\ e_0^-\end{pmatrix}_L$,	$\psi_{4L}=\begin{pmatrix}\nu_0'\\ \mu_0^-\end{pmatrix}_L$	$(\frac{1}{2}, 0, 0, -1)$
$\psi_{3R}=\begin{pmatrix}\mu_0^-\\ e_0^-\end{pmatrix}_R$		$(0, \frac{1}{2}, 0, -2)$
$\psi_{4R}=\begin{pmatrix}\nu_0\\ \nu_0'\end{pmatrix}_R$		$(0, \frac{1}{2}, 0, 0)$

(2)

Higgs bosons	Representation content
$\sigma=\begin{pmatrix}\sigma_1^+ & \sigma_2^+\\ \sigma_1^0 & \sigma_2^0\end{pmatrix}$	$(\frac{1}{2}, \frac{1}{2}, 0, 1)$
$\theta=\begin{pmatrix}\theta_{11} & \theta_{21} & \theta_{31}\\ \theta_{12} & \theta_{22} & \theta_{32}\end{pmatrix}$	$(0, \frac{1}{2}, 1, 0)$
$\pi=\begin{pmatrix}\pi_1^+ & \pi_2^+\\ \pi_1^0 & \pi_2^0\end{pmatrix}$	$(\frac{1}{2}, 0, \frac{1}{2}, 1)$
$\chi=\begin{pmatrix}\chi^0\\ \chi^{0\prime}\end{pmatrix}$	$(0, \frac{1}{2}, 0, 0)$

(3)

The following pattern of vacuum expectation values of the Higgs fields can be chosen consistent with an extremum of the potential (which can presumably be shown to be a minimum also):

$$\langle\sigma\rangle=\frac{1}{\sqrt{2}}\begin{pmatrix}0 & 0\\ \kappa & 0\end{pmatrix},\quad \langle\chi\rangle=\frac{1}{\sqrt{2}}\begin{pmatrix}a\\ 0\end{pmatrix},\quad \langle\theta\rangle=\frac{1}{\sqrt{2}}\begin{pmatrix}0 & d & 0\\ c & 0 & 0\end{pmatrix}\text{ and }\langle\pi\rangle=\frac{1}{\sqrt{2}}\begin{pmatrix}0 & 0\\ e_1 & e_2\end{pmatrix}. \tag{4}$$

To obtain the most general allowed gauge-invariant Yukawa coupling $\mathcal{L}_Y$ of fermions and Higgs bosons, we note the following transformation properties of quarks and the Higgs bosons [omitting the U(1) transformations]:

$$\begin{aligned}&\psi_{iL}\to W_2\psi_{iL},\quad \psi_{iR}\to U_2\psi_{iR},\\ &\sigma\to W_2\sigma U_2^\dagger,\quad \theta\to U_2\theta V_3^\dagger,\quad \pi\to W_2\pi V_2^\dagger,\\ &\chi\to U_2\chi,\quad \hat{\sigma}=\tau_2\sigma^*\tau_2\to W_2\hat{\sigma}U_2^\dagger,\end{aligned} \tag{5}$$

where W_n, U_n, and V_n denote the unitary representations of $SU(2)_L$, $SU(2)_R$, and $SU(2)_A$ with dimension n. This gives

$$\mathcal{L}_Y=\sum_{i=1,2}h_i\overline{\Psi}_{iL}\sigma\Psi_{1R}+\sum_{i=1,2}h_i'\overline{\Psi}_{iL}\hat{\sigma}\Psi_{2R}+\sum_{i=3,4}h_i\overline{\Psi}_{iL}\sigma\Psi_{3R}+\sum_{i=3,4}h_i'\overline{\Psi}_{iL}\hat{\sigma}\Psi_{4R}. \tag{6}$$

The quark and lepton mass matrices obtained from Eq. (6) are

$$\frac{\kappa}{\sqrt{2}}(\overline{\lambda}_{0L}\ \overline{\mathfrak{N}}_{0L})\begin{pmatrix}h_2 & 0\\ h_1 & 0\end{pmatrix}\begin{pmatrix}\lambda_{0R}\\ \mathfrak{N}_{0R}\end{pmatrix}-\frac{\kappa}{\sqrt{2}}(\overline{\mathcal{P}}_{0L}'\ \overline{\mathcal{P}}_{0L})\begin{pmatrix}h_2' & 0\\ h_1' & 0\end{pmatrix}\begin{pmatrix}\mathcal{P}_{0R}'\\ \mathcal{P}_{0R}\end{pmatrix} \tag{7}$$

for quarks, and

$$\frac{\kappa}{\sqrt{2}}(\overline{\mu}_{0L}\ \overline{e}_{0L})\begin{pmatrix}h_4 & 0\\ h_3 & 0\end{pmatrix}\begin{pmatrix}\mu_{0R}\\ e_{0R}\end{pmatrix}-\frac{\kappa}{\sqrt{2}}(\overline{\nu}_{0L}'\ \overline{\nu}_{0L})\begin{pmatrix}h_4' & 0\\ h_3' & 0\end{pmatrix}\begin{pmatrix}\nu_0'\\ \nu_0\end{pmatrix}_R\text{ for leptons.} \tag{8}$$

It is clear that the mass matrices M can be diagonalized by simple rotation of the left-handed parts and each M has only one nonzero eigenvalue. We will identify the eigenstates with zero eigenvalues with $\mathfrak{N}$, $\mathcal{P}$ and e^-, ν_e, respectively. The ψ's can be expressed in terms of the physical fields as

$$\psi_{1L}=\begin{pmatrix}\mathcal{P}_L\cos\theta+\mathcal{P}_L'\sin\theta\\ \mathfrak{N}_L\cos\theta'+\lambda_L\sin\theta'\end{pmatrix},$$

$$\psi_{2L}=\begin{pmatrix}-\mathcal{P}_L\sin\theta+\mathcal{P}_L'\cos\theta\\ -\mathfrak{N}_L\sin\theta'+\lambda_L\cos\theta'\end{pmatrix},$$

$$\psi_{1R}=\begin{pmatrix}\lambda_R\\ \mathfrak{N}_R\end{pmatrix},\quad \psi_{2R}=\begin{pmatrix}\mathcal{P}_R\\ \mathcal{P}_R'\end{pmatrix}, \tag{9}$$

$$\psi_{3L}=\begin{pmatrix}\nu_e\cos\phi'+\nu_\mu\sin\phi'\\ e^-\cos\phi+\mu^-\sin\phi\end{pmatrix}_L,$$

$$\psi_{4L}=\begin{pmatrix}\nu_\mu\cos\phi'-\nu_e'\sin\phi'\\ \mu^-\cos\phi-e^-\sin\phi\end{pmatrix}_L,$$

$$\psi_{3R}=\begin{pmatrix}\mu_R\\ e_R\end{pmatrix},\quad \psi_{4R}=\begin{pmatrix}\nu_e\\ \nu_\mu\end{pmatrix}_R,$$

where $\tan\theta=h_2'/h_1'$, $\tan\theta'=h_2/h_1$, $\tan\phi=h_4/h_3$, $\tan\phi'=h_4'/h_3'$.

Note that we have ensured the vanishing of $\mathfrak{N}$ quark, $\mathcal{P}$ quark, and the electron and ν_e mass in zeroth order in a "natural" manner, and therefore their values arising in higher orders are computable. Also $m_{\mathcal{P}}=m_{\mathfrak{N}}=0$ ensures that chiral $U(2)_L\times U(2)_R$ is an exact zeroth-order symmetry of the mass matrix and, therefore, of strong interactions [since gauge invariance and renormalizability of the weak interactions alone force the strong interactions in four-quark models to be chiral-$U(4)_L\times U(4)_R$-invariant[9]]. In higher orders contributions to the mass matrix will arise which violate $U(2)_L\times U(2)_R$ symmetry and the magnitude of this violation will be finite and computable. Now, we would like to demonstrate how the $\mathfrak{N}$ quark acquires mass in order g^4 in the gauge couplings. (Similar arguments will hold for the $\mathcal{P}$ quark and the electron and ν_e.) Notice for this purpose, that the $\mathfrak{N}$ quark remained massless in zeroth order because the $\mathfrak{N}_R$ was completely decoupled from $\mathfrak{N}_L$ and λ_L in this order (and therefore the determinant of the mass matrix $\det M$ was zero; this meant that at least one of its eigenvalues must vanish). Let us first see what happens in second order. First of all, we see that $\langle\sigma_2\rangle$, which was zero to lowest order, does become nonzero due to the diagrams shown in Figs. 1 and 2 and, incidentally, this is finite and computable. Since there is the coupling in $\mathcal{L}_Y$, i.e., $\overline{\mathfrak{N}}_R\sigma_2^0(h_1\mathfrak{N}_L+h_2\lambda_L)+\text{H.c.}$, a nonvanishing $\langle\sigma_2^0\rangle$ induces new terms into the mass matrix in the $\mathfrak{N}$-λ sector, which now becomes

$$M'=\begin{pmatrix}h_2 & \langle\sigma_2^0\rangle h_2\\ h_1 & \langle\sigma_2^0\rangle h_1\end{pmatrix}\frac{\kappa}{\sqrt{2}}. \tag{10}$$

However, this does not help in giving mass to the $\mathfrak{N}$ quark since $\det M'=0$, because $\langle\sigma_2^0\rangle$ is a function of h_1h_2 and does not depend on h_1 and h_2 separately, and therefore, since $m_\lambda\neq 0$, $m_{\mathfrak{N}}$ remains zero to this order. Now we go to fourth order in the coupling constants. In this case, the diagram shown in Fig. 3 contributes to the quark mass matrix. The new $\mathfrak{N}$-λ mass matrix now becomes

$$M''=\begin{pmatrix}h_2 & \langle\sigma_2^0\rangle h_2+Ah_1\\ h_1 & \langle\sigma_2^0\rangle h_1+Ah_2\end{pmatrix}\frac{\kappa}{\sqrt{2}}, \tag{11}$$

where A is finite and represents the magnitude of the contribution of Fig. 3 and, more importantly, is independent of h_1 and h_2. Therefore, it is clear that

$$\det M''=A(h_2{}^2-h_1{}^2)\neq 0,\quad \text{if } h_2=h_1. \tag{12}$$

This therefore gives mass to the $\mathfrak{N}$ quark, which is finite and computable and is expressible in terms of m_λ.

Since the masses of $\mathfrak{N}$, $\mathcal{P}$ quark, and the electron, in this model, arise in fourth order in the gauge couplings, one may legitimately ask: Are they too small to be physically relevant? In this paragraph, we will argue that this need not be the case. Our point is simply the following: We can choose g_L and g_R to be much larger than e, whereas g' (the Abelian gauge coupling) will be chosen to be of order e [since $e=g_Lg'/(g_L{}^2+g'^2)^{1/2}$] so that the fourth-order contribution

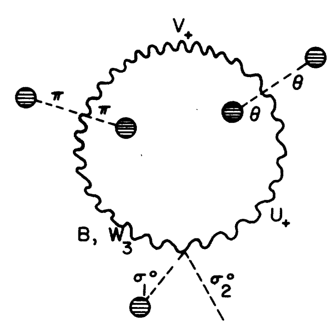

FIG. 1. The diagram that makes nonzero and finite contribution to $\langle\sigma_2^0\rangle$; W, U, and V represent the gauge bosons corresponding to $SU(2)_L$, $SU(2)_R$, and $SU(2)_A$ gauge groups.

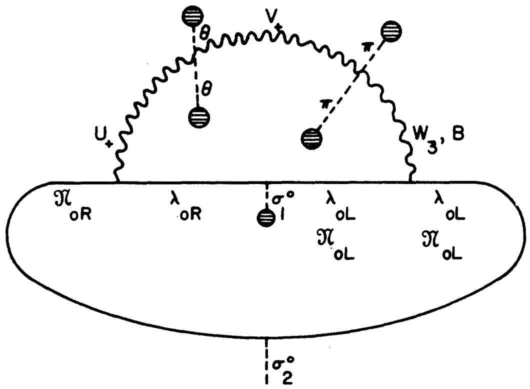

FIG. 2. Second-order diagram linking $\mathfrak{N}_{0R}$ to λ_{0L} and $\mathfrak{N}_{0L}$, and thereby making a finite contribution to $\langle\sigma_2^0\rangle$.

is really of order α (we, of course, arrange the value of $\langle\theta\rangle$ and $\langle\pi\rangle$ to be large enough, thus making this diagram numerically large). In other words, we believe, it is possible to make $m_{\mathfrak{N}}/m_\lambda \sim \alpha$ (and also $m_e/m_\mu \sim \alpha$ and $m_{\mathcal{P}}/m_{\mathcal{P}'} \sim \alpha$) by suitable choice of coupling constants and other parameters without running afoul of any known sacred principles or observations.

III. DISCUSSION OF THE MODEL AND CONCLUSION

Whether or not one believes in this model reproducing reality of the lepton-hadron interactions, it appears to provide a very economic way to understand important questions such as the origin of chiral $U(2)_L \times U(2)_R$ breaking, the approximate nature of isospin symmetry, and the muon-electron mass ratio within a gauge theory framework; moreover, we would like to stress that this model does respect all the observed weak-interaction selection rules and is certainly not in conflict with any experimental information known at present. This section will be devoted to a discussion of this and other related questions regarding the model.

(a) This model does not have any $\Delta S=1$, $\Delta Q=0$ coupling to lowest order in the left-handed sector, as is clearly seen by looking at Eq. (9). Although such a coupling will be introduced in fourth order, usual arguments will suppress it from being alarmingly large.[7] In the right-handed sector, there exists such a coupling, i.e., $g_R \bar{\mathfrak{N}}_R \gamma_\mu \lambda_R U_{+\mu}$ and this will predict a strength g_R^2/m_U^2 for the decay $K_L^0 \to \mu^+ e^-$, whose strength has an experimental upper bound of $G_F \alpha^2$; therefore, if $g_R^2 \sim e/4\pi$, m_U must be of order 10^4 to 10^5 GeV and this can be achieved by making $\langle\chi\rangle$ very large.

(b) In the lepton sector, this model has μ-e universality. However, there is also a μ-e transition as well as $\nu_e \to \mu$ and $\nu_\mu \to e$ transitions; but a suitable choice of the angle ϕ makes them consistent with the present experimental limits. Also, in our model, ν_e and ν_μ must be massive; however, since the magnitude of m_{ν_μ} is arbitrary, this can be adjusted to stay within experimental limits.

(c) The model is free of Adler-type anomalies due to mutual cancellation between lepton and quark contributions in the triangle graph.

(d) We would like to conclude with a few brief remarks about the relevance of this way of chiral-symmetry breaking. The general strong-interaction Hamiltonians in such models can be written as

$$\mathcal{H} = \mathcal{H}_0 + \epsilon_0 U_0 + \epsilon_8 U_8 + \epsilon_3 U_3 . \tag{13}$$

In the present model, to zeroth order, we should have $\epsilon_3 = 0$ and $\sqrt{2}\,\epsilon_0 + \epsilon_8 = 0$, the first implying that isospin is a natural symmetry and the second that chiral $U(2)_L \times U(2)_R$ is a natural symmetry of strong interactions. As a result of higher-order effects, we find

$$\frac{\sqrt{2}\,\epsilon_0 + \epsilon_8}{\sqrt{2}\,\epsilon_0 - 2\epsilon_8} \sim \alpha \text{ and, of course, } \epsilon_3 \sim \alpha. \tag{14}$$

Therefore, the smallness of $U(2)_L \times U(2)_R$ breaking and of the pion mass (when pion is treated as a Goldstone boson) seem to have a natural explanation in the context of gauge theories as a higher-order effect.[10] In this scheme,[11] the paradox of $\eta \to 3\pi$ decay would no longer exist, as has been discussed in Ref. 9, since a resolution of the $\eta \to 3\pi$ puzzle in gauge theories requires that in lowest order one has $\sqrt{2}\,\epsilon_0 + \epsilon_8 = 0$.

(e) Finally, we would like to remark that if $g_L^4/16\pi^2 \sim \alpha$, then the second-order electromagnetic contribution, as well as the contribution of chiral-$U(2)_L \times U(2)_R$-violating quark mass terms

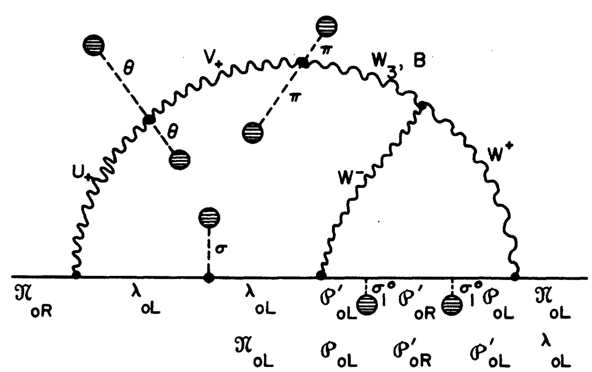

FIG. 3. The fourth-order diagram that makes a nonzero contribution to $\mathfrak{N}$-quark mass. A similar diagram contributes to e^- and $\mathcal{P}$-quark masses. The lines denote the unrotated fields.

(i.e., $m_{\mathfrak{N}}\bar{\mathfrak{N}}\mathfrak{N}+m_{\mathcal{P}}\bar{\mathcal{P}}\mathcal{P}$) to the pion mass (arising in order $g_L{}^4$ as shown in text), are of the same order of magnitude, and therefore the entire chiral $U(2)_L\times U(2)_R$ symmetry breaking in this model is of order α.

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APPENDIX A

In this section, we would like to present a simple extension of the model considered in the text, such that $U(4)_L\times U(4)_R$ is an exact zeroth-order symmetry of strong interactions. The model is based on six quarks ($\mathcal{P},\mathfrak{N},\lambda,\mathcal{P}',s,\mathcal{P}''$) and six leptons ($\nu_e, e^-, \mu^-, \nu_\mu, E^-, E^0$). The gauge group is the same as before. Denoting unrotated fields (the fields which are not eigenstates of the mass matrix) by the above symbols, we make the following assignment for the fermions:

Representation

$$\psi_{1L}=\begin{pmatrix}\mathcal{P}_L\\ \mathfrak{N}_L\end{pmatrix}\qquad (\tfrac{1}{2},0,0,1)$$

$$\psi_{2L}=\begin{pmatrix}\mathcal{P}'_L\\ \mathfrak{N}'_L\end{pmatrix}\qquad (\tfrac{1}{2},0,0,1)$$

$$\psi_{3L}=\begin{pmatrix}\mathcal{P}''_L\\ s_L\end{pmatrix}\qquad (\tfrac{1}{2},0,0,1)\qquad \text{(A1)}$$

$$\psi_{1R}=\begin{pmatrix}s_R\\ \lambda_R\\ \mathfrak{N}_R\end{pmatrix}\qquad (0,1,0,0)$$

$$\psi_{2R}=\begin{pmatrix}\mathcal{P}''_R\\ \mathcal{P}'_R\\ \mathcal{P}_R\end{pmatrix}\qquad (0,1,0,2)$$

Lepton assignment is also done exactly similarly. The Higgs boson field which has Yukawa coupling and is therefore responsible for giving the zeroth-order mass matrix is

Representation

$$\phi=\begin{pmatrix}\phi_1^+ & \phi_2^+ & \phi_3^+\\ \phi_1^0 & \phi_2^0 & \phi_3^0\end{pmatrix}\qquad (\tfrac{1}{2},1,0,1)$$

with $\qquad$ (A2)

$$\langle\phi\rangle=\begin{pmatrix}0 & 0 & 0\\ \frac{\kappa'}{\sqrt{2}} & 0 & 0\end{pmatrix}\,.$$

The other Higgs boson fields are

Representation

$$\theta=\begin{pmatrix}\theta_{11} & \theta_{12}\\ \theta_{21} & \theta_{22}\end{pmatrix}\qquad (0,\tfrac{1}{2},\tfrac{1}{2},0)\,,$$

with $\qquad$ (A3)

$$\langle\theta\rangle=\begin{pmatrix}\theta_1 & 0\\ 0 & \theta_2\end{pmatrix}$$

and π and χ as in the main text. It is clear that through the allowed Yukawa couplings one generates zeroth-order mass matrices, which have only one nonzero eigenvalue in each sector to be identified with s, $\mathcal{P}''$, E^-, and E^0 masses, respectively. As in the text, the next eigenvalue becomes nonzero in order g^4 and the following are in order g^8. Since $m_{\mathcal{P}'}=m_{\mathcal{P}}=m_{\mathfrak{N}}=m_\lambda=0$ to zeroth order, one has exact $U(4)_L\times U(4)_R$ symmetry to zeroth order and its violations arising in higher orders are computable. In the neutral-lepton sector (ν_e,ν_μ,E^0), there are two possibilities consistent with renormalizability: One is to assign $(\nu_e,\nu_\mu,E^0)_R$ to triplet under $SU(2)_R$; in that case one would predict a massive neutral lepton with a mass of a few MeV and this might contradict experiment. The other possibility is that ν_{eR}, $\nu_{\mu R}$, and E_R^0 are all singlets under the gauge group. Then, to make E_R^0 massive, one will have to put in a Weinberg-type doublet

$$\begin{pmatrix}T^+\\ T^0\end{pmatrix}$$

[which behaves like $(\tfrac{1}{2},0,0,1)$], with $\langle T\rangle=\binom{0}{\rho}$.

APPENDIX B

In this appendix, we would like to speculate on the possibility of spontaneous generation of *CP* violation as a higher-order effect in the model under consideration. The way it comes about is the following: When we examine all the allowed gauge-invariant couplings among the Higgs bosons, we find that in zeroth order, all the vacuum expectation values of Higgs fields consistent with an extremum of the Higgs potential can be made real by a gauge transformation. This is essentially because the potential at its minimum is independent of the phase of the vacuum expectation values. However, in higher orders one induces terms terms like $p\bar{\sigma}_1^0\sigma_2^0+\text{H.c.}$ (see Fig. 1) and $q(\bar{\sigma}_1^0\sigma_2^0)\times(\bar{\sigma}_1^0\sigma_2^0)+\text{H.c.}$ (see Fig. 4). Owing to the presence of these terms, a minimum of the potential seems to develop for a nonvanishing phase of $\langle\sigma_2^0\rangle$, and this in turn generates *CP*-violating terms in the Lagrangian.[12] The important thing to note is that since p and q arise in higher orders, their magnitudes are computable in terms of the basic parameters of the theory. In such a model, the magnitude of *CP* violation can in principle be predicted.

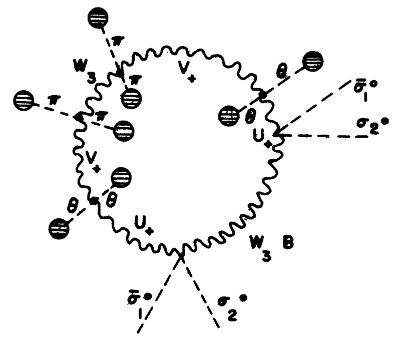

FIG. 4. The diagram that generates terms of the type $q(\bar{\sigma}_1^0\sigma_2^0)(\bar{\sigma}_1^0\sigma_2^0)$. This is responsible for spontaneous generation of *CP* violation in higher orders.

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