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π-η degeneracy problem in gauge theories*

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The quark mass matrix appears to give an inadequate description of hadron symmetry breaking. Already, for pseudoscalar masses, such an approach predicts $m_\pi = m_\eta$. We show one resolution of this old problem in terms of the explicit *strong* dynamics and the currents of $\bar{M}$-gauge models. We also discuss a second possible mechanism, applicable to a quark or M model, which generates a *calculable* breaking as a radiative correction in one loop. This second mechanism depends only on the dynamics of the *weak* interactions.

I. INTRODUCTION

It has been general knowledge for some time that "pure" quark-gluon dynamics is, in one aspect at least, inadequate for the description of hadron physics. We have in mind the fact that the $(3,\bar{3})+(\bar{3},3)$ symmetry breaking described by a quark mass matrix [plus partial conservation of axial-vector current (PCAC)] leads invariably to a degeneracy[1] of the pion with a particular combination of $\eta\eta'$, associated with the SU(3) matrix $(\sqrt{2}\lambda_0+\lambda_8)/\sqrt{3}$.

The problem is very old, and occurs in a wide panorama of quark models. It appears also in a pure S-matrix calculation: It can be checked that suppression of, say, diquark states in quark-quark scattering automatically forces a π-η (exchange) degeneracy.[2] Another way of saying this is that knowledge of quark-antiquark dynamics alone is inadequate to describe hadron symmetry breaking. From this point of view, the problem seems to involve *exotic* states.

The situation is also easy to see in the language of weak and electromagnetic currents. For simplicity, we can discuss the problem from the point of view in which hadrons start with an algebraic $SU(3)\times SU(3)$ invariance. In such a model, the quark mass matrix starts at zero. Unfortunately, if this mass matrix is our entire description of the symmetries, the model automatically possesses a full $U(3)\times U(3)$ invariance. The problem pivots around the ninth axial-vector conservation. If we imagine the quarks picking up an SU(3)-invariant mass via a dynamical spontaneous breakdown, then, by Goldstone's theorem, having spontaneously broken nine axial symmetries we must obtain nine degenerate Goldstone bosons rather than the eight we might expect from chiral SU(3). This is probably the simplest form of the problem. If, more realistically, we assumed the hadrons starting at $SU(2)\times SU(2)$, say via a quark mass matrix $m_\lambda \neq 0$, $m_{\mathcal{P}}=m_{\mathfrak{N}}=0$, then, as in the previous discussion, we find in fact the full $U(2)\times U(2)$ symmetry. Dynamical spontaneous breakdown to $m_{\mathcal{P}}, m_{\mathfrak{N}} \neq 0$ breaks *four* axial symmetries, resulting in a degeneracy of pion with an "η" whose structure is $(\sqrt{2}\lambda_0+\lambda_8)/\sqrt{3}$.

For some time, a way of alleviating this distress in some models has been a determinant interaction, as in the chiral SU(3) σ model. In this model, a term in the potential of the form $\det\Sigma+\det\Sigma^+$ [where $\Sigma=\sigma+i\phi$ is a strongly interacting $(3,\bar{3})$

field] provides an extra source of symmetry breaking for the ninth axial-vector current (beyond the quark mass matrix). This term gives a massive η', while the other eight pseudoscalars are still massless in the absence of the $(3,\bar{3})+(\bar{3},3)$ breaking. As far as it goes, this is a satisfactory solution: The determinantal term even corresponds phenomenologically to an "exotic effect," being an SU(3)-antisymmetric force in the three-quark channel.[3]

From the point of view of weak and electromagnetic currents, such a model simply starts with the ninth axial-vector current explicitly broken by non-quark-mass matrix terms. Unfortunately, this is not acceptable in a pure quark-gluon model, where it is desirable to have the strong Σ multiplet as a bound state of quarks. It is especially intolerable in an "asymptotically free theory,"[4] where elementary scalar mesons are presently not allowed at all. Attempts to circumvent the problem through anomalies are also doomed.[5]

It becomes clear that for a pure quark model there is indeed a problem, in that more symmetry breaking than just the quark mass matrix is required.

In this paper we wish to discuss two new points of view relating to this general problem. First, we want to show the existence of explicit models, which behave contrarily to general expectation: *From the point of view of the weak-electromagnetic currents there is a* $U(3)\times U(3)$ *symmetry, and yet the models do not suffer from the* π-η *degeneracy.* These are the Berkeley-type M models,[6] and they solve the problem in terms of explicit strong dynamics. It turns out that the arguments given above (for the degeneracy) are only incomplete: The Higgs mechanism of these models provides an explicit, healthy solution.

As will be discussed below, this mechanism is unorthodox. We predict that the ninth axial-vector current has no pole corresponding to the physical ninth pseudoscalar (η'), even though a physical η' exists in the theory. Equivalently, we have $\langle\eta'|J^9_{5\mu}|0\rangle=0$. Consequently, the weak currents cannot be used as interprolating fields for the physical η'. This is a complete breakdown of "ninth PCAC." We show, however, that processes like $\eta'\to 2\gamma$ are of typical magnitude. Although this M approach is a solution, it is not presently clear whether its basic features can be expressed in a pure-quark language.

Second, we discuss a different mechanism, having to do with the *weak* interactions, and therefore applicable either to quark models or M models. Here we find an extra source of ninth axial symmetry breaking without changing the customary PCAC point of view. The mechanism generates a *calculable* mass difference $m_\eta^2-m_\pi^2$ via "weak" spin-0 boson exchange. A discussion is also included of the *failure* of spin-1 gauge-boson exchange in this respect. This scalar mechanism divides into two cases: (1) All pseudoscalar masses are calculable, and the new "weak scalar" ϕ' couples to hadrons (quarks) with a semistrong coupling ($g'^2/4\pi\sim 0.1$), small enough for perturbation theory. The masses of ϕ' are relatively low (1–5 GeV^2). We also note that such scalars can, in fact, be taken compatible with known weak interactions. (2) In a parallel development where only the mass *difference* $m_\eta^2-m_\pi^2$ is taken calculable, the new scalar coupling can be as small as e, and the masses can be arbitrarily high.

As a final remark, we mention that calculable *chiral* symmetry breaking through *gauge-meson* exchange has been getting more and more complicated of late.[7] It is not clear that such an approach can converge to reality. It seems reasonable, then, to begin taking the *scalar* exchanges more seriously in general, as we have done here.

II. THE M MODELS

Because the M model has already been discussed many times,[6] we will present the model in the "graphic" form[8]

$$\begin{matrix} q_L & V_L & M_L & W_L & \cdots \\ & \Sigma & & \phi & \\ q_R & V_R & M_R & W_R & \cdots . \end{matrix} \tag{2.1}$$

Here we are describing the gauge couplings of $U(3)\times U(3)$ vector mesons $V_{L,R}$ to a $(3,\bar{3})$ Σ field. The "mixing" scalars $M_{L,R}$ ($3\times n$ complex matrices) are the usual aggregates of triplets under the strong gauge group *and* n spinors under the weak gauge group ($W_{L,R}$). "Strong" triplet quarks q_L and q_R are optional. In this paper, we make one modification relative to the standard form of these models in that we have included in the potential a determinantal term in Σ ($\det\Sigma$); thus A_9^μ cannot couple to Σ and $q_{L,R}$ but couples *only* to $M_{L,R}$ (and, if desired, to ϕ).

We arrange the weak spontaneous breakdown $\langle\phi\rangle=\lambda$ such that $U(3)\times U(3)$ is a natural hadronic symmetry, *as seen by the weak interactions.* As a result of this, the $(3,\bar{3})$ breaking term characteristic of these models,[6] $[\mathrm{Tr}(GM_L^\dagger\Sigma M_R)+\mathrm{H.c}]$, takes the form

$$G=\begin{pmatrix} 0 & & \\ & 0 & \\ & & 0 & \\ & & & x \end{pmatrix} \quad ,$$

where x is a $(n-3)\times(n-3)$ matrix. In the M models, G is the precise analog of the quark mass matrix: $G\neq 0$ is a measure of the nonconservation of the *weak* currents, defined as coupling to W_L^α W_R^α,[9]

$$J_L^{\mu\alpha} = \mathrm{Tr}([i\tfrac{1}{2}M_L^\dagger \overleftrightarrow{\partial}^\mu M_L + f M_L^\dagger V_L^\mu M_L] t_L^\alpha), \quad \text{etc.} \tag{2.2}$$

Thus, in this model, all *nine weak axial-vector currents are exactly conserved* [U(3) × U(3), not SU(3) × SU(3)]. It is noteworthy that these currents have no knowledge of the existence of the det Σ term, which breaks the global A_9 symmetry of the *strong* group, not the weak group. In particular, notice that these currents contain no Σ or q fields, and they are *not* the currents of the usual SU(3) Σ model. From the point of view of weak currents we have *exactly* the same properties that we would have encountered in the corresponding pure quark model. Our advantage here in the M models, relative to quark models, is that before spontaneous breakdown we have a *separate* strong symmetry, which we have taken as chiral SU(3) and *not* chiral U(3).

We now discuss the explicit spontaneous breakdown of hadron symmetries in the M model. (This corresponds to dynamical breaking of hadron symmetries in the quark model.) Assuming $\langle M_{L,R}\rangle = \kappa 1$, $\langle\Sigma\rangle = v1$ (1 is the 3 × 3 identity matrix), we find, in the M model only eight, not nine, physical pseudoscalars degenerate at zero mass —plus one massive η'. [In fact, there are 8 + 18 = 26 symmetries broken spontaneously, but 18 of the Goldstone particles are absorbed in the U(3) × U(3) Higgs mechanism of the strong gauge mesons.] There are many ways of counting, but we pick the simplest, focusing only on pseudoscalars and axial-vector mesons. What is happening is that the spontaneous breakdown in the $(3, \overline{3})$ Σ system gives only eight pseudoscalar Goldstone bosons (ϕ_α, $\alpha = 1, \ldots, 8$), the ninth, ϕ_9, acquiring mass proportional to the coefficient of det Σ. In the M system, we find nine pseudoscalar Goldstone bosons (ψ_α, $\alpha = 1, \ldots, 9$). Now, in the Higgs mechanism the octet of axial vectors (A_α^μ, $\alpha = 1, \ldots, 8$) absorbs a linear combination

$$\frac{\kappa\psi_\alpha + v\phi_\alpha}{(\kappa^2 + v^2)^{1/2}}, \quad \alpha = 1, \ldots, 8$$

while the ninth axial vector A_9^μ absorbs only ψ_9. What is left is the orthogonal octet of pseudoscalars (π, η, K)

$$\frac{-v\psi_\alpha + \kappa\phi_\alpha}{(\kappa^2 + v^2)^{1/2}}, \quad \alpha = 1, \ldots, 8$$

at zero mass, *plus* a massive $\eta' \equiv \phi_9$.

It is then very easy to see how the original spontaneous breakdown argument [for a $\pi(\eta\text{-}\eta')$ degeneracy in a U(3) × U(3) model] was misleadingly incomplete: It is true that nine pseudoscalar Goldstone bosons (ψ_α, $\alpha = 1, \ldots, 9$) are produced in the spontaneous breakdown of the nine *weak* axial symmetries, *but* these do not by any means have to be the physical known pseudoscalars. In fact, in this M model, the situation is very naturally much different; here the physical octet is a combination of ϕ_α and ψ_α, $\alpha = 1, \ldots, 8$, while the ninth physical pseudoscalar is pure ϕ_9 since the unphysical Goldstone boson ψ_9 is completely absorbed. The final model has only an exact chiral SU(3) Goldstone symmetry, carrying the same charges as the weak currents.

A. Consequences of the M mechanism

Here we mention some consequences that are implied by this unorthodox mechanism. In the first place, we see that, as always in the M models, the octet of pseudoscalars is representation-mixed, being nearly $\frac{1}{2}(3, \overline{3})$ (i.e., ϕ_α) and $\frac{1}{2}[(8, 1) - (1, 8)]$ (i.e, ψ_α). η', on the other hand, is pure $(3, \overline{3}) - (\overline{3}, 3)$. Intimately connected with this is the fact that the *weak* hadronic ninth axial-vector current has no matrix element between η' and the vacuum to all orders,

$$\langle\eta'|J_9^{5\mu}|0\rangle = 0\,, \tag{2.3}$$

because A_9^μ has completely absorbed all the one-particle pseudoscalar contributions of this current. What this means is that $J_9^{5\mu}$, the *weak* ninth axial-vector current, is useless as an interpolating current for η' in any PCAC-like calculation.

We may worry that Eq. (2.3) implies that processes like $\eta' \to 2\gamma$ vanish. This acutally does not happen. One can see this in an *explicit* calculation of the triangle graph using the quarks q_L, q_R just as in the SU(3) σ model. The magnitude is, of course, quite ordinary. Another way of seeing this is by defining another ninth axial-vector current associated with transformations on the *strong* side, which has as constitutents the quarks and the Σ field [the usual current of the SU(3) σ model]; this current could be used to calculate $\eta' \to 2\gamma$ decay. However, this is *not* the ninth axial-vector hadronic weak current, which, as discussed above, remains useless as an interpolating field for the η'.

There are some other, hard-to-observe consequences. For example, by its very statement (2.3) means a suppression of the ninth axial-vector weak current. However, although our mechanism represents a complete breakdown of PCAC for the ninth axial-vector current, we have been unable to find any experimental disproofs. Further thought on the matter is welcome.

Finally, we note that our M mechanism for solving the π-η-η' puzzle has very definitely to do with *exotic* states, as indicated in S-matrix theory. The M's themselves are exotic states (of the second kind), corresponding to states with two quarks and two antiquarks. We mention in particular the characteristic residual particles $P = (J^{PC} = 0^{--})$, which are pseudoscalars with *negative* charge conjugation.[10]

B. SU(3) breaking

The effect of SU(3) breaking on the π-η problem is easiest to see if we choose $U(2) \times U(2)$ as the natural invariance group rather than $U(3) \times U(3)$. Then, in our M model, the physical η' would come out as ϕ_9 plus a very *small* admixture of ϕ_8, $\eta' = \cos\theta\phi_9 + \sin\theta\phi_8$ (the smallness is a consequence of $F_\pi \cong F_K$), whose mass is still measured by the strength of $\det\Sigma$. The gauge field A_9^μ absorbs a mixture of ψ_9 and ψ_8, and the physical η comes out orthogonal to η' and the absorbed ψ's. The net result is that the weak currents still do not contain any η', which is still part of the $(3, \overline{3})$ representation. Thus, even with SU(3) breaking we still have

$$\langle \eta' | J_{5\mu}^\alpha | 0 \rangle = 0, \quad \alpha = 1, \ldots, 8, 9 \ . \tag{2.4}$$

Thus, even with broken SU(3), a PCAC approach (with weak currents) is impossible for η'.

We can also take a calculable approach to SU(3) breaking. The natural invariance group of $U(3) \times U(3)$ can be broken by using the idea of Weinberg[11] that hadron symmetry breaking is a consequence of order-α effects of the weak interactions. Although toy models of this type have been studied,[12,13] they have by no means yet reached the required degree of sophistication. Further work on this line is in progress. We emphasize, however, that M models and quark models are always in one-to-one correspondence in these issues. If a "correct model of weak interactions" is found, the M models are calculable exactly where the quark models are.[6,8,14] This is emphasized specially in the model-independent formulation of radiative hadronic symmetry breaking.[13]

Another possible approach to the SU(3) breaking is through the generalized hadron models, including higher-mass vector mesons.[8,15] Such a model is represented graphically as

$$\begin{array}{cccccccc} q_L & V_L & M_L & V_L' & M_L' & W_L & \cdots & \\ & \Sigma & & \Sigma' & & & & \\ q_R & V_R & M_R & V_R' & M_R' & W_R & \cdots, & \end{array} \tag{2.5}$$

in which again A_9 couples only to $M_{L,R}$, thus allowing a term $\det\Sigma$, while A_9' couples in the ordinary fashion to $M_{L,R}'$, Σ'. This model allows a $(3, \overline{3})'$-breaking term of the form $\mathrm{Tr}(G' M_L'^\dagger \Sigma' M_R')$, *and* SU(3) breakings (indeed, all pseudoscalar octet masses) are calculable; π, η, and η' are also split correctly. The calculation proceeds as in Refs. 8 and 15. Relative to the π, η, η' problem, our conclusions are the same as discussed above [and Eq. (2.4)].

C. Bound-state quark version of M Model

For some time now we have concerned ourselves (in collaboration with W. A. Bardeen) with the question of how close we can come to a purely bound-state picture of M models—i.e., the M's themselves arising from quarks. This is particularly intriguing with the π-η solution in sight. To have M bound states in a quark mode, we need two sets of quarks. The first are "current" quarks q, the usual quarks, transforming under the weak group. The extra set we may call "constituent" quarks Q, which transform under a strong group. (The words "current" and "constituent" quarks are here being used for convenience. These are not related by the Melosh transformation.) Binding between the two goes via gluons (Abelian or "color"). Known hadrons arise as bound states in the $\overline{Q}Q$, QQQ system, while M's arise in the $\overline{Q}q$ mass-mixing term. Weinberg-type scalars may also arise as $\overline{q}q$ states. M's arise just as pions arise in the $\overline{Q}Q$ system.[16] In fact, with such models, we get a bound-state description of all features of the M models *except*, unfortunately, the π-η mechanism discussed above. The reason for this is the same reason that the single-q model fails: Here we have no way in terms of the $\overline{Q}Q$ mass matrix of specifying the needed breaking of the ninth axial symmetry. We are hopeful, however, that our M mechanism may eventually be phraseable in some bound-state language.

III. CALCULABLE BREAKING OF THE π-η DEGENERACY

As we have seen in the previous sections, the key to the breaking of the π-η degeneracy lies with the breaking of the ninth axial symmetry, with a mechanism that goes beyond the quark mass matrix. The M model solution described above breaks the symmetry in the dynamics of the *strong* interactions. In a quark model this seems difficult if not impossible to impose; therefore, we must look elsewhere for a source of such breaking. That such a source may be found in the dynamics of the *weak* interactions has been suggested by one of us recently.[13] It is this possibility that we now wish to explore.

In unified gauge models, if the hadronic world possesses a natural zeroth-order relation[17] (here $m_\pi{}^2 = m_\eta{}^2$) which is the result of some symmetry (here A_9), and if this symmetry is not shared by the weak world (before or after weak spontaneous breaking), then the hadronic symmetry is broken as a result of radiative corrections,[11,13] and it is calculable. In the application of this idea to the case of $m_\pi{}^2 = m_\eta{}^2$ the only question is then: Can such a breaking be large enough?

In studying this question, the current-algebra formalism of Ref. 13 becomes necessary as opposed to just calculating corrections to the quark mass.[11] This is because the most general symmetry breaking that the quark mass can offer to the strong interactions is $(3,\bar{3})+(\bar{3},3)$ [assuming that the natural symmetry is $U(3)\times U(3)$]; and it is well known that such symmetry breaking does not resolve the π-η puzzle.

Pseudoscalar masses can be calculated from the general formulas,[18] based on PCAC,

$$F_a m^2{}_{ab} F_b = -i\langle 0|[Q_5^b, \partial^\mu J^a_{5\mu}]|0\rangle . \quad (3.1)$$

As a first approximation, we keep SU(3) invariance so that we have a nonet of degenerate pseudoscalars $(\vec{\pi}, K^\pm, K^0, \bar{K}^0, \eta, \eta')$, and we try to split the ninth pseudoscalar (η') away from the rest. In the one-loop correction given by Eq. (3.1), the decay constants F_a are all equal, thus $F_{\eta'} = F_\pi$. We are simply interested in

$$F_\pi{}^2(m_{\eta'}{}^2 - m_\pi{}^2) = -i\langle 0|[Q_5^9, \partial^\mu J^9_{5\mu}] - [Q_5^k, \partial^\mu J^k_{5\mu}]|0\rangle , \quad (3.2)$$

where there is no sum over $k = 1, \ldots, 8$, and $m_\pi{}^2$ represents the average mass of the pseudoscalar octet.

At this point, we must emphasize that since $m_{\eta'}{}^2 = m_\pi{}^2$ is a zeroth-order relation even when the quarks have mechanical mass, Eq. (3.2) is finite even if $m_\pi{}^2$ is not calculable. Therefore, our approach is applicable even when $U(3)\times U(3)$ is broken by hand down to U(3) with a $(3,\bar{3})+(\bar{3},3)$ term (quark mass).

We have analyzed every term in the calculated axial divergence operator[13] $\partial^\mu J^a_{5\mu}$, $a = 1, \ldots, 9$, for possible contribution to the π-η' mass splitting, with the following results:

(a) The terms proportional to the spin-0 currents S_I (tadpole graphs) do not contribute since those are $(3,\bar{3})+(\bar{3},3)$ breaking.

(b) The gauge-boson exchanges that go via the operator $\mathrm{T}_{\alpha\beta}(\kappa)$,

$$\mathrm{T}_{\alpha\beta}(\kappa) = \tfrac{1}{2} i\kappa^2 \int \frac{d\Omega_\kappa}{2\pi^2} d^4x\, e^{ik\cdot x} g^{\mu\nu} T^*(J^\alpha_\mu(x) J^\beta_\nu(0)) + (\beta \leftrightarrow \alpha) , \quad (3.3a)$$

$$\partial^\mu J^a_{5\mu} = -\tfrac{1}{2} i \int_0^\infty \kappa d\kappa (\kappa^2 + \mu^2)^{-1}{}_{\alpha\beta} [Q_5^a, \mathrm{T}_{\alpha\beta}(\kappa)] + \cdots , \quad (3.3b)$$

also do not contribute. Such exchanges can be divided into three classes: charm-charm, charm-changing, and uncharmed. The charm-charm currents always commute with the natural chiral group $SU(3)\times SU(3)$, so they cannot contribute. The uncharmed currents always commute with the ninth axial charge Q_5^9, so they cannot contribute to $m_{\eta'}{}^2$. Also, they always satisfy[8,13] Weinberg's spectral-function sum rule,[19] so their contribution to $m_\pi{}^2$ is of order $e^2/m_W{}^2$; these currents then can be neglected. Finally, the charm-changing currents can give sizable contributions to both $m_{\eta'}{}^2$ and $m_\pi{}^2$ of the form[8,13]

$$F_\pi{}^2 m_\pi{}^2 \simeq \frac{3e^2}{8\pi^2}\left(\ln\frac{\mu_1{}^2}{\mu_2{}^2}\right)\int dm^2[\rho^A(m^2) - \rho^V(m^2)] , \quad (3.4)$$

where the charm-changing spectral functions ρ^V and ρ^A do not satisfy the second spectral-function sum rule. A moment's thought will convince the reader that these currents contribute equally to $m_\pi{}^2$ and $m_{\eta'}{}^2$, so that the charm-changing currents also do not contribute to Eq. (3.2). This is an exhaustive analysis of vector mesons. Vector mesons cannot help with the π-η problem.

(c) The spin-0 boson exchanges go via the operator R_{ij},

$$\mathrm{R}_{ij}(\kappa) = -\tfrac{1}{2} i\kappa^2 \int \frac{d\Omega_\kappa}{2\pi^2} d^4x\, e^{ik\cdot x}\, T(S^i(x) S^j(0)) + (i \leftrightarrow j) , \quad (3.5a)$$

$$\partial^\mu J^a_{5\mu} = -\tfrac{1}{2} i \int_0^\infty \kappa d\kappa (\kappa^2 + M^2)^{-1}{}_{ij} [Q_5^a, \mathrm{R}_{ij}(\kappa)] + \cdots . \quad (3.5b)$$

Such exchanges can again be divided into three classes: charm-charm, charm-changing, and uncharmed. The charm-charm currents again commute with Q_5^a, so they do not contribute. The charm-changing spin-0 currents again contribute equally to $m_{\eta'}{}^2$ and $m_\pi{}^2$ just as the spin-1 currents, independently of the size of the masses $M^2{}_{ij}$ or coupling constants. Thus, we remain only with the uncharmed spin-0 currents which correspond to either all or part of the members of the $(3,\bar{3})$ classification of $SU(3)\times SU(3)$.

The contribution of these last currents would automatically be very small if their coupling constant $g = (em/m_W)$ (where m is a typical hadronic mass and m_W is a large typical weak gauge boson mass or mass difference). Although in most previous treatments of gauge models g was indeed of the order mentioned, this need not be so. The size of g is completely model-dependent and could be taken much larger provided the model is made consistent with the observed weak interactions.

Furthermore, we can show with the method of Ref. 13 that the spin-0 currents in question satisfy a spin-0 spectral-function sum rule analogous to Weinberg's second spectral-function sum rule when the mechanical quark masses are equal to zero (in the presence or absence of dynamical spontaneous breaking):

$$\int_0^\infty dm^2[\sigma^a(m^2) - \pi^b(m^2)] = 0 , \qquad (3.6)$$

for any $a, b = 1, \ldots, 9$. Here $\sigma(m^2)$, $\pi(m^2)$ are scalar and pseudoscalar spectral functions respectively for the spin-0 currents (see also below). Because of Eq. (3.6), it turns out that if m_π^2 and $m_{\eta'}^2$ are calculable separately (i.e., quark mass equal to zero), then the contribution from Eq. (3.5) has the form (g^2/M^2) rather than

$$g^2 \ln\frac{M_1^2}{M_2^2} \int_0^\infty dm^2[\sigma(m^2) - \pi(m^2)] .$$

Therefore, if all scalar masses M^2 were very large, the contribution of the $(3, \bar{3})$ currents would also be suppressed. However, if g is semistrong ($g^2/4\pi \approx 0.1$) and M^2 not too large (1–5 GeV^2), we can start to have reasonable contributions to $m_{\eta'}^2 - m_\pi^2$, as will be shown below. This size of g justifies perturbation calculations. On the other hand, if the mechanical quark mass is nonzero, Eq. (3.6) is not satisfied, and the calculable mass difference $m_{\eta'}^2 - m_\pi^2$ takes the form $g^2 \ln(M_1^2/M_2^2)$. In this case, M_i^2 can be as large as we want and g can be as small as the electric charge e.

We must emphasize that we do not have much freedom to adjust the masses and couplings of *gauge* bosons to obtain reasonable contributions from them. The electromagnetic constant e and the Fermi constant G_F provide stringent bounds for gauge-boson parameters. However, as will be discussed below, we have a lot of freedom for the spin-0 particles, enough to be able to satisfy both cases mentioned above. Since every other contribution to $m_{\eta'}^2 - m_\pi^2$ has definitely failed, and since we do not have any other hopes (except perhaps for a quark version of the M mechanism), we must explore the possibility of taking advantage of the spin-0 particles. This, as seen below, provides restrictions in model building and on the parameters that appear in the weak Lagrangian.

A. Spin-0 semistrong bosons

We must first show that spin-0 semistrong (or electromagnetic) bosons of possibly low mass—which we shall denote as ϕ'—are consistent with gauge models and the observed weak interactions. We assume that ϕ' couples to the quarks as $g'(q_L\phi' q_R + \text{H.c.})$ with semistrong g'. If ϕ' also couples to the leptons (l_L, l_R), then the leptonic coupling must be extremely small in order to be consistent with observed weak interactions (to compensate for the low mass). This is a possibility, but it is not natural. Further, if ϕ' develops a vacuum expectation value $\langle\phi'\rangle = v'$, then v' must be very small (by the scale of weak interactions) to prevent the quarks from acquiring a large mass $m = gv'$. This means that the gauge bosons should acquire their main mass from another Higgs particle ϕ (ordinary scalar). The extra ϕ should not couple to the quarks but may or may not couple to the leptons. We should have $\langle\phi\rangle = v \gg v'$.

Here we will show that there exists a rather natural and attractive possibility. We will show that in zeroth order we can have $\langle\phi'\rangle = 0$, $\langle\phi\rangle = v$ through the following general mechanism: We suppose that the quarks and the leptons are classified in the same way for simplicity. Then we allow ϕ' to couple to quarks $(q_{L,R})$, and ϕ to couple to leptons $(l_{L,R})$. We prevent cross couplings by two reflection symmetries $(\phi' \to -\phi',\ q_L \to -q_L)$ and $(\phi \to -\phi,\ l_L \to -l_L)$. Thus we have only

$$g'(\bar{q}_L\phi' q_R + \text{H.c.}) ,\ g(\bar{l}_L\phi l_R + \text{H.c.}) . \qquad (3.7)$$

Furthermore, the only interactions between ϕ and ϕ' that survive in the potential under the two separate reflection symmetries are

$$\begin{aligned} &\text{Tr}(\phi^+\phi\phi^{+\prime}\phi') , \\ &[(\text{Tr}(\phi^+\phi'))^2 + \text{H.c.}] , \\ &[\text{Tr}(\phi^+\phi'\phi^+\phi') + \text{H.c.}] . \end{aligned} \qquad (3.8)$$

Therefore, the zeroth-order vacuum expectation values can be chosen *naturally* as $\langle\phi\rangle = v$, $\langle\phi'\rangle = 0$. This gives appropriate masses to the gauge bosons and *no* mass to the quarks. Further, *it breaks the chiral symmetry of the* ϕ' *system only in the mass terms of* ϕ': From (3.8), we obtain

$$[(\text{Tr}(\phi' v))^2 + \text{H.c.}],\quad [\text{Tr}(\phi' v\phi' v) + \text{H.c.}] , \qquad (3.9)$$

which are not chiral-invariant [the chiral-invariant mass terms have the form $\text{Tr}(\phi^+\phi)$]. If we now also assume *dynamical* spontaneous breakdown, we obtain masses for quarks and massless bound-state Goldstone bosons corresponding to (π, K, η, η'). It is clear that now the masses of ϕ and ϕ' can be taken large and small, respectively, without conflicting with observed weak interactions, since ϕ does not couple to leptons. This natural state of affairs is achieved through the reflection symmetries.

The chiral-symmetry breaking in the mass matrix of ϕ' can now generate masses for the bound states (π, K, η, η') through a loop, Eqs. (3.2) and (3.5). The main point we want to make is of course that $m_{\eta'}^2 - m_\pi^2$ will not be zero.

On the other hand, if we had allowed $\langle\phi'\rangle = v' \neq 0$, the bound-state Goldstone bosons would become massive even before the radiative corrections, but the π-η degeneracy problem would persist. The degeneracy will be lifted, however, through Eqs. (3.2) and (3.5). In the following we will assume $\langle\phi'\rangle = 0$ since it is more attractive, and will return at the end to the case of $\langle\phi'\rangle \neq 0$.

B. Calculation of pseudoscalar masses

We take a "toy" model for purposes of illustration. The mechanism of the model we shall discuss can easily be embedded in a more complete and physical model as discussed in the previous sections.

We take triplets of left- and right-handed quarks q_L, q_R classified as (3, 1) and (1, 3) respectively. We couple to them a $(3, \bar{3})$ representation of spin-0 particles $\phi' = \lambda^\alpha(\chi_\alpha + i\psi_\alpha)$. We shall ignore gauge bosons, leptons, and ϕ altogether since they play no role in the mechanism. The only effect of the presence of ϕ which was important, namely the chiral symmetry breaking of the ϕ' mass matrix, will be retained, by taking the SU(3)- and parity-invariant mass matrix for scalars and pseudoscalars.[20]

$$M_9\chi_9^2 + M_8\left(\sum_{\alpha=1}^{8}\chi_\alpha^2\right) + M_{59}^2\psi_9^2 + M_{58}^2\sum_{\alpha=1}^{8}\psi_\alpha^2 . \tag{3.10}$$

The representation of the scalar S^i and the pseudoscalar S_5^i spin-0 currents is now fixed. Their commutators with the axial charge are

$$[Q_5^a, S^i] = -id_{aij}S_{5j} ,$$
$$[Q_5^a, S_5^i] = id_{aij}S_j , \tag{3.11}$$

where $a, i, j = 1, \ldots, 9$; we are taking $d_{9ij} = (\frac{2}{3})^{1/2}\delta_{ij}$ and the well-known d coefficients of SU(3).

Further, if we define the scalar and pseudoscalar spectral functions for spin-0 currents just as we would for spin-1 currents, we obtain with appropriate normalizations

$$\langle 0|R_{ij}^{S}(\kappa)|0\rangle = \delta_{ij}g^2\kappa^2\int_0^\infty dm^2\,\frac{\sigma_i(m^2)}{\kappa^2+m^2} , \tag{3.12a}$$

$$\langle 0|R_{ij}^{S_5}(\kappa)|0\rangle = \delta_{ij}g^2\kappa^2\int_0^\infty dm^2\,\frac{\pi_i(m^2)}{\kappa^2+m^2} , \tag{3.12b}$$

where R^S and R^{S_5} are constructed from scalar and pseudoscalar currents respectively as in Eq. (3.5a).

Replacing Eqs. (3.12) in Eqs. (3.2) and (3.5) and using the sum rules (3.6) to perform the κ^2 integration, we obtain finally the following formulas (these are being presented for purposes of illustration of a sample calculation; order of magnitude will be discussed later):

$$F_{\eta'}^2 m_{\eta'}^2 = \frac{g'^2}{(4\pi)^2}\int_0^\infty dm^2\left\{\frac{2}{3}\left[\frac{\ln(M_{59}^2/m^2)}{M_{59}^2/m^2-1} - \frac{\ln(M_9^2/m^2)}{M_9^2/m^2-1}\right][\sigma_9(m^2)-\pi_9(m^2)] + \frac{16}{3}\left[\frac{\ln(M_{58}^2/m^2)}{M_{58}^2/m^2-1} - \frac{\ln(M_8/m^2)}{M_8/m^2-1}\right][\sigma_8(m^2)-\pi_8(m^2)]\right\} , \tag{3.13}$$

$$F_\pi^2 m_\pi^2 = \frac{g'^2}{(4\pi)^2}\int_0^\infty dm^2\left\{\frac{2}{3}\left[\frac{\ln(M_{59}^2/m^2)}{M_{59}^2/m^2-1} - \frac{\ln(M_8^3/m^2)}{M_8/m^2-1}\right][\sigma_8(m^2)-\pi_9(m^2)] + \frac{2}{3}\left[\frac{\ln(M_{58}^2/m^2)}{M_{58}^2/m^2-1} - \frac{\ln(M_9^2/m^2)}{M_9^2/m^2-1}\right][\sigma_9(m^2)-\pi_8(m^2)] + \frac{5}{3}\left[\frac{\ln(M_{58}^2/m^2)}{M_{58}^2/m^2-1} - \frac{\ln(M_8^2/m^2)}{M_8^2/m^2-1}\right][\sigma_8(m^2)-\pi_8(m^2)]\right\} , \tag{3.14}$$

and finally

$$F_\pi^2(m_{\eta'}^2 - m_\pi^2) = \frac{g'^2}{(4\pi)^2}\int_0^\infty\left[dm^2\left\{\frac{2}{3}\left[\frac{\ln(M_{59}^2/m^2)}{M_{59}^2/m^2-1} - \frac{\ln(M_{58}^2/m^2)}{M_{58}^2/m^2-1}\right][\sigma_9(m^2)-\sigma_8(m^2)] + \frac{2}{3}\left[\frac{\ln(M_9^2/m^2)}{M_9^2/m^2-1} - \frac{\ln(M_8^2/m^2)}{M_8^2/m^2-1}\right][\pi_9(m^2)-\pi_8(m^2)] + 3\;\frac{\ln(M_{58}^2/m^2)}{M_{58}^2/m^2-1} - \frac{\ln(M_8^2/m^2)}{M_8^2/m^2-1}\right][\sigma_8(m^2)-\pi_8(m^2)]\right\} . \tag{3.15}$$

It is noteworthy that the left-hand side of these equations is naturally small because of the known small value of $F_\pi^2 \simeq 0.02$ GeV^2. It is the smallness of F_π that helps a great deal to obtain a correct order of magnitude. As discussed before, we see that if all M^2 were very large, then all contributions would be of the order of g'^2/M^2. This is because of the spectral-function sum rule of Eq. (3.6). However, with M^2 not too large (1–5 GeV^2) and g' semistrong ($g'^2/4\pi \simeq 0.1$), and assuming a rough pole dominance which satisfies Eq. (3.6),

$$\sigma(m^2) \simeq (1\text{ GeV})^4\delta(m^2 - m_\sigma^2)\,,$$
$$\pi(m^2) \simeq (1\text{ GeV})^4\delta(m^2 - 0)\,, \tag{3.16}$$

we easily get a physically reasonable $m_{\eta'}{}^2 - m_\pi{}^2$. Thus semistrong (i.e., almost weak) scalar exchange is a viable mechanism for solving the π-η problem, if the size of the spectral functions is approximately as in (3.16).

Finally, we consider the case $\langle\phi'\rangle = v' \neq 0$ in zeroth order. In this case, the over-all $m_\pi{}^2$ and $m_{\eta'}{}^2$ are not calculable. This is equivalent to not satisfying the spectral-function sum rules of Eq. (3.6). However, the mass difference $m_{\eta'}{}^2 - m_\pi{}^2$ is calculable. When Eq. (3.6) is not satisfied, then the above expressions are not valid, and we have instead

$$F_\pi{}^2(m_{\eta'}{}^2 - m_\pi{}^2) = \frac{g'^2}{(4\pi)^2}\left\{\tfrac{2}{3}\ln(M_{58}{}^2/M_{59}{}^2)\int_0^\infty dm^2[\sigma_9(m^2) - \sigma_8(m^2)] + \tfrac{2}{3}\ln(M_8{}^2/M_9{}^2)\int_0^\infty dm^2[\pi_9(m^2) - \pi_8(m^2)] + 3\ln(M_{58}{}^2/M_8{}^2)\int_0^\infty dm^2[\sigma_8(m^2) - \pi_8(m^2)]\right\}\,. \tag{3.17}$$

Therefore, for this case M^2 can be as large as we want, and g' can be as small as e, since now the logarithms combined with the spectral-function integrals could provide an extra factor of 10 relative to the previous case.[21]

Thus, we have found at least one possible mechanism to split the π-η degeneracy and argued that there is no other mechanism in quark gauge models. If no other alternative can be found that gives a correct hadronic spectrum, we must take seriously spin-0 scalars of the type considered in this paper and look for phenomenological consequences.

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[8]I. Bars, M. B. Halpern, and K. Lane, Nucl. Phys. B65, 518 (1973).

[9]These currents have been derived in Ref. 6. Here we wish to emphasize that they follow from either of two equivalent Noether variations: (1) the primed transformation on the weak side of M, $M \to MS'^{-1}$; (2) a diagonal transformation (before or after spontaneous breaking in any gauge) on both sides of M, $M \to SMS^{-1}$, which also transforms all strongly interacting particles (including a gauge transformation on the strong vector mesons). Thus, these currents satisfy current algebra and, after spontaneous breakdown, provide appropriate hadronic charges.

[10]The structure of the 3×3 part of the M's and the 3×3 Σ, in terms of J^{PC} eigenstates, is

$$\Sigma = \sigma + i\phi\,,$$
$$(M_L)_{3\times3} = (\chi - P) + i(S - \psi)\,,$$
$$(M_R)_{3\times3} = (\chi + P) + i(S + \psi)\,,$$

with $J^{PC} = 0^{++}$, 0^{++}, 0^{--}, 0^{+-}, 0^{-+}, and 0^{-+} for σ, χ, P, S, ψ, and ϕ, respectively. S is absorbed by the strong vector mesons V, and the strong axial-vector mesons A absorb a combination of the $(3,\bar{3}) - (\bar{3},3)$ ϕ and the $(8,1)-(1,8)$ ψ. The particles χ and P and the combination $\pi = \cos\theta\,\phi + \sin\theta\,\psi$ remain as physical spin-0 fields in the Lagrangian. The P multiplet has the unusual quantum numbers ($J^{PC} = 0^{--}$). All of its interactions are determined by the Lagrangian, but the mass is unknown A phenomenological study of these particles would be most welcome.

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[20]Note that this model, using only scalars, can stand on its own since global symmetry breaking in terms of dimension 2 is renormalizable according to Symanzik's theorem. See K. Symanzik, in *Fundamental Interactions at High Energies II*, edited by A. Perlmutter, R. W. Williams, and G. J. Iverson (Gordon and Breach, New York, 1970); see also T. Hagiwara and B. W. Lee, Phys. Rev. D 7, 459 (1973).

[21]We remark that Eq. (3.17) is obtained by assuming PCAC for *massive* π and η'. This may not be neccessarily correct, and we remark that no good method exists to replace usual PCAC for such a *mass difference* calculation (analogous to π^+-π^0 mass difference). Therefore, Eq. (3.17) should be taken only as an estimate.

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Is $\eta \to 3\pi$ a short-distance problem?

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Wilson has suggested that electromagnetic ultraviolet radiative corrections can induce a u_3 tadpole into the effective electromagnetic Lagrangian for $\eta \to 3\pi$ which would then eliminate Sutherland's soft-pion theorem. We have investigated this proposal within the framework of the σ model in perturbation theory and find no such effect. All induced tadpole counterterms leave the neutral isovector–axial-vector current conserved. We speculate that the "true" u_3 needed for understanding $\eta \to 3\pi$ may have its origin in the infrared structure of the weak interaction.

I. INTRODUCTION

It is now recognized that the singular behavior of of radiative corrections can radically alter some of the apparent properties of unrenormalized Lagrangians. A familiar example is the breakdown of the $\pi \to \gamma\gamma$ low-energy theorem and associated Ward identities due to the triangle anomaly.[1] Another process which could in principle fall into this category is the $\eta \to 3\pi$ decay, since it involves a closed-photon-loop integration. This would be very desirable because of Sutherland's well-known theorem that the decay rate should vanish in the soft-pion limit.[2] Indeed, Wilson[3] has speculated that the operator-product expansion of $J_\mu^{em}(x)J_\nu^{em}(0)$ contains a singular u_3 tadpole-type term which is induced as an electromagnetic renormalization counterterm. Such a tadpole, being in the $(3,\bar{3}) \oplus (\bar{3},3)$ representation of SU(3)×SU(3), would then break the Sutherland theorem. This approach has been further investigated by Loodts, Mannheim, and Brout,[4] who suggested that the tadpole was a consequence of Bjorken scaling and could thus provide a link between the absence of the neutral-pion Adler zero in $\eta \to 3\pi$ and the problem of the finiteness of the electromagnetic mass differences.

In view of the somewhat speculative nature of this proposal, we have decided to make a study of electromagnetic perturbations of the σ model, which is the most convenient framework for treating chiral symmetry and spontaneous breakdown. We find that though electromagnetism does induce tadpoles, they belong to representations such as $(8,1)\oplus(1,8)$ and hence do not affect the conservation of A_μ^3 at all. This then makes it quite unlikely that electromagnetism provides the chiral-invariant strong interaction with a preferred direction, and we may have to look elsewhere, for instance to the weak interaction, in order to lift the degeneracy of the vacuum and understand the $\eta \to 3\pi$ process. This opinion has also been expressed recently by Weinberg,[5] from a somewhat different standpoint.

We start in Sec. II by working in a simplified