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[21]We remark that Eq. (3.17) is obtained by assuming PCAC for *massive* π and η'. This may not be neccessarily correct, and we remark that no good method exists to replace usual PCAC for such a *mass difference* calculation (analogous to π^+-π^0 mass difference). Therefore, Eq. (3.17) should be taken only as an estimate.

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Is $\eta \to 3\pi$ a short-distance problem?

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Wilson has suggested that electromagnetic ultraviolet radiative corrections can induce a u_3 tadpole into the effective electromagnetic Lagrangian for $\eta \to 3\pi$ which would then eliminate Sutherland's soft-pion theorem. We have investigated this proposal within the framework of the σ model in perturbation theory and find no such effect. All induced tadpole counterterms leave the neutral isovector-axial-vector current conserved. We speculate that the "true" u_3 needed for understanding $\eta \to 3\pi$ may have its origin in the infrared structure of the weak interaction.

I. INTRODUCTION

It is now recognized that the singular behavior of of radiative corrections can radically alter some of the apparent properties of unrenormalized Lagrangians. A familiar example is the breakdown of the $\pi \to \gamma\gamma$ low-energy theorem and associated Ward identities due to the triangle anomaly.[1] Another process which could in principle fall into this category is the $\eta \to 3\pi$ decay, since it involves a closed-photon-loop integration. This would be very desirable because of Sutherland's well-known theorem that the decay rate should vanish in the soft-pion limit.[2] Indeed, Wilson[3] has speculated that the operator-product expansion of $J_\mu^{em}(x)J_\nu^{em}(0)$ contains a singular u_3 tadpole-type term which is induced as an electromagnetic renormalization counterterm. Such a tadpole, being in the $(3,\bar{3}) \oplus(\bar{3},3)$ representation of SU(3)×SU(3), would then break the Sutherland theorem. This approach has been further investigated by Loodts, Mannheim, and Brout,[4] who suggested that the tadpole was a consequence of Bjorken scaling and could thus provide a link between the absence of the neutral-pion Adler zero in $\eta \to 3\pi$ and the problem of the finiteness of the electromagnetic mass differences.

In view of the somewhat speculative nature of this proposal, we have decided to make a study of electromagnetic perturbations of the σ model, which is the most convenient framework for treating chiral symmetry and spontaneous breakdown. We find that though electromagnetism does induce tadpoles, they belong to representations such as $(8,1)\oplus(1,8)$ and hence do not affect the conservation of A_μ^3 at all. This then makes it quite unlikely that electromagnetism provides the chiral-invariant strong interaction with a preferred direction, and we may have to look elsewhere, for instance to the weak interaction, in order to lift the degeneracy of the vacuum and understand the $\eta \to 3\pi$ process. This opinion has also been expressed recently by Weinberg,[5] from a somewhat different standpoint.

We start in Sec. II by working in a simplified four-state $(\sigma_0\pi_3, \sigma_3\pi_0)$ σ model. In this model there is the possibility that in the Goldstone mode ($\langle\sigma_0\rangle \neq 0$) the σ_3 field also can go to the vacuum through a tadpole graph in the presence of electromagnetism. This would correspond to an over-all shift of the σ_3 potential but not to a double-well potential and degenerate vacuum. (That radiative corrections can cause spontaneous breakdown has been pointed out recently by Coleman and Weinberg.[6] However, this is a consequence of summing a particular infinite class of infrared-divergent graphs, and cannot arise in the case we are considering here of a few low-order ultraviolet-divergent graphs.) Should the σ_3 field acquire a vacuum expectation value we would then have to induce a σ_3 counterterm in order to maintain the stability of the model, since we must continually perturb order by order about the minimum in the σ_3 potential. This would then be a good candidate for Wilson's u_3 tadpole. However, we find below that we can take out the infinite $\langle\sigma_3\rangle$ and all other relevant self-energy divergences in a chiral-invariant manner, and that the Goldstone and Sutherland theorems continue to hold.

As well as examining the Wilson u_3-type anomaly, for completeness we have also studied the more traditional type of anomalies, and we demonstrate their absence for all the graphs considered in our analysis by constructing a gauge- and chiral-invariant regulator scheme (even with a translated σ_0 field). In Sec. III we extend the analysis to the more general SU(3)×SU(3) σ model with the same results, and we conclude with some comments in Sec. IV, where we discuss a possible role for the weak interaction. For brevity we shall actually only discuss $\eta\pi$ mixing in the text and leave the details of the $\eta \to 3\pi$ calculation to an appendix, where we also raise a possible difficulty for the pion-pole-model interpretation of the process.

II. TREATMENT OF THE SIMPLIFIED σ MODEL

The model we consider first consists of an isodoublet of fermions, isoscalar σ_0' and π_0, and isovector σ_3 and π_3. Electromagnetism will then be minimally coupled to the fermions only. The model is essentially the neutral-meson part of the SU(2)×SU(2) σ model. The bare Lagrangian is

$$\begin{aligned} L' = {} & \tfrac{1}{2}i(\bar\psi\gamma_\mu\overleftrightarrow{\partial}_\mu\psi) + \tfrac{1}{2}(\partial_\mu\sigma_0'\partial^\mu\sigma_0' + \partial_\mu\pi_3\partial^\mu\pi_3 + \partial_\mu\sigma_3\partial^\mu\sigma_3 + \partial_\mu\pi_0\partial^\mu\pi_0) + g_0\bar\psi(\sigma_0' + i\pi_3\gamma_5\tau_3)\psi \\ & + g_0\bar\psi(\sigma_3\tau_3 + i\pi_0\gamma_5)\psi - \frac{\lambda}{4!}(\sigma_0'^2 + \pi_3^2 + \sigma_3^2 + \pi_0^2)^2 - \frac{\mu_0^2}{2!}(\sigma_0'^2 + \pi_3^2 + \sigma_3^2 + \pi_0^2) - \frac{\mu^2}{2!}(\sigma_3^2 + \pi_0^2)\ . \end{aligned} \tag{1}$$

The model has a conserved neutral isovector-axial-vector current

$$A_\mu^3 = \tfrac{1}{2}\bar\psi\gamma_\mu\gamma_5\tau_3\psi - \pi_3\overleftrightarrow{\partial}_\mu\sigma_0' - \pi_0\overleftrightarrow{\partial}_\mu\sigma_3\ , \tag{2}$$

which is generated by the transformations

$$\begin{aligned} &\psi \to (1 + \tfrac{1}{2}i\gamma_5\epsilon_3\tau_3)\psi\ , \\ &\pi_3 \to \pi_3 - \epsilon_3\sigma_0'\ , \quad \pi_0 \to \pi_0 - \epsilon_3\sigma_3\ , \\ &\sigma_0' \to \sigma_0' + \epsilon_3\pi_3\ , \quad \sigma_3 \to \sigma_3 + \epsilon_3\pi_0\ . \end{aligned} \tag{3}$$

The vector current is given by

$$V_\mu^3 = \tfrac{1}{2}\bar\psi\gamma_\mu\tau_3\psi \tag{4}$$

and commutes with the axial-vector current. Thus if we choose $\mu_0^2 < 0$, we will give σ_0' a vacuum expectiation value in the tree approximation, make the π_3 a Goldstone boson, and have a Sutherland theorem, i.e., no $\pi_0\pi_3$ mixing. (The $\pi_0 \to 3\pi_3$ decay itself is discussed in the Appendix.)

Before introducing electromagnetism we recall briefly how the Goldstone theorem is maintained in perturbation theory (see, e.g., Refs. 7 and 8). We translate the σ_0' by introducing

$$\sigma_0' = \sigma_0 + v\ , \tag{5}$$

where $v = (-6\mu_0^2/\lambda)^{1/2}$ so that $\langle\sigma_0\rangle = 0$ in the tree approximation. The translated Lagrangian is now

$$\begin{aligned} L = {} & E_K + g_0\bar\psi(\sigma_0 + i\pi_3\gamma_5\tau_3)\psi \\ & + g_0\bar\psi(\sigma_3\tau_3 + i\pi_0\gamma_5)\psi + g_0 v\bar\psi\psi \\ & - \frac{\lambda}{4!}(\sigma_0^2 + \pi_3^2 + \sigma_3^2 + \pi_0^2)^2 \\ & - \frac{\lambda}{3!}\sigma_0 v(\sigma_0^2 + \pi_3^2 + \sigma_3^2 + \pi_0^2) \\ & - \frac{\lambda}{3!}v^2\sigma_0^2 - \frac{\mu^2}{2!}(\sigma_3^2 + \pi_0^2)\ , \end{aligned} \tag{6}$$

where E_K is the kinetic-energy operator, and is ready for perturbing about the classical minimum. It is important, however, that we do not normal-order L, as this would cause trouble with the stability of the Euler-Lagrange equations[7] or the chiral transformation of the fermion fields.[8] Anyway, it is unnecessary to normal-order the fermion part of a chiral theory, because the Dirac sea (which gives rise to an infinite electrostatic energy) is filled up according to the Pauli principle, leaving the vacuum with zero chirality ($\langle\bar\psi\gamma_\mu\gamma_5\psi\rangle_0 = 0$ for a free fermion). This lack of normal ordering then allows tadpole graphs of the type exhibited in Fig. 1.

If we now calculate the π_3 self-energy, $\Sigma_{\pi_3}(q^2)$, in order g_0^2 we have the graph of Fig. 2 and a quadratic divergence. Also, the tadpole graph of Fig. 1 gives a quadratically divergent contribution to $\langle\sigma_0\rangle$ in order g_0^2. A chiral-invariant counterterm

$$L_1 = -\frac{1}{2!}\delta\mu_0^2(\sigma_0^2 + 2\sigma_0 v + \pi_3^2) \tag{7}$$

has to be introduced, with $\delta\mu_0^2$ chosen so as to bring $\langle\sigma_0\rangle$ back to zero in order to maintain the stability of the minimum of the potential consistently in this order of perturbation. Note that L_1 is not a genuine renormalization counterterm since it would be required even in a cutoff theory so as to maintain the stability of the tree-approximation minimum. Moreover, the coefficient $\delta\mu_0^2$ is unique, since the requirement of stability does not allow us to add on a further arbitrary finite piece to L_1. The counterterm L_1 would not be necessary at all, of course, had we chosen to translate σ_0' by $v(1+\delta\mu_0^2)$ and then perturbed consistently about the tree + one-loop minimum. However, this choice of $\delta\mu_0^2$ precisely cancels the quadratic divergence in the π_3 self-energy, leaving $\Sigma_{\pi_3}(q^2) \sim q^2\times$log divergence. A chiral-invariant wave-function renormalization (a genuine removal of of an infinity this time) will then make $\Sigma_{\pi_3}(q^2)$ finite and still proportional to q^2 so that $\bar{\Sigma}_{\pi_3}(0)=0$. Hence the π_3 stays massless. Further, L_1 automatically takes out the quadratic divergence in the σ_0 self-energy, the reason for this being that the short-distance behavior of the theory is chiral-symmetric and not affected by the soft operator (the $\mu_0^2<0$ mass term) which was used to convert the theory into the Goldstone mode. After wave-function renormalization, however, $\Sigma_{\sigma_0}(q^2)$ is still not finite but has a piece proportional to $m^2\times$log divergence [see Eq. (A5)]. This infinity is then taken out simultaneously with all the logarithmically divergent 3- and 4-point functions of the theory by one chiral-invariant counterterm (restricting ourselves to the σ_0, π_3 model only):

$$L = C[(\sigma_0^2+\pi_3^2)^2 + 4\sigma_0 v(\sigma_0^2+\pi_3^2) + 4v^2\sigma_0^2] \;. \tag{8}$$

Note that this counterterm has no term linear in the σ_0 field; hence it does not affect the stability of the model, and is in fact the only counterterm we could introduce with this feature. This completes the discussion of the methodology of renormalizing the σ model at the one-loop level in the Goldstone mode.

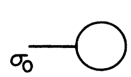

FIG. 1. The basic σ_0 tadpole graph containing one fermion loop.

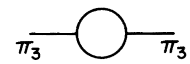

FIG. 2. The order-g_0^2 π_3 self-energy graph.

Electromagnetism is now introduced minimally as usual by adding an interaction $\frac{1}{2}e\bar{\psi}\gamma_\mu(1+\tau_3)\psi A^\mu$. The relevant divergent diagrams which we need to consider in order $g_0^2e^2$ are exhibited in Figs. 3 and 4. The counterterm of Eq. (7) will continue to take out the $(\pi_3|\pi_3)$, $(\sigma_0|\sigma_0)$, and $\langle\sigma_0\rangle$ infinities. We now also have the $(\pi_3|\pi_0)$, $(\sigma_3|\sigma_0)$, and $\langle\sigma_3\rangle$ divergences of Fig. 4 as well. For these we have to induce a new isospin-violating counterterm

$$L_2 = -\delta\mu_{03}^2(\pi_3\pi_0 + \sigma_3\sigma_0')\,, \tag{9}$$

which upon translating the σ_0' field becomes

$$L_2 = -\delta\mu_{03}^2(\pi_3\pi_0 + \sigma_3\sigma_0 + \sigma_3 v)\,. \tag{10}$$

It is easy to see that L_2 is invariant under the axial-vector transform of Eq. (3), though it is an isovector under vector transforms. Now the only difference between the diagrams of Figs. 3 and 4 is in the trace on fermion loops in the isospin space, the Feynman integrations being the same. But the couplings of the mesons to the charged fermion are the same, so the cancellations required of L_2 will take place identically to those of L_1 with $\delta\mu_{03}^2 = \delta\mu_0^2$.

So we see that the translation of σ_0' induces a linear term in the σ_3 field in L_2 which takes out $\langle\sigma_3\rangle$ so as to keep the stability of the σ_3 potential. Thus, though we induce a term $-\delta\mu_{03}^2\sigma_3 v$ [which itself would appear to be in the $(2,\bar{2})$ representation if it were erroneously classified as though the vacuum were still unique], it always comes together with the $\pi_0\pi_3$ and $\sigma_0\sigma_3$ mixing terms, so that we retain the condition $\partial_\mu A_\mu^3=0$ and continue to have a massless π_3 and no $\pi_0\pi_3$ mixing. Strictly speaking, so far we have only derived that there is no $\pi_0\pi_3$ mixing at $q^2=0$, i.e., when the π_3 is on shell, though there is still mixing at $q^2=m_{\pi_0}^2$. We dis-

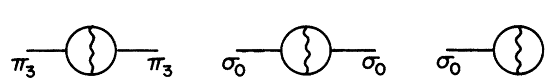

FIG. 3. Some order-$g_0^2e^2$ self-energy and tadpole graphs. These graphs have counterparts in the theory without electromagnetism.

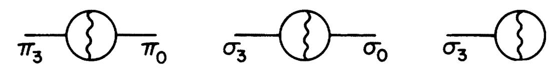

FIG. 4. Mixing and tadpole graphs of order $g_0^2e^2$ which are not present without electromagnetic isospin breaking.

cuss in the Appendix why this does not affect the $\pi_0 \to \pi_3\pi_3\pi_3$ process. The $(\pi_0|\pi_0)$ and $(\sigma_3|\sigma_3)$ divergences can also be taken out invariantly, and we have made a thorough search of other divergences such as in 3- and 4-point functions and also allowed for higher-order effects in g_0^2, λ^2, and e^2. No violations of the Goldstone theorem were found, as there is always sufficient residual chiral symmetry in the theory.

Having exhibited the cancellation mechanism, we now construct a regulator scheme to ensure that we can attach a meaning to the diagrams we have just discussed. Since a normal fermion regulator mass would break chiral symmetry we employ the parity-doublet scheme of Gervais and Lee.[9] In order to impose gauge invariance we look at diagrams such as σ going to 2γ through a fermion loop, regulate the diagram using the Pauli-Villars technique, and then close the photon loop to obtain the tadpole graph. We have collected together in Fig. 5 the primitively divergent one-fermion-loop graphs which need regulating. We note first that the graphs $\langle\sigma_0\rangle$, $(\sigma_0|\sigma_0\sigma_0)$, and $(\sigma_0|\gamma\gamma)$ are all proportional to the fermion mass on dimensional grounds and are thus only nonvanishing in the translated model. Thus, for instance, $\langle\sigma_0\rangle$ is proportional to g_0 from the vertex and to g_0v from the fermion propagator, i.e., $\langle\sigma_0\rangle \sim g_0^2v$. The graphs with an even number of external lines also exist in the normal case ($v=0$).

Following Ref. 9 we introduce sets χ_i of parity doublets which transform according to

$$\chi_i = \begin{pmatrix}\phi_i^1\\ \phi_i^2\end{pmatrix} \to \chi_i - \tfrac{1}{2}i\rho_2\vec{\epsilon}\cdot\vec{\tau}\chi_i\,, \tag{11}$$

where ρ_2 is a Pauli matrix which acts in the doublet space and $\vec{\tau}$ is the usual isospin. Then $\bar{\chi}\chi$ and $\bar{\chi}\rho_2\chi$ are chiral invariants with σ_0' coupling to $\bar{\chi}\rho_3\chi$ and $\vec{\pi}$ coupling to $\bar{\chi}\rho_1\vec{\tau}\chi$. Thus, we have

$$\begin{aligned} L_R = \sum_i \{ &\bar{\phi}_i^1(i\gamma\cdot\partial - m_i)\phi_i^1 + \bar{\phi}_i^2(i\gamma\cdot\partial - m_i)\phi_i^2 \\ &- g_i[\sigma_0'(\bar{\phi}_i^1\phi_i^1 - \bar{\phi}_i^2\phi_i^2) \\ &+ \pi_3(\bar{\phi}_i^1\tau_3\phi_i^2 + \bar{\phi}_i^2\,\tau_3\phi_i^1)]\}, \end{aligned} \tag{12}$$

where L_R is a chiral-invariant regulator Lagrangian. After translation each parity doublet will get a mass splitting of $2g_iv$. More importantly, in tadpole graphs such as that of Fig. 1 the normal parts of the doublet mass, m_i, cancel in the summed fermion loops, while the translated parts add, giving an over-all factor proportional to g_i^2v from the doublet. This factor then stays finite when we send m_i to infinity. [This will be made more clear by Eq. (17).]

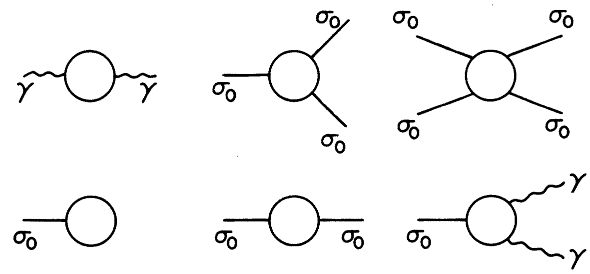

FIG. 5. The primitively divergent single-fermion-loop graphs which require regulating.

Returning to Fig. 5 we calculate the graph

$$(\sigma_0(q_\mu=0)|\gamma_\mu(k),\gamma_\nu(k))$$

for a fermion of mass parameter m and obtain

$$J_{\mu\nu}(k^2,m) \propto me^2g_0\left[\left(g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right)A(m^2) + g_{\mu\nu}\right]\,, \tag{13}$$

where

$$\begin{aligned} A(m^2) = &\frac{8m^2}{(-k^2)}[1+4m^2/(-k^2)]^{-1/2} \\ &\times \ln\left\{\frac{[1+4m^2/(-k^2)]^{1/2}+1}{[1+4m^2/(-k^2)]^{1/2}-1}\right\} - 4\,, \end{aligned} \tag{14}$$

so that $J_{\mu\nu}$, while completely finite, is not yet gauge-invariant. [$J_{\mu\nu}$ is found to be finite because we performed the angular integration first. If we first integrate over the time component of the loop momentum then we need to introduce the regulators immediately. Either procedure leads to Eq. (18).] Thus our regulator scheme must make $(\sigma_0|\gamma\gamma)$ and the vacuum polarization gauge-invariant while taking out all the other infinites of Fig. 5 as well at the same time. As a solution we try two sets of doublets, one with charge αe and coupling g_α which is quantized with the negative metric and a second one one (βe and g_β) which is quantized normally. Thus the couplings are fixed as follows:

$$\begin{aligned} &(\gamma|\gamma):\ e^2 - 2\alpha^2e^2 + 2\beta^2e^2 = 0\,, \\ &(\sigma_0|\sigma_0\sigma_0) \text{ and } (\sigma_0\sigma_0|\sigma_0\sigma_0):\ g_0^4 - 2g_\alpha^4 + 2g_\beta^4 = 0\,, \\ &\langle\sigma_0\rangle \text{ and } (\sigma_0|\sigma_0):\ g_0^2 - 2g_\alpha^2 + 2g_\beta^2 = 0\,, \\ &(\sigma_0|\gamma\gamma):\ g_0^2 - 2g_\alpha^2\alpha^2 + 2g_\beta^2\beta^2 = 0\,, \end{aligned} \tag{15}$$

with solution $\alpha^2 = \frac{3}{4}$, $\beta^2 = \frac{1}{4}$, $g_\alpha^2 = \frac{3}{4}g_0^2$, $g_\beta^2 = \frac{1}{4}g_0^2$. Hence we now have, for instance for $(\sigma_0|\gamma\gamma)_{\text{reg}}$,

$$J^{\text{reg}}_{\mu\nu}(k^2) \propto e^2 g_0 \int d^4p \, \text{Tr}\left[\frac{1}{(\not{p}-g_0 v)^2}\gamma_\mu \frac{1}{(\not{p}+\not{k}-g_0 v)}\gamma_\nu\right]$$
$$- e^2 g_\alpha \alpha^2 \Big\{ \int d^4p \, \text{Tr}\left[\frac{1}{(\not{p}-m_\alpha-g_\alpha v)^2}\gamma_\mu \frac{1}{(\not{p}+\not{k}-m_\alpha-g_\alpha v)}\gamma_\nu\right]$$
$$- \int d^4p \, \text{Tr}\left[\frac{1}{(\not{p}-m_\alpha+g_\alpha v)^2}\gamma_\mu \frac{1}{(\not{p}+\not{k}-m_\alpha+g_\alpha v)}\gamma_\nu\right]\Big\} + e^2 g_\beta \beta^2\{\beta \text{ contribution}\} \tag{16}$$

$$\propto e^2 v\left(g_{\mu\nu}-\frac{k_\mu k_\nu}{k^2}\right)[g_0{}^2 A(g_0{}^2 v^2) - 2\alpha^2 g_\alpha{}^2 A(m_\alpha{}^2) + 2\beta^2 g_\beta{}^2 A(m_\beta{}^2)] + e^2 v g_{\mu\nu}(g_0{}^2 - 2\alpha^2 g_\alpha{}^2 + 2\beta^2 g_\beta{}^2)\,, \tag{17}$$

for large m_α, m_β. Now $A(m_\alpha{}^2)\to 0$ as $m_\alpha{}^2$, the untranslated part of the doublet mass in Eq. (12), tends to infinity. Thus

$$J^{\text{reg}}_{\mu\nu}(k^2) = \left(g_{\mu\nu}-\frac{k_\mu k_\nu}{k^2}\right)e^2 v g_0{}^2 A(g_0{}^2 v^2)\,, \tag{18}$$

and is now manifestly gauge-invariant. The other diagrams of Fig. 5 are simultaneously regulated in the same manner by Eqs. (15). Further the solution in the σ_3 sector is identical so we do not present it here. Thus we have constructed an explicit set of chiral-invariant and gauge-invariant regulators which takes out all divergences including those to be subsequently met in the Appendix, to confirm the absence of anomalies for the class of diagrams considered in this paper.[10]

III. EXTENSION TO THE COMPLETE σ MODEL

In this section we extend the calculation to the more realistic SU(3)×SU(3) σ model, which is not an Abelian model and possesses the nonlinear constraints of current algebra. This then gives the electromagnetic current a slightly better chance of knowing about the axial-vector current. Also, the η and π (which will now be referred to as π_8 and π_3 respectively) are this time put in the same SU(3)×SU(3) multiplet. In the full model electromagnetism can couple to both the quarks and the charged mesons. Following the notation of Ref. 11, we generate an axial-vector current

$$A^k_\mu = \bar{q}\gamma_\mu\gamma_5 \tfrac{1}{2}\lambda_k q - (\tfrac{2}{3})^{1/2}(\pi_k \overleftrightarrow{\partial}_\mu \sigma_0' + \pi_0 \overleftrightarrow{\partial}_\mu \sigma_k) - d_{klm}\pi_m \overleftrightarrow{\partial}_\mu \sigma_l\,, \tag{19}$$

through the transformations [$\vec{\lambda}$ represents the 8 SU(3) matrices]

$$\begin{aligned} q &\to (1+\tfrac{1}{2}i\gamma_5 \vec{\lambda}\cdot\vec{\epsilon})q\,,\\ \sigma_0' &\to \sigma_0' + (\tfrac{2}{3})^{1/2}\vec{\epsilon}\cdot\vec{\pi}\,,\\ \pi_k &\to \pi_k - (\tfrac{2}{3})^{1/2}\epsilon_k\sigma_0' - d_{klm}\epsilon_l\sigma_m\,,\\ \pi_0 &\to \pi_0 - (\tfrac{2}{3})^{1/2}\vec{\epsilon}\cdot\vec{\sigma}\,,\\ \sigma_k &\to \sigma_k + (\tfrac{2}{3})^{1/2}\epsilon_k\pi_0 + d_{klm}\epsilon_l\pi_m\,. \end{aligned} \tag{20}$$

We introduce a Lorentz-scalar SU(3) nonet

$$d_i = \sum_0^8 d_{ipq}(\sigma_p\sigma_q + \pi_p\pi_q)\,, \tag{21}$$

where d_i $(i=1,8)\in(8,1)\oplus(1,8)$ and $d_0\in(1,1)$ under SU(3)×SU(3). In particular,

$$\begin{aligned} d_3 &= \frac{2}{\sqrt{3}}\sigma_3\sigma_8 + \tfrac{1}{2}(\sigma_4{}^2+\sigma_5{}^2-\sigma_6{}^2-\sigma_7{}^2) + \frac{2\sqrt{2}}{\sqrt{3}}\sigma_3\sigma_0' + (\text{pions})\,,\\ d_8 &= \frac{2\sqrt{2}}{\sqrt{3}}\sigma_8\sigma_0' + \frac{1}{\sqrt{3}}(\sigma_1{}^2+\sigma_2{}^2+\sigma_3{}^2-\sigma_8{}^2) - \frac{1}{2\sqrt{3}}(\sigma_4{}^2+\sigma_5{}^2+\sigma_6{}^2+\sigma_7{}^2) + (\text{pions})\,,\\ d_0 &= \frac{\sqrt{2}}{\sqrt{3}}\left[\sum_1^8(\sigma_i{}^2+\pi_i{}^2) + \sigma_0'^2 + \pi_0{}^2\right]. \end{aligned} \tag{22}$$

We also need

$$\begin{aligned} M_A &= \sigma_1{}^2+\sigma_2{}^2+\sigma_4{}^2+\sigma_5{}^2 + (\text{pions})\,,\\ M_B &= \sigma_6{}^2+\sigma_7{}^2+(\text{pions})\,; \end{aligned} \tag{23}$$

M_A and M_B belong to linear combinations of the $\underline{1}$, $\underline{8}$, and $\underline{27}$ [under SU(3)] Lorentz-scalar pieces of the $(8,\overline{8})$ representation. It is then seen that d_3, d_8, M_A, and M_B, while not chiral invariants, are still left invariant under the ϵ_3 transformation (and also under ϵ_8).

We proceed as before and translate the σ_0' first at the strong-interaction level so that we have to introduce a stability counterterm $\propto d_0$ in order $g_0{}^2$. In the presence of electromagnetism we will have to restabilize the σ_0, σ_8, and σ_3 potentials. This is done by a counterterm

$$L_c = D\left(d_3 + \frac{1}{\sqrt{3}}d_8\right) + Ed_0, \tag{24}$$

where D and E are of order $e^2 g_0{}^2$. In this order (diagrams analogous to Figs. 3 and 4) the D and E terms simultaneously take out the quadratic divergences in $(\pi_3\pi_3)$, $(\pi_0\pi_0)$, $(\pi_8\pi_8)$, $(\pi_3\pi_0)$, $(\pi_0\pi_8)$, and $(\pi_3\pi_8)$. However, the fermion loop contributions to the charged-meson self-energies are still quadratically divergent after restabilizing the potentials and so we have to add on a normal infinity-removing counterterm $AM_A + BM_B$ to take out the $(\pi_1\pi_1)$, $(\pi_2\pi_2)$, $(\pi_4\pi_4)$, $(\pi_5\pi_5)$, $(\pi_6\pi_6)$, and $(\pi_7\pi_7)$ infinities.[12] The situation is then exactly the same as that considered in Sec. II.

Thus there will still be massless π_3 and π_8 states and no $\pi_3\pi_8$ mixing after renormalizing the SU(3) $\times$SU(3)-translated σ model in the presence of electromagnetism.

IV. COMMENTS

The intuitive reason why Wilson's proposal does not seem to work in perturbation theory is best stated by asking how we can break the degeneracy of the vacuum, or more correctly, what made the vacuum degenerate in the first place. Proceeding by analogy with the example of a ferromagnetic phase transition we would expect spontaneous breakdown to be a consequence of long-range order and hence basically an infrared problem. Moreover, since it is a cooperative phenomenon it should be a consequence of the dynamics, i.e., output (or bootstrap),[13] rather than the fake tree-approximation destabilization traditionally used in the σ model or the Higgs-Englert-Brout mechanism. Indeed there have been a few attempts[14-17] to pursue the dynamical (i.e., without input scalar tachyons) Goldstone program of Nambu and Jona-Lasinio,[18] and presumably this underlies the interesting results of Casher, Kogut, and Susskind,[19] who studied a particular infrared-singular situation, the Schwinger mechanism in two-dimensional quantum electrodynamics (QED). Since we require infrared nonpertubative effects to make the vacuum degenerate it would seem to be extremely difficult to lift that degeneracy using perturbative ultraviolet (UV) effects (though the results of the work of this paper certainly do not exclude possible nonperturbative ultraviolet effects), and even when we destabilize the vacuum by hand we see that this does not happen either. Essentially, if Wilson's proposal is to work then the u_3 tadpole must be induced nonperturbatively, and for the moment we see no particular reason why a nonperturbative UV term would want to break $\partial_\mu A_\mu{}^3 = 0$ at all.[20]

We thus see a basic distinction between the u_3 tadpole anomaly and the usual triangle anomaly, since the latter is perturbative and already present even in the normal vacuum case, so that it does not affect the degeneracy of the vacuum at all. In passing we should then remark that whatever does take place in $\eta \to 3\pi$ will also take place in $\pi^0 \to \gamma\gamma$, though its effect may well be masked by the the triangle anomaly. (This remark would not be so academic if the triangle anomaly were to go away, say, at the finite QED eigenvalue.)

Given the above remarks it appears that the natural place to look for an explanation of $\eta \to 3\pi$ would be in a theory in which the axial-vector current plays a major role and whose infrared structure is not of the mild form of conventional fermion QED. Indeed, the Weinberg theory of weak interactions (see Ref. 5 and references therein) which is built out of Yang-Mills fields has these features.[21] We can at this stage only speculate on the economical possibility that a dynamical Higgs mechanism takes place which gives the intermediate vector bosons their masses by providing dynamical u_0, u_8, and u_3 tadpoles [i.e., in $(3, \bar{3}) \oplus (\bar{3}, 3)$]. At the same time, these then give the strong interaction [whose vacuum we hope became SU(3)$\times$SU(3)-degenerate because of its (infrared-unstable) Yang-Mills structure[22]] its preferred direction. This means that the Hamiltonian due to Gell-Mann, Oakes, and Renner[23] plays a double role. It has a piece in $(8,1)\oplus(1,8)$ [the D term of Eq. (24) only, once we have understood octet enhancement] coming from the ultraviolet (which would be of the original Coleman-Glashow type, that is, pure electromagnetic, and would make electromagnetic masses finite, and not just mass differences[24]), and a second piece coming from the weak-interaction infrared in $(3, \bar{3}) \oplus (\bar{3}, 3)$ (which gives the pion its mass and contributes a finite piece to, say, the n-p mass difference). This viewpoint is not the normal one, namely that the terms which break chiral symmetry also break scale invariance (i.e., that the same soft operator which removes the degeneracy of the chiral vacuum also destroys the scale invariance of the Lagrangian—over and above the hidden scale breaking due to the ultraviolet regulators which are accounted for by the Ward identities for broken scale invariance, the Callan-Symanzik equations). Here scale invariance is broken by a degenerate chiral-invariant vacuum through the nonvanishing of a vacuum expectation value which sets the scale for the masses. (I.e., the mechanism which makes the chiral vacuum degenerate necessarily introduces a scale. Note that this is not a dilaton theory, there being no spontaneous breakdown of the dilatation current. Here scale invariance is broken by infrared regulation since it is infrared instabilities which generate spontaneous breakdown of the chiral group.) The chiral degeneracy is then removed by the weak interaction, which tells us which of the degenerate vacua to use, i.e., how to label the strong-interaction quantum numbers.[25]

What we have suggested here is highly tentative, but would seem to be what is required of the Weinberg theory when contact is made with the hadrons, and in particular with the pion.

ACKNOWLEDGMENTS

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APPENDIX: DETAILS OF THE $\eta \to 3\pi$ CALCULATION

Any tadpole mechanism which breaks the Sutherland theorem for $\eta \to 3\pi$ would have to eliminate the Adler zero in $\eta\pi$ mixing as well, since the same current-algebra and partial-conservation-of-axial-vector-current techniques used to demonstrate the Sutherland theorem would give an Adler zero to the $\eta\pi$-mixing matrix element at $q^2 = 0$. Our analysis in Sec. II has shown that this $\eta\pi$ Adler zero is maintained in renormalized perturbation theory. Since the η is not introduced as a massless Goldstone boson in the simplified model of Sec. II, where it belongs to a different multiplet than the pion [in the full SU(3)×SU(3) σ model the η would have to be a potential Goldstone particle as it is in the same multiplet as the pion], we do not have a no-mixing theorem at the η mass. We discuss in this appendix how $\eta\pi$ mixing at $q^2 = m_\eta^2$ still does not affect the Sutherland theorem for $\eta \to 3\pi$, and how the theorem itself is maintained for arbitrary η mass.[26]

Before we discuss the $\pi_3\sigma_0\pi_0\sigma_3$ model we discuss first how the Adler zero is obtained in one loop for the $\pi\pi \to \pi\pi$ and $\pi\pi \to \sigma\sigma$ on-shell scattering amplitudes when one π is softened in the most simple model, i.e., just a pion and a σ without electromagnetism, a purely strong interaction. It is convenient to set $\lambda = 3K^2$, to introduce the σ mass M and the fermion mass m, and to rewrite the translated Lagrangian of the model [similar to Eq. (6)] as ($v = M/K = m/g_0$)

$$L = E_K + m\bar\psi\psi + m\frac{K}{M}\bar\psi(\sigma + i\pi\gamma_5\tau_3)\psi - \tfrac{1}{8}K^2(\pi^2+\sigma^2)^2 - \tfrac{1}{2}KM\sigma(\pi^2+\sigma^2) - \tfrac{1}{2}M^2\sigma^2 \,. \tag{A1}$$

The loop expansion is now obtained by perturbing in K, holding M and m fixed. We discuss first the fermion sector in order K^2. We have from Fig. 1

$$\langle\sigma\rangle = -\frac{m^2K}{M}\int\frac{d^4k}{(k^2-m^2)} \,, \tag{A2}$$

where we have omitted a factor $8/(2\pi)^4$ coming from the spin-isospin trace and from the phase-space factor of the integration. This obliges us to introduce a stability counterterm as described in Sec. II,

$$L_1 = \frac{m^2K^2}{2M^2}\int\frac{d^4k}{(k^2-m^2)}\left(\sigma^2 + 2\sigma\frac{M}{K} + \pi^2\right) . \tag{A3}$$

From now on we shall use $L' = L + L_1$ as the unrenormalized or bare Lagrangian, since as we explained previously L_1 is not to be treated as a conventional renormalization counterterm. The theory built on L' as an input Lagrangian is no more than logarthmically divergent, and to demonstrate the Adler zero requires that we show it first in the L' theory with a cutoff, with no further counterterms, and then show that the cutoff can be removed in a chiral-invariant manner so as to have the zero in the renormalized theory.

We calculate first the pion and σ propagators in order K^2 from L', so that

$$\Sigma_\pi(q^2) = \frac{K^2m^2}{M^2}\int\frac{d^4k(q^2+q\cdot k)}{(k^2-m^2)[(k+q)^2-m^2]} = \frac{K^2m^2}{2M^2}q^2 I_1(q^2) \,, \tag{A4}$$

$$\Sigma_\sigma(q^2) = \frac{K^2m^2}{2M^2}(q^2 - 4m^2)I_1(q^2) \,, \tag{A5}$$

where

$$I_1(q^2) = \int\frac{d^4k}{(k^2-m^2)[(k+q)^2-m^2]} \,. \tag{A6}$$

[Technically Eqs. (A4) and (A5) involve the translation of a linearly divergent integral, so strictly we should introduce the parity-doublet regulator scheme immediately before translating. However, since we have shown that the regulator scheme preserves chiral symmetry, we know that no anomaly can occur so we can safely use Eqs. (A4) and (A5) in their unregulated forms.] It is now an easy matter to check that both the tree graphs and the one-loop graphs for $\pi\pi \to \pi\pi$ (which are collected together in Fig. 6) add to zero when any one pion is softened and the others are on shell. Thus the logarithmic divergences in the 2-, 3-, and 4-point functions cancel among themselves at the Adler point. In any other momentum configuration we would have to take out these infinities using the counterterm of Eq. (8). However, this counterterm is chiral-invariant, as is the regulator scheme, so we also have the Adler zero in the renormalized theory at the one-fermion-loop level.

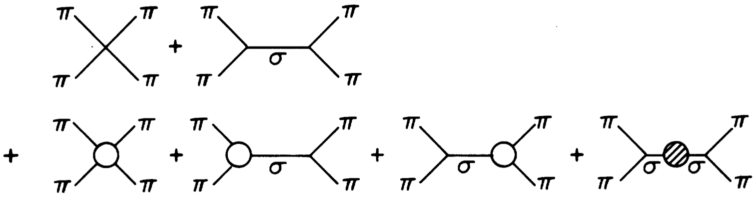

FIG. 6. The tree graphs and one-fermion-loop graphs which give the Adler zero to $\pi\pi \to \pi\pi$ scattering through order K^4. The shaded blob σ propagator is that of Eq. (A5).

The one-meson-loop graphs which are presented in Fig. 7 are handled analogously. The fermion-loop graphs for $\pi\pi \to \sigma\sigma$ are presented in Fig. 8. Again the cancellation is achieved. In this case there is a subtlety, however. Though the pion mass shell does not move at all, the σ mass shell does move to $M_R^2 = M^2 + \Sigma_\sigma(M_R^2)$, i.e., a shift of order K^2. Thus at the Adler point ($t = u = M_R^2$, $s = 0$) there is a contribution of order K^4 from the tree graphs which are superficially of order K^2, and that contribution precisely cancels the other graphs.

We now proceed to the $\pi_3\sigma_0\pi_0\sigma_3$ system, and discuss it first without electromagnetism. As well as the Lagrangian of Eq. (6), other chiral-invariants for this system are $\mathrm{Tr} M_i M_j^\dagger M_i M_j^\dagger$ and $\mathrm{Re}\,|M|$, where $M_i = \sigma_i + i\pi_i$, which are

$$\begin{aligned} \text{Trace} &= \sigma_0^2\sigma_3^2 + 2\sigma_0\pi_0\sigma_3\pi_3 + \pi_0^2\pi_3^2 , \\ \text{Determinant} &= \sigma_0\pi_0 - \sigma_3\pi_3 . \end{aligned} \tag{A7}$$

Thus the most general translated chiral-invariant Lagrangian for the system is

$$\begin{aligned} L_3 = E_K &+ m\bar\psi\psi + m\frac{K}{M}\bar\psi(\sigma_0 + i\pi_3\tau_3\gamma_5 + \sigma_3\tau_3 + i\pi_0\gamma_5)\psi \\ &- \tfrac{1}{8}K^2(\pi_0^2+\sigma_3^2+\pi_3^2+\sigma_0^2)^2 - \tfrac{1}{2}KM\sigma_0(\pi_0^2+\sigma_3^2+\pi_3^2+\sigma_0^2) - \tfrac{1}{2}M^2\sigma_0^2 - \tfrac{1}{2}\mu^2(\sigma_3^2+\pi_0^2) \\ &- T(K^2\sigma_0^2\sigma_3^2 + K^2\pi_0^2\pi_3^2 + 2K^2\sigma_0\pi_0\sigma_3\pi_3 + 2MK\sigma_0\sigma_3^2 + 2MK\pi_0\sigma_3\pi_3 + M^2\sigma_3^2) \\ &- D(K^2\sigma_0^2\pi_0^2 + K^2\sigma_3^2\pi_3^2 - 2K^2\sigma_0\pi_0\sigma_3\pi_3 + 2MK\sigma_0\pi_0^2 - 2MK\pi_0\sigma_3\pi_3 + M^2\pi_0^2) . \end{aligned} \tag{A8}$$

Note that L_3 is a strictly chiral-invariant strong-interaction Lagrangian. To L_3 we must still add L_1 of Eq. (A3) for stability. The T and D terms do not contribute in fixing the tree-approximation minimum, since they contain no term which is a pure power in σ_0. They are necessary to provide new vertices such as $\pi_0\pi_3\sigma_3$ so that the allowed strong-interaction processes $\pi_3\pi_3 \to \pi_0\pi_0$, $\pi_3\pi_3 \to \sigma_3\sigma_3$, and $\pi_3\pi_0 \to \sigma_3\sigma_0$ may all have the Adler zero for a softened π_3 at the one-loop level. Also they would be induced as renormalization counterterms anyway to take out infinities in diagrams such as $\pi_3\pi_0\sigma_3\sigma_0$ and $\pi_3\pi_0\sigma_3$ in one loop.

We now switch on electromagnetism. Immediately the σ_3 goes to the vacuum in one loop, i.e.,

$$\langle\sigma_3\rangle = -\int \frac{d^4k}{k^2} J_\mu^\mu(k^2, m^2) , \tag{A9}$$

where $J_\mu^\mu(m^2)$ is defined in Eq. (18). To obtain Eq. (A9) we performed the fermion integration first. It is more convenient to perform the photon integration first and so we set

$$\langle\sigma_3\rangle = -e^2 m^2 \frac{K}{M}\int \frac{d^4k}{(k^2-m^2)} F(k^2) . \tag{A10}$$

Moreover, we shall set $F(k^2) = 1$, since it will factor out as a universal factor in all of the remaining graphs to be considered in this appendix. We must now restabilize $\langle\sigma_3\rangle$, so we add

$$L_2 = \frac{e^2m^2K^2}{M^2}\int \frac{d^4k}{(k^2-m^2)}\left(\sigma_0\sigma_3 + \frac{M}{K}\sigma_3 + \pi_0\pi_3\right) . \tag{A11}$$

We now have the following mixing matrix elements:

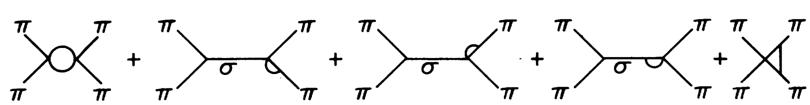

FIG. 7. The additional graphs needed for the Adler zero in $\pi\pi \to \pi\pi$ scattering in the meson-loop sector in order K^4. Here the loops are both π and σ. Also needed are the graphs of Fig. 6 with meson loops.

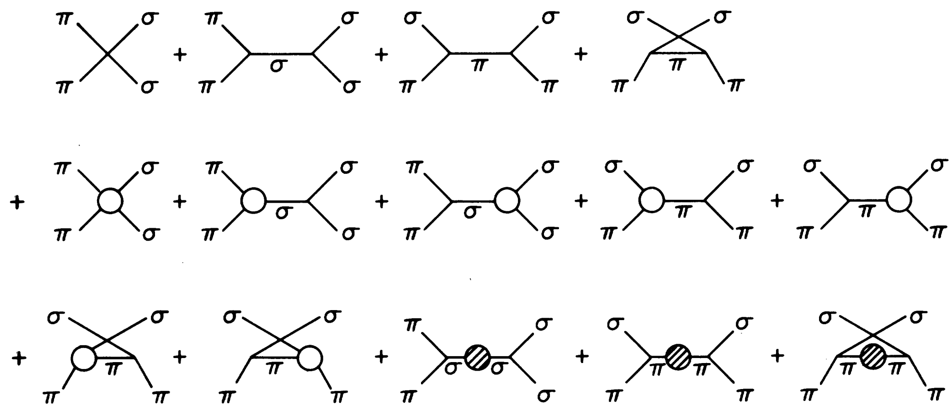

FIG. 8. The Adler-zero graphs for $\pi\pi \to \sigma\sigma$ scattering through order K^4 in the fermion sector.

$$\Sigma_{\pi_3\pi_0}(q^2) = \frac{e^2K^2m^2}{2M^2} q^2 I_1(q^2),$$
$$\Sigma_{\sigma_3\sigma_0}(q^2) = \frac{e^2K^2m^2}{2M^2} (q^2 - 4m^2) I_1(q^2), \tag{A12}$$

and we are finally ready to study $\pi_0 \to \pi_3\pi_3\pi_3$, using $L_3 + L_1 + L_2$ as the input Lagrangian. All the required graphs in order e^2K^4 are collected together in Fig. 9. The $\tilde{\pi}_3$ denotes which pion is softened.

The photon line is a shorthand for all possible permutations around the fermion loops. We need not consider graphs of the type shown in Fig. 10, where $\pi_3\pi_0$ mixing takes place at the π_3 mass shell [because of Eq. (A12)], but only those which have $\pi_3\pi_0$ mixing for an external π_0. We need to calculate a fermion loop with two external pions and an external σ, $I_2(p,q,r)$, and a loop with four external pions, $I_3(p,q,r,w)$. We find that

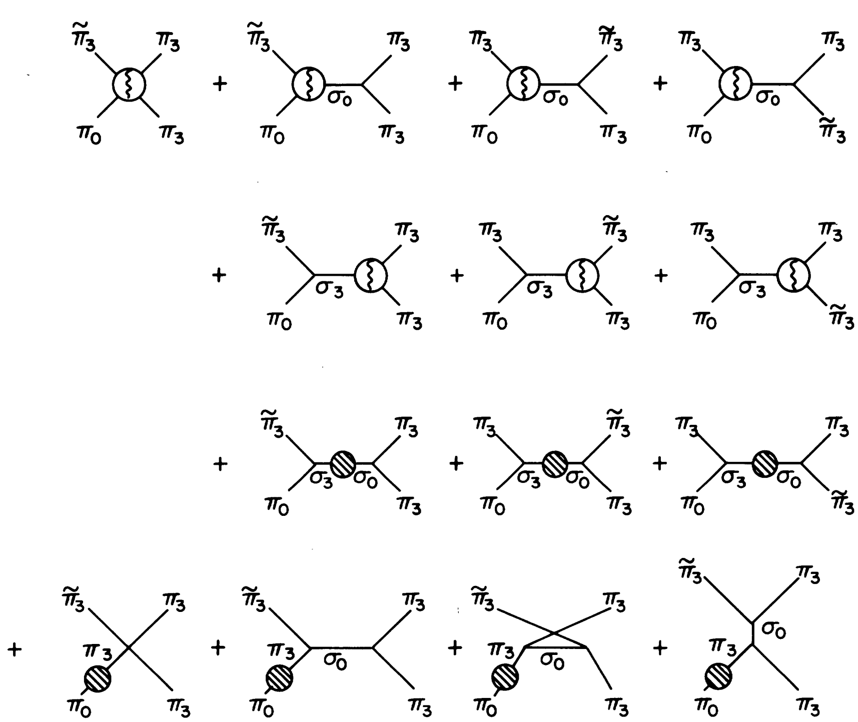

FIG. 9. The Adler-zero graphs for $\pi_0 \to \pi_3\pi_3\pi_3$. The $\pi_0\pi_3$ and $\sigma_0\sigma_3$ propagators (shaded blobs) are given by the mixing matrix elements of Eq. (A12). The graphs are of order e^2K^4, the loops are fermion loops, and $\tilde{\pi}_3$ denotes the pion which is softened.

$$I_2(p,q)=\int\frac{d^4k}{(k^2-m^2)[(k+p+q)^2-m^2]}+\int\frac{d^4k\,p\cdot q}{(k^2-m^2)[(k+p)^2-m^2][(k+p+q)^2-m^2]}\,, \tag{A13}$$

$$\begin{aligned} I_3(p_\mu=0,q,r)&=\int\frac{d^4k}{(k^2-m^2)[(k+q+r)^2-m^2]}+\int\frac{d^4k\,r\cdot(k+q)}{(k^2-m^2)[(k+q)^2-m^2][(k+q+r)^2-m^2]}\\ &=I_3(q,r)\,. \end{aligned} \tag{A14}$$

Adding all the graphs of Fig. 9 at the Adler point then gives [we use $I_2(q)$ to denote $I_2(p_\mu=0,q)$]

$$\begin{aligned} &M(\pi_0(w^2=\mu^2+2M^2D)+\pi_3(p_\mu=0)+\pi_3(r^2=0)+\pi_3(q^2=0)\to 0)\\ &\quad=-e^2m^4\frac{K^4}{M^4}[I_3(q,r)+I_3(r,q)+I_3(q,w)+I_3(w,q)+I_3(r,w)+I_3(w,r)]\\ &\qquad-e^2m^4\frac{K^4}{M^2}\left\{\frac{2I_2(w)}{(\mu^2+2M^2D-M^2)}-\frac{[I_2(w,q)+I_2(q,w)+I_2(w,r)+I_2(r,w)]}{M^2}\right\}\\ &\qquad-2e^2m^4\frac{K^4}{M^2}(T-D)\left\{\frac{[I_2(r,q)+I_2(q,r)]}{2M^2(D-T)}-\frac{2[I_2(r)+I_2(q)]}{(\mu^2+2M^2T)}\right\}\\ &\qquad+e^2m^2K^4(T-D)\left[\frac{(\mu^2+2M^2D-4m^2)I_1(w^2)}{2M^2(D-T)(\mu^2+2M^2D-M^2)}-\frac{8m^2I_1(0)}{(\mu^2+2M^2T)M^2}\right]\\ &\qquad+e^2\frac{m^2K^2}{2M^2}I_1(w^2)\left[3K^2+\frac{M^2K^2}{(\mu^2+2M^2D-M^2)}-\frac{2M^2K^2}{M^2}\right]\,. \end{aligned} \tag{A15}$$

Simple algebra then shows that the above sum vanishes identically with $M_{\pi_0}{}^2=\mu^2+2M^2D$ unconstrained.

We still have to renormalize the theory, so we add an isospin-violating term which leaves the neutral axial-vector current untouched (it transforms the same way as L_2 under the chiral group):

$$L_4=EK^2\left[(\pi_0{}^2+\sigma_3{}^2+\pi_3{}^2+\sigma_0'^2)-\frac{M^2}{K^2}\right](\pi_0\pi_3+\sigma_3\sigma_0')\,, \tag{A16}$$

which upon translation becomes

$$\begin{aligned} L_4&=EK^2(\pi_0{}^2+\sigma_3{}^2+\pi_3{}^2+\sigma_0{}^2)(\pi_0\pi_3+\sigma_3\sigma_0)\\ &\quad+EMK\sigma_3(\pi_0{}^2+\sigma_3{}^2+\pi_3{}^2+\sigma_0{}^2)\\ &\quad+2EMK\sigma_0(\pi_0\pi_3+\sigma_3\sigma_0)+2EM^2\sigma_0\sigma_3\,, \end{aligned} \tag{A17}$$

where E is a suitably chosen logarithmically divergent constant of order $e^2m^4K^2/M^4$. The $-M^2/K^2$ term has been chosen in Eq. (A16) so that the translated form of Eq. (A17) will not contain a term which is linear in the σ_3 field in order to keep the stability of the σ_3 potential; then L_4 is found to cancel precisely all the required divergences, including the logarithmic divergence that was left over in $\Sigma_{\sigma_3\sigma_0}(q^2)$ of Eq. (A12) after wave-function renormalization. (Also it does not introduce any new $\pi_0\pi_3$ mixing.) Thus we finish with the Sutherland theorem still holding in the renormalized theory.

We conclude with a final observation. The mechanism for maintaining the Sutherland theorem in order e^2K^4 is by a cancellation between the two different classes of diagrams exhibited in Fig. 9, i.e., between diagrams containing internal photons and diagrams with electromagnetic $\pi_0\pi_3$ mixing on an external leg followed by purely strong interactions. A popular way of discussing the $\pi_0\to\pi_3\pi_3\pi_3$ process is the π_3 pole model which keeps only $\pi_0\pi_3$ mixing followed by $\pi_3\pi_3\to\pi_3\pi_3$ described, say, by the Lovelace-Veneziano form. The reasons for the success of the pole model[4] have always been obscure (i.e., how the Lovelace-Veneziano model is clever enough to get rid of the neutral pion Adler zeros while retaining the charged pion ones, and why the rate seems to be normalized to the psuedoscalar octet electromagnetic mass differences). Now we must add a new challenge, namely what happens to the internal photon diagrams in the full theory.

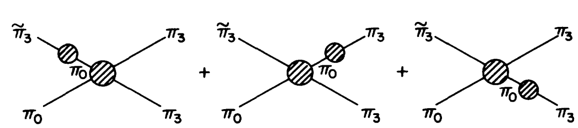

FIG. 10. Another class of graphs for $\pi_0\to\pi_3\pi_3\pi_3$ having the Adler zero.

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[12]Noting that only M_A and M_B contain the 27 representation of SU(3), we thus bring out a feature of octet enhancement, namely that the D term is on a completely different dynamical footing than the A and B terms—D is pure octet and restabilizes the vacuum, A and B only remove ultraviolet divergences. We also note that Eq. (24) may be relevant to the question of singlet-octet mixing of the natural-parity trajectories, since, though it eliminates on-shell $\pi_0\pi_8$ mixing, it does not (cannot, in fact) avoid $\sigma_0\sigma_8$ mixing—as is also reflected in the role of the scalar 2-point function in Eqs. (8) and (A17). This is reminiscent of the situation with regard to the 35 representation of SU(6), where the ω and ϕ mix while the η and η' do not.

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[20]There is an example in the literature of a chirally symmetric theory in which a mass term may be induced nonperturbatively through a short-distance anomaly (so that no accompanying Goldstone boson is necessary). This is the Johnson-Baker-Willey finite QED as discussed by M. Baker and K. Johnson, Phys. Rev. D 3, 2516 (1971). Here the bare mass $m_0 \sim (\Lambda^2)^{-\epsilon}$ and vanishes as $\Lambda^2 \to \infty$, so that the pseudoscalar-vertex renormalization constant Z_5 is cutoff-dependent even in a gauge where Z_2 is finite (since $m_0 Z_2/Z_5$ is finite according to the renormalized axial-vector Ward identity). Hence the axial charge does not create a single-pseudoscalar-particle state out of the vacuum. We would like to stress that the intention of this paper is not to show that Wilson's proposal is necessarily wrong, but that if it is to be correct then ultraviolet information alone is not sufficient. We have recently extended the Baker-Johnson evasion of the Goldstone theorem to the non-Abelian case. What happens is that the vacuum becomes degenerate through the infrared (the bootstrap of Ref. 13), but the ultraviolet anomaly remains. The object $\bar{\psi}\lambda_3\psi$ obtains a dynamical infrared vacuum expectation value and then contributes as the u_3 tadpole in the ultraviolet current-current operator-product expansion. Thus Wilson's proposal can work while satisfying the infrared requirements of Sec. IV. This mechanism will be discussed elsewhere.

[21]This is the recent observation of D. J. Gross and F. Wilczek, [Phys. Rev. Lett. 30, 1343 (1973)] and of H. D. Politzer, [*ibid.* 30, 1346 (1973)] that Yang-Mills theories are asymptotically free and hence infrared-unstable.

[22]A possible mechanism for this could be that the same operator ($\langle\bar{\psi}\lambda_i\psi\rangle$?) that acquires a vacuum expectation value would simultaneously spontaneously break the local Yang-Mills gauge and the global γ_5 invariance of the strong-interaction vacuum, thus relating f_π to the Regge slope (the Goldberger-Treiman relation). The weak interaction (again Yang-Mills) then removes the γ_5 degeneracy.

[23]M. Gell-Mann, R. J. Oakes, and B. Renner, Phys. Rev. 175, 2195 (1968).

[24]This of course presupposes that the masses were nonzero before electromagnetism was added. However, if electromagnetic radiative corrections act before spontaneous breakdown gives an otherwise conformal-invariant theory its mass scale, then the Cottingham formula reduces to the relation $0=0$, so that the whole of the n-p mass difference is due to the infrared. Then, as there are no renormalization counterterms, the mass difference is in principle calculable (Ref. 5).

[25]We make here an additional remark about Yang-Mills theories. As is known (Ref. 21), the Callan-Symanzik β function has the opposite sign to that of QED in the domain of the origin. A heuristic reason for this has been suggested to the author by G. 't Hooft (unpublished). In QED the vacuum is a dielectric and sets up a depolarizing field in the presence of an applied field. However, the Yang-Mills vacuum contains magnetic dipole moments and is paramagnetic, so that the vacuum is reinforced by an applied field. The question which is raised then is whether the vacuum is also ferromagnetic, so that spontaneous breakdown automatically takes place in an asymptotically free theory. It is very suggestive to note that the famous Landau ghost (negative-norm, negative-mass-squared state) whose removal in Johnson-Baker-Willey QED is achieved by an eigenvalue away from the origin, should become a tachyon (positive norm, negative mass squared) in a Yang-Mills theory because of this change in sign of β, and this invites spontaneous breakdown so as to shift the state into the physical region, without apparently needing a nontrivial infrared-stable eigenvalue. Moreover, the Yang-Mills field is self-coupled so that it is its own source (unlike QED). Thus it always carries its own "Weiss mean field" with it, and

acts simultaneously as the external and internal fields. Moreover, it is only in the non-Abelian case that the field can be its own source, can carry a charge as well as a spin, and can thus have a dipole moment to perhaps become "ferromagnetic" (superconductivity would ultimately be a better analogy—perhaps with Cooper pairs of magnetic monopoles), so in this picture it would be quite natural for the Gell-Mann–Nishijima formula to be that particular vacuum of the strong interaction which the weak interaction picks.

[26] I am indebted to Professor S. B. Treiman and Professor D. Z. Freedman for raising this point.

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Dual scattering of particles and Pomeranchukons*

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We propose an SU(1, 1)-invariant vertex for the emission of Pomeranchuk resonances in the generalized Veneziano model. Known amplitudes for the decay of spin-l Pomeranchukons on the leading trajectory are recovered and an integral representation is given for multi-Pomeranchukon-Reggeon N-point functions. The Regge limits of elastic Pomeranchukon-particle scattering are investigated. Applications to inclusive reactions are suggested.

I. INTRODUCTION

Since its conception in 1968 the Veneziano model[1] has developed from a phenomenological model for a few particle reactions to the point where it represents a serious hope for a strong-interaction theory adequate to deal with the steadily increasing numbers of resonant states. As a consequence of the group-theoretical understanding of duality it was at one point possible to construct vertices for the emission of high-spin particles (Reggeons) at arbitrary positions in the multiperipheral chain.[2] Recently the resonances of the Pomeranchuk sector were successfully factorized, first for the conventional Veneziano model[3] (CVM) and then for more general models.[4] It was found that in any ghost-free model based on SU(1, 1), barring unknown and pathological gauge mechanisms, the Pomeranchuk trajectory would have twice the intercept and half the slope of the leading Reggeon.

The purpose of the present paper is to repeat the procedure of Ref. 2 to construct invariant vertices for the emission of Pomeranchuk resonances. We will then be in a position to write the amplitude for M Pomeranchukons scattering with N Reggeons. For simplicity, we deal only with the CVM in the present work.

It has been shown[4] that the amplitude for a spin-l Pomeranchukon of momentum k on the leading trajectory to decay into N ground-state Reggeons is

$$A_{P_l \to NR_0} = \int \prod_{i=1}^{N-1} d\theta_i \left\langle k, 0 \right| (a_\mu^1)^{l/2} (a_\mu^{1\prime})^{l/2} \exp(a^\dagger \cdot a'^\dagger) \prod_{i=1}^{N} : \exp[ik_i \cdot Q(z_i, \tfrac{1}{2}a + a'^\dagger, \tfrac{1}{2}a' + a^\dagger)] : \left| 0 \right\rangle \Big|_{\theta_N = 0} , \quad (1.1)$$

where $z_i = \exp(i\theta_i)$ and $(a_\mu)^l \equiv a_{\mu_1} a_{\mu_2} \cdots a_{\mu_l}$. The problem at hand is to construct invariant vertices such that

$$A_{P_l \to NR_0} = \frac{1}{C} \left\langle 0 \right| V_{P_l}(k) \prod_{i=1}^{N} V_{R,0}(k_i) \left| 0 \right\rangle , \quad (1.2)$$

and to investigate the multi-Pomeranchuk amplitudes suggested thereby. For completeness we review briefly the covariance properties of the conventional vertex for ground-state emission:

$$V_0(k, z, a, a^\dagger) = \exp[ik \cdot Q(z, a, a^\dagger)] . \quad (1.3)$$

Q is the generalized coordinate four-vector

$$Q_\mu(z, a, a^\dagger) = F_\mu(z, a^\dagger) + F_\mu(z^{-1}, a) , \quad (1.4)$$

where

$$F_\mu(z, \alpha) \equiv \sum_{n=0}^{\infty} \gamma_n z^{n+\epsilon} \alpha_{\mu, n} \quad (1.5)$$

and