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Currents and local gauge symmetries*

B. de Wit†

Institute for Theoretical Physics, State University of New York at Stony Brook, Stony Brook, New York 11790

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The current-algebra properties of gauge field theories are investigated. First, we consider a unified gauge field model of strong, weak, and electromagnetic interactions, which is a natural extension of the σ model combined with the Weinberg-Salam model. The violation of CP invariance is forbidden, and isospin is only broken by electromagnetic interactions. The pion is possibly a pseudo-Goldstone boson, which picks up its mass from weak and electromagnetic interactions. In the physical gauge the weak axial-vector currents are not of the canonical form, thus invalidating the current-algebra hypothesis. However, further analysis based on generalized Ward-Takahashi identities shows that the divergence equations are not affected. Furthermore, we discuss in which case the partially conserved axial-vector current approximation can be justified.

I. INTRODUCTION

The current-algebra hypothesis has been one of the most fruitful ideas in the theory of weak and electromagnetic interactions.[1] According to this hypothesis, the weak and electromagnetic currents can be expressed in terms of currents that are directly related to the internal symmetry of the hadronic system. The latter are the so-called canonical, or Noether, currents, which can be generated directly from the internal-symmetry transformations. The second part of the hypothesis states that the charges of these canonical currents generate a closed algebra under equal-time commutation relations in all orders of the strong interactions. In addition to this hypothesis one often assumes certain properties for the current divergences, such as conserved vector currents (CVC) and partially conserved axial-vector currents (PCAC), which, again, are supposed to hold in all orders of the strong interactions.

In the context of Lagrangian field theory the second part of the hypothesis has been studied in simple models.[2] Perturbative calculations in most cases confirmed the validity of the various assumptions for canonical currents, although there were sometimes difficulties, for example, the so-called PCAC anomalies.[3] But in this approach one is still left with the really surprising fact that weak and electromagnetic currents seem to be directly related to the canonical currents. Possibly, a natural explanation for this phenomenon can be found in gauge field theories, because gauge fields are coupled to currents that are associated with the gauge symmetry. Hence, this would suggest that in a gauge field theory of weak and electromagnetic interactions the current-algebra hypothesis is actually no longer an assumption, but it is an intrinsic part of the theory.

In this paper we will analyze the current-algebra properties of gauge field theories. To get some idea of the possible difficulties that one can encounter, we first investigate a unified gauge field model for strong, weak, and electromagnetic interactions. Models of this type[4] were recently proposed by de Wit[5] and Bars, Halpern, and Yoshimura.[6] One of them allowed the suppression of the neutral strangeness-changing currents without enlarging the number of fermion fields.[6] In another model[5] a mechanism was discovered that could explain the origin of the Cabibbo angle. Moreover, the recent result that gauge field theories can be asymptotically free is an additional argument for the investigation of completely unified models.[7]

The particular model that we will consider is the most direct extension of the σ model to a gauge field model for strong interactions,[2] combined with the Weinberg-Salam model of weak and electromagnetic interactions.[8] The strongly interacting particles of the model are two triplets of vector and axial-vector mesons, presumably the ρ and A_1 mesons, a triplet of pions, the nucleon doublet, and two scalar mesons and one pseudoscalar meson. The weak and electromagnetic interactions are mediated by massive vector bosons and by a massless photon. Apart from leptons, there is one additional scalar particle, as in the Weinberg-Salam model, which interacts only weakly. Although in this paper we introduce the model only as a guide in discussing the current-algebra properties of gauge field models, it certainly has its own merits. For instance, the isospin breaking comes purely from electromagnetic interactions, which means that the mass differences within isospin multiplets are calculable. It can also be shown that the gauge symmetry implies CP invariance. Moreover, under certain circumstances the pion is a pseudo-Goldstone boson, which receives its mass from weak and electromagnetic closed-loop corrections, a possibility that has been put forward by Weinberg.[9]

When the above-mentioned model is considered in the physical (unitary) gauge, it turns out that the current-algebra hypothesis is not fulfilled in lowest-order (tree) approximation. The deviations concern only the axial-vector currents, which have no resemblance to any canonical current. In fact, there does not even exist an infinitesimal transformation from which they could be constructed in the usual manner. This may be seen as an indication that certain gauge field models do not confirm the validity of the current-algebra approach. Nevertheless, it is not excluded that many results derived by current-algebra methods remain unaffected.

Subsequently, we analyze the generalized Ward-Takahashi identities, which generally follow from gauge invariance and hold in all orders of perturbation theory. From these identities we derive the so-called divergence equations, which depend only on the gauge symmetry that has originally been chosen for the weak and electromagnetic interactions, which was $SU(2)\otimes U(1)$ in our case. A characteristic feature of these equations is that all the strongly interacting particles are on the mass shell. We find that, as soon as strongly interacting particles are off the mass shell, the results will in general depend on the specific structure of the strong interactions.

An important observation is that these divergence equations have a similar structure as those equations that can be derived from the current-algebra hypothesis. In fact, the latter were sufficient for the derivation of the main current-algebra results for the vector currents.[10] For the axial-vector currents one additionally needed the notion of PCAC in order to find results that are experimentally testable. We show that in our unified gauge field model the result of the PCAC assumption is confirmed up to orders of some parameter b, for terms that enter in the divergence equations, provide that we keep only terms of first order in b in the so-called σ term. It turns out that the same parameter b causes the chiral-symmetry breaking and a nonvanishing pion mass. This implies that all current-algebra results that could be derived from divergence equations and PCAC, such as the Goldberger-Treiman relation, the Adler-Weisberger relation, and the Adler consistency relation,[11] are ensured to be valid in an appropriate (chiral) limit. Let us stress that, although we have analyzed the PCAC assumption only in the context of our unified model, our arguments apply for a larger class of gauge field models. The divergence equations, as already mentioned, are completely independent of the specific structure of the strong interactions.

Finally, we will make a few remarks about a possible pseudo-Goldstone nature of the pion.

This paper is organized as follows: Section II introduces the unified gauge field model of strong, weak, and electromagnetic interactions, and reviews its general properties. In Sec. III we consider the model in the tree approximation. We evaluate some quantities of interest, and determine the axial-vector currents, which turn out to have no relation to the canonical currents. Section IV contains an analysis of the generalized Ward-Takahashi identities. From these identities we generally derive the divergence equations in Sec. V. Section VI gives an analysis of PCAC. In the Appendix we give the propagators needed for our considerations.

II. A UNIFIED MODEL OF STRONG, WEAK, AND ELECTROMAGNETIC INTERACTIONS

In this section we will introduce a natural extension of the σ model[2] to a gauge field model of strong interactions, combined with the Weinberg-Salam model of weak and electromagnetic interactions.[8] The underlying gauge group of the strong interactions is the chiral SU(2)⊗SU(2) group, and we denote the chiral gauge fields belonging to this group by X_μ^a and Y_μ^a $(a=1,2,3)$. In principle, X_μ^a and Y_μ^a could well correspond to the sum and the difference, respectively, of the ρ and A_1 vector-meson fields. Under the chiral gauge group X_μ^a and Y_μ^a transform according to

$$\begin{aligned} X_\mu(x) &\to U(x)X_\mu(x)U^\dagger(x)+ig_X{}^{-1}U(x)\partial_\mu U^\dagger(x)\ , \\ Y_\mu(x) &\to V(x)Y_\mu(x)V^\dagger(x)+ig_Y{}^{-1}V(x)\partial_\mu V^\dagger(x), \end{aligned} \tag{1}$$

where we have used the notation $X_\mu \equiv \frac{1}{2}X_\mu^a\tau_a$, $Y_\mu \equiv \frac{1}{2}Y_\mu^a\tau_a$. The corresponding coupling constants are g_X and g_Y, and $U(x)$ and $V(x)$ are local SU(2) matrices. In order to acquire massive gauge fields without disturbing the gauge invariance, we will make use of the Higgs-Kibble mechanism.[12] This implies that, in addition to the pion and σ fields of the σ model, we need additional spinless fields. When some of these fields acquire nonzero vacuum expectation values, the gauge fields will become massive without affecting the gauge invariance, and thus preserving the renormalizability.[13]

Let us write the original fields of the σ model as a 2×2 matrix:

$$K_\Sigma = \frac{1}{\sqrt{2}}(\sigma_\Sigma + i\psi_\Sigma^a\tau_a),$$

where σ_Σ and ψ_Σ^a correspond to the σ particle and the pions, respectively. We add two complex doublet fields, which are denoted by K_X and K_Y. As is well known, K_Σ transforms under the chiral gauge group as

$$K_\Sigma(x) \to U(x)K_\Sigma(x)V^\dagger(x)\ , \tag{2a}$$

and the new fields, K_X and K_Y (in the same 2×2 notation) will transform according to

$$\begin{aligned} K_X(x) &\to U(x)K_X(x)\,, \\ K_Y(x) &\to V(x)K_Y(x)\ . \end{aligned} \tag{2b}$$

A nucleon doublet field can also be added, which transforms under the chiral group as

$$N(x) \to \tfrac{1}{2}(1+\gamma_5)U(x)N(x)+\tfrac{1}{2}(1-\gamma_5)V(x)N(x)\ . \tag{2c}$$

We can now write down the most general gauge-invariant Lagrangian, impose invariance under parity, and find a completely renormalizable model for hadrons. Such a model was introduced by Bardakci.[14] However, we will first consider the extension of this hadron model with weak and electromagnetic interactions, and combine it with the Weinberg-Salam model.[8] The underlying gauge group for the weak and electromagnetic interactions in that model is SU(2)⊗U(1). The gauge fields belonging to this group are denoted by Z_μ^a and Z_μ^0, and they transform in the following way:

$$\begin{aligned} Z_\mu(x) &\to S(x)Z_\mu(x)S^\dagger(x)+ig_Z{}^{-1}S(x)\partial_\mu S^\dagger(x)\ , \\ Z_\mu^0(x) &\to Z_\mu^0(x)+q^{-1}\partial_\mu\Lambda^0(x)\ , \end{aligned} \tag{3}$$

where g_Z and q are the coupling constants, $Z_\mu \equiv \frac{1}{2}Z_\mu^a\tau_a$, and $S(x)$ is a local SU(2) matrix. The spinless doublet field of the Weinberg-Salam model is again written as a 2×2 matrix, denoted by K_Z. Its transformation properties are given by

$$K_Z(x) \to S(x)K_Z(x)T^\dagger(x)\ , \tag{4}$$

where $T(x)=\exp[\frac{1}{2}i\Lambda^0(x)\tau_3]$.

As is well known the vacuum expectation value of K_Z, which is supposed to be very large, gives rise to three very massive vector bosons and one massless photon mediating the weak and electromagnetic interactions. The assignment of the leptons into representations of the weak and electromagnetic gauge group is completely the same as in the Weinberg-Salam model.

In order to have weak and electromagnetic interactions with the previously constructed hadron model, the hadronic fields must in addition transfrom under the weak and electromagnetic gauge group. Because these gauge transformations must commute with the previously defined transformations of the gauge group that governs the strong interactions, there are only a few possibilities. The only one which makes sense is

$$\begin{aligned} K_X(x) &\to K_X(x)S^\dagger(x) , \\ K_Y(x) &\to K_Y(x)T^\dagger(x) , \\ N(x) &\to \exp[\tfrac{1}{2}i\Lambda^0(x)]N(x) . \end{aligned} \tag{5}$$

Once all the transformation properties are determined, the construction of the model is rather straightforward. The covariant antisymmetric tensors of the gauge fields are defined as

$$\begin{aligned} G^0_{\mu\nu} &= \partial_\mu Z^0_\nu - \partial_\nu Z^0_\mu, \\ G^X_{\mu\nu} &= \partial_\mu X_\nu - \partial_\nu X_\mu - ig_X[X_\mu, X_\nu], \end{aligned}$$

and similarly for $G^Y_{\mu\nu}$ and $G^Z_{\mu\nu}$. The covariant derivatives of the remaining fields are given by

$$\begin{aligned} D_\mu K_X &= \partial_\mu K_X - ig_X X_\mu K_X + ig_Z K_X Z_\mu, \\ D_\mu K_Y &= \partial_\mu K_Y - ig_Y Y_\mu K_Y + \tfrac{1}{2}iqZ^0_\mu K_Y\tau_3 , \\ D_\mu K_\Sigma &= \partial_\mu K_\Sigma - ig_X X_\mu K_\Sigma + ig_Y K_\Sigma Y_\mu , \\ D_\mu K_Z &= \partial_\mu K_Z - ig_Z Z_\mu K_Z + \tfrac{1}{2}iqZ^0 K_Z \tau_3 , \\ D_\mu N &= \partial_\mu N - \tfrac{1}{2}ig_X X_\mu(1+\gamma_5)N \\ &\quad - \tfrac{1}{2}ig_Y Y_\mu(1-\gamma_5)N - \tfrac{1}{2}iqZ^0_\mu N. \end{aligned}$$

We divide the most general Lagrangian, invariant under the chiral SU(2)⊗SU(2) gauge group of the strong interactions and under the SU(2)⊗U(1) gauge group of the weak and electromagnetic interactions, into five parts:

$$\mathcal{L} = \mathcal{L}_S + \mathcal{L}_{\text{WEM}} + \mathcal{L}_\lambda + \mathcal{L}_b + \mathcal{L}_{\text{pv}} . \tag{6a}$$

The first part, $\mathcal{L}_S$, contains only the strongly interacting fields, together with their interactions with the weak and electromagnetic gauge fields:

$$\begin{aligned} \mathcal{L}_S = &-\tfrac{1}{2}\text{Tr}\{G^X_{\mu\nu}G^X_{\mu\nu} + G^Y_{\mu\nu}G^Y_{\mu\nu} + D_\mu K_X^\dagger D_\mu K_X + D_\mu K_Y^\dagger D_\mu K_Y + D_\mu K_\Sigma^\dagger D_\mu K_\Sigma\} \\ &- \bar{N}\gamma_\mu D_\mu N - G_N \bar{N}(\sigma_\Sigma - 2i\psi_\Sigma\gamma_5)N + \mu_1(|K_X|^2 + |K_Y|^2) + \mu_2|K_\Sigma|^2 + \mu_3(|K_X|^4 + |K_Y|^4) \\ &+ \mu_4|K_X|^2|K_Y|^2 + \mu_5|K_\Sigma|^4 + \mu_6|K_\Sigma|^2(|K_X|^2 + |K_Y|^2) . \end{aligned} \tag{6b}$$

We used the definitions $\psi_\Sigma = \tfrac{1}{2}\psi^a_\Sigma\tau_a$, $|K_X|^2 = \sigma_X{}^2 + (\psi^a_X)^2 = \text{Tr}\{K_X^\dagger K_X\}$, etc. The fields that have only weak and electromagnetic interactions are contained in $\mathcal{L}_{\text{WEM}}$:

$$\begin{aligned} \mathcal{L}_{\text{WEM}} = &-\tfrac{1}{4}G^0_{\mu\nu}G^0_{\mu\nu} - \tfrac{1}{2}\text{Tr}\{G^Z_{\mu\nu}G^Z_{\mu\nu} + D_\mu K_Z^\dagger D_\mu K_Z\} \\ &+ \rho_1|K_Z|^2 + \rho_2|K_Z|^4 + \text{leptons} . \end{aligned} \tag{6c}$$

The last three terms contain the remaining interactions among the spinless fields:

$$\begin{aligned} \mathcal{L}_\lambda &= |K_Z|^2[\lambda_1(|K_X|^2 + |K_Y|^2) + \lambda_2|K_\Sigma|^2], \\ \mathcal{L}_b &= b\,\text{Tr}\{K_Z^\dagger K_X^\dagger K_\Sigma K_Y \exp(i\rho'\tau_3)\}, \\ \mathcal{L}_{\text{pv}} &= (|K_X|^2 - |K_Y|^2)[\delta_1 + \delta_2(|K_X|^2 + |K_Y|^2) \\ &\quad + \delta_3|K_\Sigma|^2 + \delta_4|K_Z|^2]. \end{aligned} \tag{6d}$$

Let us now discuss some important features of the the Lagrangian (6). First, the spinless fields are expected to acquire nonzero vacuum expectation values in order to have massive gauge fields. As noticed before, the vacuum expectation value of K_Z, $\langle K_Z\rangle_0$ must be very large, such that the intermediate vector bosons of the weak interactions are very massive. In fact, $\langle K_Z\rangle_0$ must be of the order of $G_F{}^{-1/2}$, where G_F is the Fermi coupling constant. This also necessarily implies that certain couplings with the fields K_Z must be small, such that the large value of $\langle K_Z\rangle_0$ will not induce too strong effects. Hence, λ_1 and λ_2 are of order G_F, whereas b must be of order $G_F{}^{1/2}$.[15,16]

The nonzero vacuum expectation values will make the gauge fields massive.[12] However, in order to have still one massless gauge field, which will be identified as the photon field, one local U(1) subgroup of the total SU(2)⊗SU(2)⊗SU(2)⊗U(1) gauge group of the strong, weak, and electromagnetic interactions must remain unaffected by the presence of these nonzero vacuum expectation values. In that case, one can show that after a suitable redefinition of the spinless fields, only $\sigma_{X,Y,\Sigma,Z}$ will acquire nonzero vacuum expectation values. The electromagnetic local gauge transformations are then defined by

$$U(x) = V(x) = S(x) = T(x) = \exp[\tfrac{1}{2}i\Lambda^{\text{EM}}(x)\tau_3] . \tag{7}$$

In the case that all vacuum expectation values were zero, the parameter ρ' contained in $\mathcal{L}_b$ could be absorbed into the field K_Z by redefining $K_Z \to K_Z\exp(i\rho'\tau_3)$. One must realize, however, that once the vacuum expectation value of K_Z is chosen such that σ_Z is the only component with a nonzero vacuum expectation value, ρ' can no longer be absorbed into the definition of the fields. In fact, ρ' is now determined by the tadpole conditions, the equations that determine the magnitudes of the various vacuum expectation values.[17]

However, the term proportional to $\sin\rho'$ is the only one in the Lagrangian (6) that breaks CP invariance. Under CP the fields transform in the following way:

$$X_\mu(\vec{x}, x_0) \to X_\mu^*(-\vec{x}, x_0),$$

and similarly for Y_μ and Z_μ,

$$Z_\mu^0(\vec{x}, x_0) \to (1-2\delta_{\mu 4})Z^0(-\vec{x}, x_0),$$

$$K_X(\vec{x}, x_0) \to K_X^*(-\vec{x}, x_0),$$

and similarly for K_Y, K_Σ, and K_Z.

This implies that, unless a peculiar cancellation between different orders of perturbation theory takes place, $\sin\rho'$ must be equal to zero, and the system is not allowed to have CP violation. Hence, we may choose $\rho'=0$.

Subsequently, we consider the behavior of the total Lagrangian (6) under the parity transformation. This transformation is defined by

$$X_\mu(\vec{x}, x_0) \leftrightarrow -(1-2\delta_{\mu 4})Y_\mu(-\vec{x}, x_0),$$

$$Z_\mu(\vec{x}, x_0) \to -(1-2\delta_{\mu 4})Z_\mu(-\vec{x}, x_0),$$

$$Z_\mu^0(\vec{x}, x_0) \to -(1-2\delta_{\mu 4})Z_\mu^0(-x, x_0),$$

$$K_X(\vec{x}, x_0) \leftrightarrow K_Y(-\vec{x}, x_0),$$

$$K_{\Sigma,Z}(\vec{x}, x_0) \to K_{\Sigma,Z}^\dagger(-\vec{x}, x_0).$$

It turns out that parity is broken by interactions with the massive weak gauge fields [the interaction with the photon is obviously parity-conserving, as follows from the structure of the electromagnetic gauge group (7)]. Furthermore, parity is broken by terms proportional to $g_X - g_Y$ and by the terms contained in $\mathcal{L}_{pv}$. Therefore, we must take $g_X - g_Y$ and the coupling constants δ_i of $\mathcal{L}_{pv}$ to be at least of order G_F.[15]

Finally, let us analyze the structure of the purely hadronic Lagrangian. That is, $\mathcal{L}_S + \mathcal{L}_\lambda + \mathcal{L}_b$, where we disregard all terms of order g_Z, q, and $g_X - g_Y$ and replace the field K_Z by its vacuum expectation value $\langle K_Z\rangle_0$, which as argued before, is proportional to the identity matrix. Consider now the effect of the following global transformations:

$$K_X \to UK_XS^\dagger, \quad X_\mu \to UX_\mu U^\dagger,$$

$$K_Y \to VK_YT^\dagger, \quad Y_\mu \to VY_\mu V^\dagger,$$

$$K_\Sigma \to UK_\Sigma V^\dagger, \quad N \to \tfrac{1}{2}(1+\gamma_5)UN + \tfrac{1}{2}(1-\gamma_5)VN,$$

when U, V, S, and T are independent global SU(2) transformations. The only term that violates invariance under these transformations is $\mathcal{L}_b$. Suppose that $b=0$, and that we choose a gauge which does not disturb this invariance. Then in the presence of nonzero vacuum expectation values of the fields σ_X, σ_Y, and σ_Σ, the Goldstone theorem[18] ensures that all the fields ψ_X^a, ψ_Y^a, and ψ_Σ^a are massless. As the physical pion field must be a linear combination of these fields, the pion mass, a gauge-independent quantity, must be zero. Hence we have proved that the pion mass is of order $b\langle\sigma_Z\rangle_0$ in all orders of the strong interactions.

Consider now those transformations with $S=U$ and $T=V$. These transformations are the usual global chiral SU(2)⊗SU(2) transformations. And the chiral-symmetry breaking is again $\mathcal{L}_b$, and thus of order $b\langle\sigma_Z\rangle_0$, just as the pion mass.

Finally, the transformations $U=V=T=S$ are identified as the isospin SU(2) group. Extending these transformations for the total Lagrangian (6), we find that the only isospin violation comes from the electromagnetic interactions. This implies that all mass differences within isospin multiplets are calculable in this model.

In conclusion, we have introduced a unified gauge field model of strong, weak, and electromagnetic interactions, which, due to the gauge invariance, is renormalizable. It has strongly interacting vector and axial-vector mesons, and two scalar and one pseudoscalar isosinglets. The remaining spinless isotriplet fields are unphysical.[19] The pseudoscalar particle $(1/\sqrt{2})(\sigma_X - \sigma_Y)$ cannot correspond to the η meson because of its behavior under CP. Finally, we mention the presence of triangle anomalies in this model coming from the electromagnetic interactions.[3] Although there are several ways to get rid of them, they will be ignored in this paper.

III. SOME RESULTS IN LOWEST-ORDER APPROXIMATION

As argued before, only the fields $\sigma_{X,Y,\Sigma,Z}$ will acquire nonzero vacuum expectation values, which we will denote by F_X, F_Y, F_Σ, and F_Z, respectively. Neglecting parity violation, which was supposed to be of order G_F, we have $F_X = F_Y = F$ and $g_X = g_Y = g$. In this section we will consider the effects of the nonzero vacuum expectation values in the tree approximation.

The first result will be that certain linear combinations of gauge fields become massive. To account for the main effects of mixing between the gauge fields, it is convenient to make the following substitutions:

$$X_\mu = \frac{1}{\sqrt{2}}(U_\mu + V_\mu) + \frac{1}{2}\frac{e}{g}A_\mu\tau_3 + \frac{1}{2}\frac{g_W}{g}W_\mu(1+\epsilon),$$

$$Y_\mu + \frac{1}{\sqrt{2}}(U_\mu - V_\mu) + \frac{1}{2}\frac{e}{g}A_\mu\tau_3 + \frac{1}{2}\frac{g_W}{g}W_\mu(1-\epsilon),$$

$$Z_\mu = \frac{g_W}{g_Z}W_\mu + \frac{1}{2}\frac{e}{g_Z}A_\mu\tau_3, \tag{8a}$$

$$Z_\mu^0 = \frac{e}{q}A_\mu.$$

We have used the following definitions:

$$
\begin{aligned}
e &= g g_Z q(g^2q^2+g^2g_Z{}^2+2g_Z{}^2q^2)^{-1/2}\,,\\
g_W &= g g_Z[g^2+\tfrac{1}{2}(1+\epsilon^2)g_Z{}^2]^{-1/2}\,,\\
\epsilon &= t_\Sigma{}^2(2+t_\Sigma{}^2)^{-1},\\
t_\Sigma &= F F_\Sigma{}^{-1} \text{ and } t_Z = F F_Z{}^{-1}.
\end{aligned}
\tag{8b}
$$

With these substitutions the gauge field propagators are diagonal at zero momentum.[20] The parameters e and g_W are the electromagnetic and weak coupling constants, respectively. The latter is related to the Fermi coupling constant by $G_F=\frac{1}{8}\sqrt{2}\,g_W{}^2M_W{}^{-2}$. Notice that the quantity t_Z is of order $G_F{}^{1/2}$.

The calculation of the masses is now straightforward in this approximation. The field A_μ, corresponding to the photon, remains massless. The remaining masses are given (in lowest order in G_F) by

$$
\begin{aligned}
M_U &= \tfrac{1}{2}gF,\\
M_V &= \epsilon^{-1/2}M_U,\\
M_W &= \tfrac{1}{2}g_W F_Z\,.
\end{aligned}
\tag{9}
$$

Another effect of the nonzero vacuum expectation values concerns the physical states. This is obvious for the gauge fields, as the massive vector particles acquire one additional polarization. For the four spinless isotriplet fields, it implies that only one linear combination of them will correspond to physical states.[19] This is the pion field, which has the following form (in lowest order in G_F):

$$
\pi=\left(\frac{1-\epsilon}{2}\right)^{1/2}(\psi_X-\psi_Y-t_\Sigma\psi_\Sigma+t_Z\psi_Z)\,. \tag{10a}
$$

The remaining linear combinations correspond to unphysical states:

$$
\begin{aligned}
\psi_U &= \frac{1}{\sqrt{2}}(\psi_X+\psi_Y)\,,\\
\psi_V &= \tfrac{1}{2}(1-\epsilon)^{1/2}(t_\Sigma\psi_X-t_\Sigma\psi_Y+2\psi_\Sigma)\,,\\
\psi_W &= \tfrac{1}{2}(1-\epsilon)(t_Z\psi_X-t_Z\psi_Y-t_Zt_\Sigma\psi_\Sigma)-\psi_Z\,,
\end{aligned}
\tag{10b}
$$

where we used again the notation $\psi_X=\frac{1}{2}\psi_X^a\tau_a$, etc. In order to calculate the masses of these spinless fields, we must first determine the so-called tadpole conditions. In the tree approximation, these conditions are simply found by requiring that, after the substitutions,

$$
\begin{aligned}
\sigma_{X,Y} &\to \sigma_{X,Y}+F\,,\\
\sigma_\Sigma &\to \sigma_\Sigma+F_\Sigma,\\
\sigma_Z &\to \sigma_Z+F_Z\,,
\end{aligned}
\tag{11}
$$

the coefficients of the terms linear in the fields vanish. This yields three equations for F, F_Σ, and F_Z, and making use of them, we can calculate the masses of the spinless mesons. It turns out that the fields $\psi_{U,V,W}$ are massless, as they should be according to the Goldstone theorem.[18] The pion, however, picks up a mass, which is given by (in lowest order in G_F)

$$
m_\pi{}^2=bF_ZF_\Sigma(1-\epsilon)^{-1}.
$$

As was generally argued in the previous section, this is indeed of order bF_Z.

Let us now consider the purely hadronic part of the total Lagrangian (6), and subsequently analyze the first-order weak and electromagnetic interactions with the hadrons. It is obvious that this can be done the most appropriately in the physical gauge, as in this gauge all fields will correspond to physical particles. In the tree approximation this simply implies that all terms containing ψ_U, ψ_V, or ψ_W can be disregarded. If we do so, and moreover use the substitutions (8) and (11), we find the following result for the purely hadronic part of the Lagrangian (in lowest order in G_F):

$$
\begin{aligned}
\mathcal{L} = &-\mathrm{Tr}\{\tfrac{1}{2}U_{\mu\nu}U_{\mu\nu}+M_U^2U_\mu{}^2+\tfrac{1}{2}V_{\mu\nu}V_{\mu\nu}+M_V{}^2V_\mu{}^2+(\partial_\mu\pi)^2+m_\pi{}^2\pi^2\}\\
&-\tfrac{1}{2}[(\partial_\mu\sigma_1)^2+m_1{}^2\sigma_1{}^2+(\partial_\mu\sigma_\Sigma)^2+m_\Sigma{}^2\sigma_\Sigma{}^2+(\partial_\mu\eta)^2+m_\eta{}^2\eta^2]\\
&-\overline{N}(\gamma_\mu\partial_\mu+m_N)N-\tfrac{1}{2}gM_U\sqrt{2}\,\mathrm{Tr}\{\sigma_1U_\mu{}^2+(\sigma_1+2\sqrt{2}\,t_\Sigma{}^{-1}\sigma_\Sigma)V_\mu{}^2+2\eta U_\mu V_\mu+2i(1-\epsilon)^{1/2}\pi[U_\mu,V_\mu]\}\\
&+\tfrac{1}{2}\sqrt{2}\,g\,\mathrm{Tr}\{(1-\epsilon)^{1/2}\eta\overleftrightarrow{\partial}_\mu\pi U_\mu+(1-\epsilon)^{1/2}(\sigma_1-\sqrt{2}\,t_\Sigma\sigma_\Sigma)\overleftrightarrow{\partial}_\mu\pi V_\mu-i(1+\epsilon)\partial_\mu\pi[\pi,U_\mu]\}\\
&-\tfrac{1}{2}g^2\,\mathrm{Tr}\{\tfrac{1}{4}[\sigma_1{}^2+\eta^2+4(1+\epsilon)\pi^2](U_\mu{}^2+V_\mu{}^2)+\sigma_\Sigma{}^2V_\mu{}^2+\sigma_1\eta U_\mu V_\mu+i[2(1-\epsilon)]^{1/2}t_\Sigma\sigma_\Sigma\pi[U_\mu,V_\mu]-2\epsilon\,\pi(U_\mu\pi U_\mu-V_\mu\pi V_\mu)\}\\
&+\tfrac{1}{2}ig\sqrt{2}\,\overline{N}(\gamma_\mu U_\mu+\gamma_\mu\gamma_5V_\mu)N-G_N\overline{N}\{\sigma_\Sigma+i[2(1-\epsilon)]^{1/2}t_\Sigma\pi\gamma_5\}N+\mathcal{L}'(\pi,\sigma_1,\sigma_\Sigma,\eta)\,.
\end{aligned}
\tag{12}
$$

$\mathcal{L}'$ gives the interactions of the spinless fields among themselves. We have used the following definitions:

$$U_{\mu\nu}=\partial_\mu U_\nu-\partial_\nu U_\mu-\frac{1}{\sqrt{2}}ig[U_\mu,U_\nu]-\frac{1}{\sqrt{2}}ig[V_\mu,V_\nu],$$

$$V_{\mu\nu}=\partial_\mu V_\nu-\partial_\nu V_\mu-\frac{1}{\sqrt{2}}ig[U_\mu,V_\nu]-\frac{1}{\sqrt{2}}ig[V_\mu,U_\nu],$$

$$\sigma_1=\frac{1}{\sqrt{2}}(\sigma_X+\sigma_Y),$$

$$\eta=\frac{1}{\sqrt{2}}(\sigma_X-\sigma_Y).$$

When we now consider the coupling of the hadrons to the weak and electromagnetic gauge fields, W_μ and A_μ, it turns out that the vector part of this interaction can be completely accounted for (apart from a less relevant term $g^{-1}\partial_\mu U_{\mu\nu}$, the derivative of an antisymmetric tensor) by simply replacing the derivatives by "minimal" derivatives:

$$\partial_\mu \pi^a \to \partial_\mu^{\mathrm{WEM}}\pi^a=\partial_\mu\pi^a-e\epsilon_{ab3}\pi^b A_\mu-\tfrac{1}{2}g_W\epsilon_{abc}\pi^b W_\mu^c,$$

and similarly for U_μ^a and V_μ^a,

$$\partial_\mu N\to\partial_\mu^{\mathrm{WEM}}N=\partial_\mu N-\tfrac{1}{2}ieA_\mu(1+\tau_3)N-\tfrac{1}{2}ig_W W_\mu N.$$

This clearly shows that the weak and electromagnetic vector currents are indeed related to isospin transformations in a way that is prescribed by the CVC hypothesis. The remaining interactions with the weak gauge fields W_μ will define the weak axial-vector currents:

$$\begin{aligned}&ig_W\epsilon\,\mathrm{Tr}\{W_\mu[V_\nu,U_{\mu\nu}]+W_\mu[U_\nu,V_{\mu\nu}]+i\sqrt{2}g^{-1}W_\mu\partial_\nu V_{\mu\nu}\}\\&-\tfrac{1}{2}g_W(1-\epsilon)^{1/2}\mathrm{Tr}\{W_\mu[(1-\epsilon)\sigma_1+\epsilon t_\Sigma\sqrt{2}\sigma_\Sigma]\overleftrightarrow{\partial}_\mu\pi+2iM_U(1+\epsilon)W_\mu[\pi,U_\mu]+\tfrac{1}{2}i\sqrt{2}g[W_\mu,\pi](\sigma_1U_\mu+\epsilon\sqrt{2}t_\Sigma\sigma_\Sigma U_\mu+\eta V_\mu)\}\\&+g_WM_U\,\mathrm{Tr}\{(1-\epsilon)W_\mu U_\mu\eta+W_\mu V_\mu[(1-\epsilon)\sigma_1-2\sqrt{2}\epsilon t_\Sigma^{-1}\sigma_\Sigma]\}\\&-\tfrac{1}{8}\sqrt{2}g_Wg\,\mathrm{Tr}\{2(\epsilon-1)W_\mu U_\mu\eta\sigma_1+W_\mu V_\mu[(\epsilon-1)(\sigma_1^2+\eta^2)+4\epsilon\sigma_\Sigma^2+4\epsilon(1+\epsilon)\pi^2]-4(1-\epsilon-2\epsilon^2)W_\mu\pi V_\mu\pi\}\\&+\tfrac{1}{2}i\epsilon g_W\overline{N}\gamma_\mu\gamma_5W_\mu N.\end{aligned}$$

These axial-vector currents are certainly not of the canonical form. Their scale is not fixed, and, moreover, there exists no infinitesimal transformation of the fields that can generate these currents from the hadronic Lagrangian (12).

We conclude that this raises some doubt about the validity of the current-algebra hypothesis in this type of gauge field models. In the next section we will investigate the possible validity of the current-algebra approach in more detail. This will be done by analyzing the constraints of gauge invariance as given by generalized Ward-Takahashi identities. Finally, we will argue that, although the current-algebra hypothesis in its original formulation may or may not be true, the divergence equations, which can be derived from it, are not invalidated.

IV. GENERALIZED WARD-TAKAHASHI IDENTITIES

The constraints that follow from gauge invariance can be expressed in generalized Ward-Takahashi identities. These identities are valid in every order of perturbation theory, and depend explicitly on the gauge in which the calculations are carried through. Let us first briefly summarize in a general way how to proceed in higher orders of perturbation theory, and then give the generalized Ward-Takahashi identities.[21,22]

In a gauge field theory higher-order calculations must be performed in a certain gauge. One way of fixing a gauge is by choosing functions of the fields, $C_a(x)$, where the index a labels the generators of the gauge group, and replacing the original gauge-invariant Lagrangian $\mathcal{L}_{\mathrm{inv}}$ by

$$\mathcal{L}_{\mathrm{inv}}-\tfrac{1}{2}\sum_a C_a{}^2\,. \qquad (13)$$

The functions C_a are chosen in such a way that this replacement removes completely the original gauge freedom. We will suppose that the C_a are linear combinations of the various fields.

The next step is to add a Faddeev-Popov Lagrangian to (13), which is defined as

$$\mathcal{L}_{\mathrm{FP}}=\phi_a^*(x)\frac{\delta C_a(x)}{\delta\Lambda^b(x')}\,\phi_b(x')\,.$$

where $\delta C_a/\delta\Lambda^b$ represents the change of C_a under an infinitesimal gauge transformation, described by parameters Λ^b. The fields ϕ_a are unphysical and occur only in closed loops. These so-called Faddeev-Popov ghost fields obey Fermi statistics, which implies that every closed ghost loop has an additional minus sign.

The starting point for our further discussion is the generalized Ward-Takahashi identity, as it was formulated by 't Hooft and Veltman.[19] Suppose the fields A_i of the original Lagrangian exhibit the following behavior under infinitesimal gauge transformations:

$$A_i(x)\to A_i(x)+t_i^a\Lambda^a(x)+gs_{ij}^aA_j(x)\Lambda^a(x)\,. \qquad (14)$$

In our case, t_i^a is always a constant or a derivative, and it is of zeroth order in g, the coupling constant belonging to the gauge group. The quantities s_{ij}^a are simple constants. With these definitions, the generalized Ward-Takahashi identities can be graphically represented as in Fig. 1. A solid line with index i belongs to the field A_i. As the gauge factors C_a were supposed to be linear combinations of the fields, the first diagram is simply a linear combination of the Green's functions of the fields A. A dashed line with index a represents the Faddeev-Popov ghost ϕ_a. The additional vertices $t_i^b\phi_b(x)$ and $gs_{im}^b A_m(x)\phi_b(x)$, which do not occur in the S matrix, are completely defined by the transformation properties of the fields A_i, as given by (14). The "etc." in Fig. 1 represents similar diagrams as the second and the third one, with the Faddeev-Popov ghost connected to one of the other external lines, labeled by $j\cdots k$. Notice that for any given set of external lines, we have as many identities as the number of generators of the gauge group. For further details and a proof of these identities, we refer to 't Hooft and Veltman.[19]

After this rather general discussion we turn again to our unified gauge field model. Let us first define the infinitesimal transformations of our SU(2)⊗SU(2)⊗SU(2)⊗U(1) gauge group. They are directly related to the local transformations $U(x)$, $V(x)$, $S(x)$, and $T(x)$ which were introduced in Sec. II. We choose the following parametrization for the gauge group of the weak and electromagnetic interactions:

$$S(x)\sim 1+\tfrac{1}{2}ig_W\Lambda_W^a(x)\tau_a+\tfrac{1}{2}ie\Lambda_A(x)\tau_3\,,\qquad T(x)\sim 1+\tfrac{1}{2}ie\Lambda_A(x)\tau_3\,, \tag{15a}$$

and for the chiral gauge group of the strong interactions:

$$U(x)\sim 1+\tfrac{1}{2}ig(\Lambda_U^a+\Lambda_V^a)\tau_a\;,\qquad V(x)\sim 1+\tfrac{1}{2}ig(\Lambda_U^a-\Lambda_V^a)\tau_a\;. \tag{15b}$$

We will partly take into account the mixing of the photon field with the neutral weak gauge field by substituting

$$Z_\mu=\frac{g_W}{g_Z}W_\mu+\frac{1}{2}\frac{e}{g_Z}A_\mu\tau_3\;,\qquad Z_\mu^0=\frac{e}{q}A_\mu\;. \tag{16a}$$

The parameters e and g_W are defined by

$$e=qg_W(q^2+g_W{}^2)^{-1/2}\;,\qquad g_W=g_Z\;. \tag{16b}$$

Notice that these definitions differ from the previous ones (8b) only by terms that are of higher order in the weak and electromagnetic interactions.

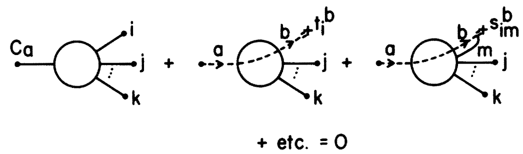

FIG. 1. The graphical representation of the generalized Ward-Takahashi identities.

It is of interest to give the transformation properties of these newly defined fields under the infinitesimal weak and electromagnetic transformations (15a):

$$\begin{aligned}W_\mu &\to W_\mu+\partial_\mu\Lambda_W+ig_W[\Lambda_W,W_\mu]\\&\quad+\tfrac{1}{2}ieA_\mu[\Lambda_W,\tau_3]+\tfrac{1}{2}ie\Lambda_A[\tau_3,W_\mu],\\ A_\mu&\to A_\mu+\partial_\mu\Lambda_A\,,\end{aligned} \tag{17}$$

where $\Lambda_W\equiv\frac{1}{2}\Lambda_W^a\tau_a$.

Henceforth, we fix the weak and electromagnetic gauge by choosing

$$C_W=\rho_W\partial_\mu W_\mu\,,\qquad C_A=\rho_A\partial_\mu A_\mu\,. \tag{18}$$

For the strong gauge we will consider several possibilities, one of them being

$$C_U=-\rho_U M_U\psi_U\,,\qquad C_V=-\rho_V M_V\psi_V\,, \tag{19}$$

with $M_{U,V}$ and $\psi_{U,V}$ as defined in Eqs. (9) and (10). The relevance of this particular gauge comes from the fact that in the limit $\rho_{U,V}\to\infty$, the fields ψ_U and ψ_V become infinitely massive. This implies that they will no longer contribute to internal lines. And as ψ_U and ψ_V were by definition unphysical fields, this limit will give us the physical gauge, as far as the strongly interacting particles are concerned.

Finally, we add the following terms to the Lagrangian:

$$\mathcal{L}_c=-\mathrm{Tr}\{C_U{}^2+C_V{}^2+C_W{}^2\}-\tfrac{1}{2}C_A{}^2\,. \tag{20}$$

The Faddeev-Popov Lagrangian is determined by the change of the various factors C under infinitesimal gauge transformations. For example, for C_W and C_A we find the following transformation properties:

$$\begin{aligned}C_W&\to C_W+\rho_W\partial^2\Lambda_W+ig_W\rho_W\partial_\mu[\Lambda_W,W_\mu]\\&\quad+\tfrac{1}{2}ie\rho_W\partial_\mu[\Lambda_W,A_\mu\tau_3]+\tfrac{1}{2}ie\rho_W\partial_\mu[\Lambda_A\tau_3,W_\mu],\\ C_A&\to C_A+\rho_A\partial^2\Lambda_A\,.\end{aligned}$$

This result gives rise to the following terms in $\mathcal{L}_{FP}$:

$$\mathcal{L}_{\rm FP}^{(1)} = -\rho_A \partial_\mu \phi_A^* \partial_\mu \phi_A - 2\rho_W \,\mathrm{Tr}\{\partial_\mu \phi_W^* \partial_\mu \phi_W + i g_W [\partial_\mu \phi_W^*, \phi_W] W_\mu + \tfrac{1}{2} i e [\partial_\mu \phi_W^*, \phi_W] A_\mu \tau_3\},$$
$$\mathcal{L}_{\rm FP}^{(2)} = i e \rho_W \,\mathrm{Tr}\{[\partial_\mu \phi_W^*, W_\mu]\tau_3\}\phi_A . \quad (21)$$

An analogous calculation for C_U and C_V, with, for simplicity, $\sqrt{2}\rho_U = \sqrt{2}\rho_V = \rho$, gives the remaining terms:

$$\mathcal{L}_{\rm FP}^{(3)} = -2\rho\, \mathrm{Tr}\{M_U{}^2 \phi_U^* \phi_U + M_V{}^2 \phi_V^* \phi_V\}$$
$$- \tfrac{1}{2}\rho g M_U \,\mathrm{Tr}\{\sqrt{2}\, \phi_U^* \phi_U \sigma_1 + \phi_V^* \phi_V (\sqrt{2}\,\sigma_1 + 4 t_\Sigma{}^{-1} \sigma_\Sigma) + \sqrt{2}\,(\phi_U^* \phi_V + \phi_V^* \phi_U)\eta$$
$$+ i([\phi_U^*, \phi_U] + [\phi_V^*, \phi_V])(\psi_X + \psi_Y) + i[\phi_U^*, \phi_V](\psi_X - \psi_Y) + i[\phi_V^*, \phi_U](\psi_X - \psi_Y + 4 t_\Sigma{}^{-1}\psi_\Sigma)\}, \quad (22)$$
$$\mathcal{L}_{\rm FP}^{(4)} = \tfrac{1}{4}\rho e M_U \,\mathrm{Tr}\{4 g^{-1} M_U \phi_U^* \tau_3 + \sqrt{2}(\phi_U^* \sigma_1 + \phi_V^* \eta)\tau_3 + i[\phi_U^*, \psi_X + \psi_Y]\tau_3 + i[\phi_V^*, \psi_X - \psi_Y]\tau_3\}\phi_A$$
$$+ \rho g_W M_U \,\mathrm{Tr}\left\{\frac{1}{\sqrt{2}}(\phi_U^* + \phi_V^*)\phi_W (2\sqrt{2}\, g^{-1} M_U + \sigma_1 + \eta) + i[\phi_U^* + \phi_V^*, \phi_W]\psi_X\right\}.$$

The total Faddeev-Popov Lagrangian is given by the sum of these terms: $\mathcal{L}_{\rm FP}^{(1)} + \mathcal{L}_{\rm FP}^{(2)} + \mathcal{L}_{\rm FP}^{(3)} + \mathcal{L}_{\rm FP}^{(4)}$. Furthermore, we used the definitions $\phi_{U,V,W} = \frac{1}{2}\phi_{U,V,W}^a \tau_a$ and $\phi_{U,V,W}^* = \frac{1}{2}\phi_{U,V,W}^{*a}\tau_a$.

In S-matrix calculations, where the Faddeev-Popov ghosts occur only in closed loops, the terms in $\mathcal{L}_{\rm FP}^{(2)}$ and $\mathcal{L}_{\rm FP}^{(4)}$ cannot contribute. The ghost field ϕ_A can even be ignored completely, because it is a free field essentially. Hence we are only left with $\mathcal{L}_{\rm FP}^{(1)}$ and $\mathcal{L}_{\rm FP}^{(3)}$, which can be evaluated separately.

However, the terms in $\mathcal{L}_{\rm FP}^{(2)}$ and $\mathcal{L}_{\rm FP}^{(4)}$ can no longer be ignored, when we consider the generalized Ward-Takahashi identities. As an example, we consider such an identity for two nucleons, which is given in Fig. 2. In theories where the strong interactions are not governed by local gauge groups, so that ϕ_W and ϕ_A are the only Faddeev-Popov ghosts, the ghost field in the second and third diagram will not interact with the blob in lowest order in g_W. And in this order the blob is described by strong interactions alone. In that case, one easily derives identities like

$$(p_2 - p_1)_\mu \Gamma_{a\mu}(p_2, p_1) = \tau_a S^{-1}(p_1) - S^{-1}(p_2)\tau_a ,$$
$$(p_2 - p_1)_\mu \Gamma_{a\mu}^5(p_2, p_1) + \Gamma_a^5(p_2, p_1) = \tau_a \gamma_5 S^{-1}(p_1) + S^{-1}(p_2)\tau_a\gamma_5 , \quad (23)$$

where $\Gamma_{\mu a}$, $\Gamma_{\mu a}^5$, and Γ_a^5 are proportional to the irreducible vertex functions of the vector current, the axial-vector current, and its divergence, respectively. The nucleon propagator is denoted by S, and the momenta of the incoming and outgoing nucleon are p_1 and p_2. Notice that this result is valid in all orders of the strong interactions. The first identity is the one originally derived by Ward and Takahashi for the case of quantum electrodynamics,[23] from which one can show that the charge is not renormalized.

However, in our case the ghost field will interact with the blob, in first order in g_W, through terms which were contained in $\mathcal{L}_{\rm FP}^{(4)}$. An example of one of those contributions is depicted in Fig. 3. Obviously, as the ghost ϕ_W goes into ϕ_U and the latter can strongly interact with the blob, the identities (23) will be affected by strong interactions. Furthermore, as $\mathcal{L}_{\rm FP}^{(4)}$ is actually determined from the behavior of C_U and C_V under the weak and electromagnetic gauge transformations, the identities (23) depend on the choice of the "strong" gauge. For instance, if we had fixed the gauge by choosing functions C_U and C_V which do not transform under the weak and electromagnetic gauge transformations (18), the right-hand sides of Eqs. (23) would vanish. On the other hand, in the previous gauge (19) in the limit $\rho \to \infty$, the physical gauge as far as hadrons are concerned, one can still show that the first identity (23) for the vector current is correct.

Hence, we have observed that in unified gauge field theories of strong, weak, and electromagnetic interactions, the Ward-Takahashi identities that involve hadrons off the mass shell are affected by

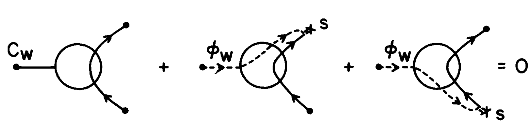

FIG. 2. Generalized Ward-Takahashi identity for two external nucleons.

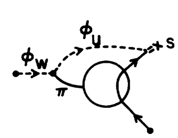

FIG. 3. One of the diagrams that will affect Eq. (23).

strong interactions in lowest order in e and g_W. Furthermore, they depend on the gauge that has been chosen for the strong interactions. In the next section we will consider generalized Ward-Takahashi identities where the hadrons are on the mass shell. The resulting identities turn out to depend only on the structure of the weak and electromagnetic gauge group, which is $SU(2) \otimes U(1)$ in our case.

V. DIVERGENCE EQUATIONS FOR THE SU(2) ⊗ U(1) WEAK AND ELECTROMAGNETIC GAUGE GROUP

In this section we will derive a class of divergence equations for the hadronic matrix elements of weak and electromagnetic currents. Our starting point is the generalized Ward-Takahashi identity, as it was depicted in Fig. 1. However, henceforth we will take all the strongly interacting particles on the mass shell. This has the consequence that, due to the pole structure, all the attachments of the Faddeev-Popov ghosts to the external physical hadron lines will vanish.[24] Hence, the physical hadrons give no explicit contribution to the generalized Ward-Takahashi identities.

The first class of Ward-Takahashi identities we will consider are those with only physical hadrons on the mass shell. They are depicted in Fig. 4(a), where "h" denotes the external hadrons. This expression will be considered in first order of the weak and electromagnetic interactions, in which case the blob is given by the strong interactions alone. The identities represented by Fig. 4(a) lead to what we will call first-order divergence equations.

Another class of generalized Ward-Takahashi identities gives rise to the second-order divergence equations. These identities which are pictorially represented in Fig. 4(b) contain, apart from the external hadrons h, an additional line ω, which corresponds to a field that transforms only under weak and electromagnetic gauge transformations. In our model, this can only be one of the fields W^a_μ, A_μ, and K_Z. Due to this requirement, the Faddeev-Popov ghosts that are attached to vertices s and t are necessarily the fields ϕ_W or ϕ_A.

Henceforth, we take the weak and electromagnetic gauge transformations as defined in Eq. (15a), and the gauge fields W^a_μ and A_μ as given by Eq. (16). The weak and electromagnetic gauge is fixed by choosing C_W and C_A as given in Eq. (18). In this gauge the propagators of interest are calculated in the Appendix, whereas the pertinent part of the Faddeev-Popov Lagrangian, $\mathcal{L}^{(1)}_{FP} + \mathcal{L}^{(2)}_{FP}$, was determined in Eq. (21).

After these definitions, the derivation of the divergence equations is rather straightforward. Using the expressions for the propagators, we find directly from Fig. 4(a):

$$\begin{aligned} k_\mu J^a_\mu(k) &= i\frac{M_W}{g_W} J^a_\psi(k), \\ k_\mu J^A_\mu(k) &= 0. \end{aligned} \tag{24}$$

We call these equations the first-order divergence equations. $J^a_\mu(k)$ and $J^A_\mu(k)$ are the hadronic matrix elements of the weak and the electromagnetic currents, respectively. These currents are defined by the coupling of W^a_μ or A_μ with (incoming) momentum k to the hadrons, disregarding the corresponding coupling constants g_W and e. The quantity $J^a_\psi(k)$ is the hadronic matrix element of the hadronic source that is coupled to the fields ψ^a_Z. As argued in Sec. II, J^a_ψ must be of first order in the weak interactions. The origin of this term in the divergence equations comes from W^a_μ, proceeding through ψ^a_Z, before it couples to the hadrons.[25]

The derivation of the second-order divergence equations is somewhat more involved. When we consider only second order in g_W or e, we can replace Fig. 4(b) by Fig. 5. The ghost fields ϕ_W and ϕ_A can no longer entangle with the strong ghost fields, because ω was required to transform only under the weak and electromagnetic group. This is then also the case for ω' in Fig. 5.

Let us now introduce the notion of "weak irreducibility." A diagram is called weakly irreducible when it cannot be divided into two nontrivial parts by cutting a line that corresponds to one of the fields that transforms purely under the weak and electromagnetic gauge group. The weakly reducible graphs contributing to the first diagram in Fig. 5 are given in Fig. 6. The lower part of the graph is the hadronic matrix element of the source of the field ω''. For the top part of the diagram, consisting of a vertex with three lines $C_{W,A}$, ω, and ω'', we can write down a generalized Ward-Takahashi identity. It is depicted in Fig. 7, and

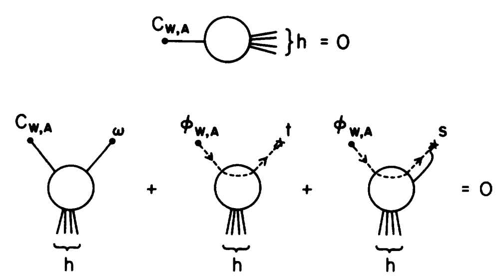

FIG. 4. Generalized Ward-Takahashi identities that lead to the divergence equations.

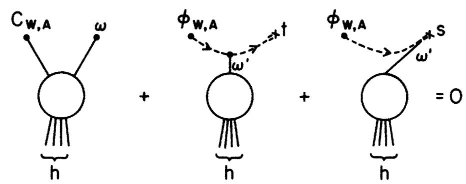

FIG. 5. Generalized Ward-Takahashi identity in second order in g_W or e.

of first order in g_W or e. Subsequently, we subtract the weakly reducible diagrams in Fig. 6 from the expression in Fig. 5, using Fig. 7. It turns out, that the second and third graph cancel the contributions from the second and third graph of Fig. 7. The contributions from the fourth diagram in Fig. 7 for the various possible ω'' cancel among themselves, due to the first-order divergence Eq. (24). Hence, we are left with the expression depicted in Fig. 8, where the left-hand side is confined to only weakly irreducible diagrams. The fields ω and ω'' are by definition fields that purely transform under the weak and electromagnetic gauge group, in our model given by W^a_μ, A_μ, ψ^a_Z, and σ_Z. Notice that the blob contains only contributions from strong interactions.

Finally, after removing the various propagators from Fig. 8 we find the following result:

$$p_\mu U^a_{\mu\omega}(p,q) - i\frac{M_W}{g_W}T^a_\omega(p,q) = \sum_{\omega''} s^a_W(\omega,\omega'')J_{\omega''}(p+q)\,,$$

$$p_\mu U^A_{\mu\omega}(p,q) = \sum_{\omega''} s_A(\omega,\omega'')J_{\omega''}(p+q)\,, \tag{25a}$$

where $\omega, \omega'' = W^a_\mu$, A_μ, ψ^a_Z, or σ_Z. $U^a_{\mu\omega}(p+q)$ is the hadronic amplitude of a vector boson W^a_μ and a field ω with incoming momenta p and q, respectively, where both W^a_μ and ω interact directly with the hadrons. $U^A_{\mu\omega}$ and T^a_ω are similar quantities with W^a_μ replaced by A_μ and ψ^a_Z, respectively. The quantity $J_\omega(k)$ denotes the hadronic matrix element of the source that is coupled to the field ω with corresponding (incoming) momentum k.

The behavior of ω'' under infinitesimal gauge transformations determines $s^a_W(\omega,\omega'')$ and $s_A(\omega,\omega'')$ Explicit calculation gives the following results for the identity (25a) in the case that $\omega = W^b_\nu$, ψ^b_Z, σ_Z,

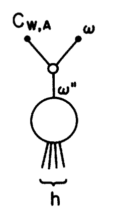

FIG. 6. Weakly reducible graphs contributing in Fig. 5.

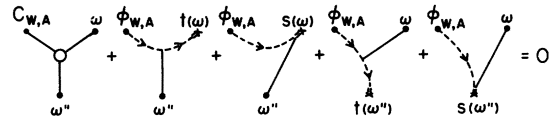

FIG. 7. Generalized Ward-Takahashi identity for the top part of the weakly reducible diagrams of Fig. 6.

and A_ν, respectively:

$$p_\mu U^{ab}_{\mu\nu}(p,q) - i\frac{M_W}{g_W}T^{ab}_\nu(p,q) = \epsilon_{abc}J^c_\nu(p+q)\,,$$

$$p_\mu U^{ab}_\mu(p,q) - i\frac{M_W}{g_W}T^{ab}(p,q) = \tfrac{1}{2}\delta_{ab}J_\sigma(p+q) + \tfrac{1}{2}\epsilon_{abc}J^c_\psi(p+q)\,, \tag{26a}$$

$$p_\mu U^a_\mu(p,q) - i\frac{M_W}{g_W}T^a(p,q) = -\tfrac{1}{2}J^a_\psi(p+q)\,,$$

$$p_\mu U^{aA}_{\mu\nu}(p,q) - i\frac{M_W}{g_W}T^{aA}_\nu(p,q) = \epsilon_{a3c}J^c_\nu(p+q)\,.$$

A similar calculation gives the identity (25b) for $\omega = W^b_\nu$, ψ^b_Z:

$$\begin{aligned} &p_\mu U^{Ab}_{\mu\nu}(p,q) = \epsilon_{3bc}J^c_\nu(p+q)\,,\\ &p_\mu U^{Ab}_\mu(p,q) = \epsilon_{3bc}J^c_\psi(p+q)\,. \end{aligned} \tag{26b}$$

For $\omega = A_\nu$ or σ_Z the right-hand side of the identity (25b) is simply zero. The definitions of the various functions in these identities is obvious. Notice that we have already extracted factors g_W and e from those functions where the fields W_μ and A_μ are involved.

Equations (26) are called second-order divergence equations. Not quite unexpectedly, they show the same structure as the divergence equations that can be derived from the usual current-algebra assumptions, provided we identify the divergence of the axial-vector current as ψ_Z and the so-called σ term as J_σ, up to appropriate constants.

Hence, we have proved an important set of identities purely from the gauge invariance of the weak and electromagnetic interactions. The derivation did not depend on the structure of the strong interactions, and is also valid when those

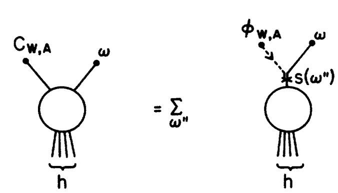

FIG. 8. The second-order divergence equations.

interactions are governed by a local gauge group, as in the unified model of Sec. II. Compared with the current-algebra derivation, our approach needs only one assumption: that of weak and electromagnetic gauge invariance. The generalized Ward-Takahashi identities, which were the main ingredient in the derivation, are proved to be valid in all orders of perturbation theory. And they are manifestly Lorentz-covariant, so that Schwinger and seagull terms do not play a role.[26]

VI. THE PCAC HYPOTHESIS AND THE PION

As is well known,[10] most of the results of the current-algebra approach are essentially based on the divergence equations which were found in the previous section. It is obvious that these equations were independent of the specific structure of the strong interactions. The only additional information one usually needs concerns the isospin and parity properties of the various matrix elements.

However, the divergence equations contain hadronic amplitudes involving ψ_Z, and it is desirable to relate them to purely hadronic matrix elements in order to find experimentally testable predictions. As argued before, in the current-algebra approach ψ_Z corresponds to the divergence of the axial-vector current. In that case, by making use of the PCAC hypothesis, one is able to find results for the corresponding matrix elements of the pion field.

In this section we will analyze the question of how to relate the hadronic matrix elements of ψ_Z to the corresponding ones of the pion field, in order to find a similar result as that given by PCAC. The starting point of our discussion is the total Lagrangian, written as

$$\mathcal{L} = \mathcal{L}_S(\pi_s, W_\mu, A_\mu) + \lambda |K_Z|^2 \mathcal{L}'_\lambda(\pi_s) + b\mathcal{L}_b(K_Z, \pi_s) + \mathcal{L}_{\text{WEM}}(K_Z, W_\mu, A_\mu) \,. \tag{27}$$

$\mathcal{L}_S$ contains only the strongly interacting particles together with their interactions with the weak and electromagnetic gauge fields, W_μ and A_μ. Apart from a quadratic, respectively, linear dependence on the field K_Z, the second and third terms contain only hadronic fields. As argued in Sec. II, λ is of order G_F, whereas b must be of order $G_F^{1/2}$. However, due to the large vacuum expectation value of the field σ_Z, which is given by $F_Z = 2g_W^{-1}M_W$, these terms still give rise to contributions of zeroth order in the weak and electromagnetic interactions. These contributions taken together with the first term $\mathcal{L}_S$ are denoted by $\mathcal{L}_h$ (π_s, W_μ, A_μ). The last term in Eq. (27), $\mathcal{L}_{\text{WEM}}$, contains terms that only depend on K_Z, A_μ, and W_μ.

As far as the hadron fields are concerned, we only made the dependence on the field π_s explicit. This field is defined as the physical pion field in the absence of weak and electromagnetic interactions. In the presence of these interactions, the physical pion field can be written as

$$\pi_p = \pi_s - \frac{g_W}{M_W} a\psi_Z + O(g_W^2) \,, \tag{28}$$

where the parameter a can in principle be determined in perturbation theory. In lowest order we have $a = -\frac{1}{4}F[2(1-\epsilon)]^{1/2}$, where F and ϵ were defined in Sec. III.

Let us first consider the irreducible diagrams with one external ψ_Z and one external physical pion line in first order in g_W. By irreducible, we mean that they cannot be divided into two parts by cutting one pion line. From the substitution (28) it is clear that all contributions purely from $\mathcal{L}_h$ are given by $-g_W M_W^{-1} a D_\pi^{-1}(s)$, where $D_\pi(s)$ is the pion propagator. The only remaining contributions contain $\mathcal{L}_b$, as b is of the order $g_W M_W^{-1}$. Hence, we can write the total contribution as

$$-\frac{g_W}{M_W} aD_\pi^{-1}(s) + ibd(s) \,, \tag{29}$$

where the last term comes from $\mathcal{L}_b$. On the mass shell, this quantity is related to the matrix element of the weak current between a pion and the vacuum through the first-order divergence equations (24). This leads to the result

$$b\, d(-m_\pi^2) = \frac{1}{2} \frac{g_W}{M_W} m_\pi^2 f_\pi \,, \tag{30}$$

where the current matrix element was defined as $iq_\mu f_\pi$, with q_μ the pion momentum. Using this relation and the propagators given in the Appendix, one can show that f_π corresponds to the conventional pion decay constant.

Consider now the diagrams where an external ψ_Z line is connected to a blob with an arbitrary number of external hadrons. In first order of the weak and electromagnetic interactions, the blob contains only strong-interaction effects. In the case that ψ_Z proceeds through a pion which interacts with the blob, the corresponding contribution is given by

$$i\left[-\frac{g_W}{M_W} aD_\pi^{-1}(s) + ibd(s)\right] D_\pi(s) J_\pi(s) \,,$$

where $J_\pi(s)$ is the hadronic matrix element of the pion source. When ψ_Z is directly coupled to the blob, thus without exhibiting a pole at $s = -m_\pi^2$, by using the previous arguments the result can be written as

$$i\frac{g_W}{M_W} aJ_\pi(s) + ibj(s) \,.$$

The first term represents the contributions purely from $\mathcal{L}_h$, whereas the second one is the remaining contribution from $\mathcal{L}_b$. Hence we find the result that diagrams with one external ψ_Z line and an arbitrary number of external hadrons, are given by

$$-\frac{1}{2}\frac{g_W}{M_W} m_\pi^2 f_\pi D_\pi(s) J_\pi(s) + b\{[d(-m_\pi^2) - d(s)] D_\pi(s) J_\pi(s) + ij(s)\},$$

where we have used Eq. (30). The first term of this result represents the usual PCAC term. The second term has no pole at $s=-m_\pi^2$ and is of order b.

The analysis for weakly irreducible diagrams with two lines, corresponding to ψ_Z^a and ψ_Z^b, goes completely analogously. They can be expressed as

$$\frac{1}{4}\left(\frac{g_W}{M_W}\right)^2 m_\pi^4 f_\pi^2 D_\pi^a(s_1) D_\pi^b(s_2) J_{\pi\pi}^{ab}(s_1, s_2) + \lambda\delta_{ab} J_\lambda(t) + O(b),$$

where we have used the definitions $s_1 = q_1^2$, $s_2 = q_2^2$, and $t=(q_1+q_2)^2$, with q_1 and q_2, the incoming momenta of ψ_Z^a and ψ_Z^b. Again, the first term reflects the PCAC result, and the terms of order b are less singular than the first one, having at least one propagator less. However, there is an additional term, which comes from the part of the Lagrangian (27) proportional to λ. This term has a simple structure: It is symmetric in a and b and depends only on t. Moreover, in the divergence equations (26) this term does not contribute in fact. This is because $T^{ab}(p, q)$, to which it contributes, is always accompanied by $-\frac{1}{2}i(g_W/M_W) \times\delta_{ab} J_\sigma(p+q)$. The contribution to J_σ proportional to λ cancels exactly the term $\lambda\delta_{ab}J_\lambda$ in $T^{ab}(p, q)$. Hence, in the divergence equations these terms of order λ can simply be ignored. This implies that $J_\sigma(k)$ is effectively of order b, since its remaining contributions, in this order, can only come from $\mathcal{L}_b$.

Hence, we have found that the hadronic, weakly irreducible amplitudes with one or two ψ_Z fields, are related to the corresponding matrix elements of the pion field as prescribed by PCAC, up to orders of the parameter b.[27] $J_\sigma(k)$, which in the current-algebra approach corresponds to the σ term, is effectively of first order in b. This, together with the divergence equations of the previous section, is sufficient to derive many of the important results that were found by current-algebra methods. To be more specific: What we have shown is that certain gauge field theories confirm results as the Goldberger-Treiman relation, the Adler-Weisberger relation, and the Adler consistency relation[11] in zeroth order $(g_W^{-1}M_W b)$, the parameter which causes the nonvanishing pion mass. We stress that these results are not confined to the model of Sec. II, since we made almost no reference to the specific structure of that model. It turns out, that in a large class of models the gauge invariance of the weak and electromagnetic interactions is sufficient to ensure the validity of these results in the appropriate (chiral) limit, in all orders of the strong interactions.

Finally, let us discuss the case where $b=0$. Because $\mathcal{L}_b$, as it was defined in Eq. (6d), was the only term of the total Lagrangian that is linear in the various spinless fields, b will remain zero in every order of perturbation theory. This implies that the pion will be a pseudo-Goldstone boson, which picks up its mass from closed-loop contributions of the weak and electromagnetic interactions. This possibility was recently put forward by Weinberg.[9] Numerically, this is not included *a priori*, since the mass ratio m_π^2/m_A^2, where m_A is the mass of the A_1 axial-vector meson, is close to the fine-structure constant. It is certainly an appealing possibility, which allows the calculation of the pion mass, and moreover tends to explain the good experimental confirmation of the previously mentioned current-algebra results.

APPENDIX

In this appendix we will give the various propagators that were used in Secs. V and VI. The pertinent part of the Lagrangian, in lowest order of weak and electromagnetic interactions, is given by

$$\mathcal{L} = -\tfrac{1}{4}(\partial_\mu W_\nu^a - \partial_\nu W_\mu^a)^2 - \tfrac{1}{2}\rho_W^2(\partial_\mu W_\mu^a)^2 - \tfrac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)^2 - \tfrac{1}{2}\rho_A^2(\partial_\mu A_\mu)^2$$
$$-\frac{1}{2}\frac{e}{g_W}(\partial_\mu A_\nu - \partial_\nu A_\mu)(\partial_\mu W_\nu^3 - \partial_\nu W_\mu^3) - \tfrac{1}{2}(\partial_\mu \psi_Z^a)^2 - M_W \psi^a \partial_\mu W_\mu^a - \tfrac{1}{2}M_W^2(W_\mu^a)^2 - \rho_A \partial_\mu \phi_A^* \partial_\mu \phi_A - \rho_W \partial_\mu \phi_W^{*a} \partial_\mu \phi_W^a.$$

An explicit calculation of the propagators gives the following results:

$$W^{1,2}:\ \delta_{ab}(q^2+M_W^2-i\epsilon)^{-1}\{\delta_{\mu\nu} + q_\mu q_\nu \rho_W^{-2}(q^2-i\epsilon)^{-2}[M_W^2 + (1-\rho_W^2)q^2]\},$$

$$W^3:\ (1-\kappa^2)^{-1}(q^2+M_0^2-i\epsilon)^{-1}\{\delta_{\mu\nu} + q_\mu q_\nu \rho_W^{-2}(q^2-i\epsilon)^{-2}[M_W^2 + (1-\rho_W^2-\kappa^2)q^2]\},$$

where $\kappa = e g_W^{-1}$ and $M_0^2 = (1-\kappa^2)^{-1}M_W^2$,

$$A:\ (1-\kappa^2)^{-1}(q^2-i\epsilon)^{-1}(q^2+M_0^2-i\epsilon)^{-1}(q^2+M_W^2)[\delta_{\mu\nu} - q_\mu q_\nu(q^2-i\epsilon)^{-1}] + \rho_A^{-2} q_\mu q_\nu (q^2-i\epsilon)^{-2},$$

W^3-A transition : $-\kappa(1-\kappa^2)^{-1}(q^2-i\epsilon)^{-1}(q^2+M_0^2-i\epsilon)^{-1}(q^2\delta_{\mu\nu}-q_\mu q_\nu)$,

ψ : $\delta_{ab}(q^2-i\epsilon)^{-2}(q^2+M_W^2\rho_W^{-2})$,

W-ψ_Z transition : $-iM_W\rho_W^{-2}(q^2-i\epsilon)^{-2}q_\mu$ (q_μ is the outgoing ψ_Z momentum) ,

ϕ_A : $(\rho_A q^2-i\epsilon)^{-1}$,

ϕ_W : $\delta_{ab}(\rho_W q^2-i\epsilon)^{-1}$.

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[15]It is possible that higher-order effects will induce terms that are not of the appropriate order in G_F. In that case the counterterms must be adjusted such that the net effect remains of that order of G_F. This implies that the smallness of these terms has no natural origin in these models. This problem was also mentioned in Ref. 5. However, we wish to point out that this phenomenon is pertinent only in models of this type. For instance, this problem was also found in Ref. 16. See also S. Weinberg, Phys. Rev. D 8, 605 (1973); M. Veltman, invited paper presented at the International Symposium on Electron and Photon Interactions at High Energies, Bonn, 1973 (unpublished). We thank Professor Veltman for an important discussion on these points.

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[21]This procedure, as well as the derivation of the generalized Ward-Takahashi identities, was generally discussed by 't Hooft and Veltman, and Lee and Zinn-Justin. Further references can be found in their papers (Refs. 19 and 22). In this paper we will use the diagrammatic formulation of 't Hooft and Veltman.

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[25]We wish to point out that the definition of the currents is somewhat ambiguous, as we could as well separate the terms that proceed through a field other than ψ_Z^a. An example of such a field is ψ_W^a, as defined in (10), but in principle it can be any field differing from ψ_Z^a by terms of order $G_F^{1/2}$. Our choice of ψ_Z with its corresponding currents has the advantage that the second-order divergence equations have a simple structure.

[26]It should be mentioned that such a result was put forward by Veltman (Ref. 10) and Bell, who imposed generalized gauge invariance on certain S-matrix elements. See J. S. Bell, Nuovo Cimento 50, 129 (1967).

[27]Notice that the PCAC hypothesis is not justified for any number of ψ_Z lines, or for diagrams with additional weak and electromagnetic gauge fields.