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Quark statistics and octonions*

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It is suggested that quark fields could be regarded as transverse octonionic fields. The resulting scheme unifies the existing three-triplet quark models within an "exceptional" paraquark scheme. We give a new reason for the observability of singlet states of the exact SU(3) group of three-triplet quark models and the nonobservability of quarks and diquarks.

I. INTRODUCTION

The unobservability of quark states has been the subject of renewed speculations[1] since it was noticed that quarks would act like partons in a gauge field theory of strong interactions in which the gauge vector bosons are associated with an exact non-Abelian group.[2] The candidates for such a group would be the "color" SU(3) group of Gell-Mann,[3] the SU(3)″ group of the Han-Nambu model[4,5] or the intrinsic invariance group SU(3) [or SO(3)] of the paraquark model.[5-7] These groups not only provide the para-Fermi property of order 3 to quarks,[3-5] a property required to account for the experimentally observed SU(6) multiplets,[6] but they also explain the $\pi^0 \to 2\gamma$ decay in a quark-parton model.[5,8] However, the present experimental data on the total e^+e^- annihilation ratio $R\ [=\sigma(e^+e^- \to \text{hadrons})/\sigma(e^+e^- \to \mu^+\mu^-)]$ seem to favor the Han-Nambu model. The success of the three-triplet models makes the unobservability problem even more acute, since one has to account not only for the unobservability of quarks and diquarks but also for that of their associated massless color gauge bosons. Various dynamical reasons have been invoked[9] to make the fundamental constituents that appear in the gauge field theory unobservable, but so far such considerations are at a preliminary stage. In this note we would like to suggest a supplementary solution to the quark puzzle in the form of a mathematical model that realizes the "fictitious Hilbert space" in which, according to Gell-Mann, quarks and color bosons should operate.[3] The scheme we shall present unifies the existing three-triplet models within a general formalism.

Our proposal is to describe quarks (and their associated color gauge bosons) in an octonionic Hilbert space.[10,11] States in such a space will not all be observable because the propositional calculus of observable states as developed by Birkhoff and von Neumann[12] can *only* have realizations as projective geometries corresponding to Hilbert spaces over associative composition algebras,[13] while octonions are nonassociative. This can be seen simply by considering the decomposition of a unit matrix I into products of kets with octonion elements with bras obtained from kets by octonion conjugation. If $I=\sum|n\rangle\langle n|$ is inserted in the octonion amplitude $\langle\alpha|\beta\rangle=\langle\alpha|I|\beta\rangle$, where $\langle\alpha|$ is an octonionic bra and $|\beta\rangle$ is an octonionic ket, then the equality of $\langle\alpha|[(\sum|n\rangle\langle n|)|\beta\rangle]$ with $\sum(\langle\alpha|n\rangle)(\langle n|\beta\rangle)$, which requires associativity, will not be true in general in an octonionic Hilbert space. Another theory of observables due to Jordan, von Neumann, and Wigner[14] is based on their algebraic properties. In that case octonion-valued observables become admissible only in the case of three degrees of freedom.[15] Octonion-valued fields with an infinite number of degrees of freedom can only operate in a nonobservable Hilbert space. A field-theoretical dynamics, however, could well have a simpler formulation in such a generalized (or fictitious) Hilbert space. The octonion field equations would then imply definite dynamical relations in the observable subspace (representable as an ordinary Hilbert space) in which the postulates of quantum mechanics become valid. In this octonionic approach to field theory, the exactly conserved group which provides both the parafermion and the parton properties of quarks and the invariance of observable states under this group follow naturally from the theory.

In a previous publication[10] it was shown how an octonionic Hilbert space with complex scalar products incorporates an automorphism group SU(3), which leaves the complex subspace and the scalar product invariant. It would be tempting to identify this group with the physical unitary-spin group $SU_P(3)$ as suggested in Ref. 10 were it not for the fact that such a group cannot be broken without altering the underlying algebraic structure and the scalar product defined over it. Therefore we identify the exact SU(3) of the octonionic Hilbert space with the exactly conserved SU(3) group of three-triplet quark models. We shall call this exactly conserved SU(3) group the C-spin group and denote it as $SU_C(3)$, where C alludes to color, charm, and Cayley (since in our formalism it arises as the automorphisms of Cayley algebra). The exactness of $SU(3)_C$ in the color-quark scheme of Gell-Mann makes it equivalent to a theory of para-Fermi quarks.[3,5,7] However, the Han-Nambu model is not equivalent to the usual para-Fermi theory of quarks since the Green component fields of a paraquark field operator are assigned different quantum numbers.[5] Below we shall show that it is possible to incorporate the Han-Nambu model as well as the other three-triplet models within an "exceptional" realization of para-Fermi field theory.

In the octonion approach, the split octonion algebra provides us with a new version of the Klein factors in the Green decomposition of a para-Fermi field of order 3.[16] The three Klein factors are replaced by the three "transverse" split octonion units u_i discussed previously.[10,11] They are defined by fixing a "longitudinal" imaginary unit e_7 for the purpose of representing translations in an octonionic Hilbert space. Thus, a transverse split octonionic field can be regarded as a parafermion of order 3. Because of the nonassociativity of the underlying composition algebra, the states created by such a field will not be observable. This follows not only from the postulates satisfied by observables[12-15] but also from the absence of a satisfactory method for defining tensor product states among nonassociative state vectors to represent composite systems in such a way that the observables associated with different constituents commute with each other. This difficulty even arises for which associativity holds.[17] On the other hand, if quarks are described as parafermions in the normal complex Hilbert space with the usual Klein operators, they will obey the cluster property[7] and be observable, barring very special dynamical conditions.[1,18]

Let us now see how an observable subspace arises. Within the Fock space spanned by vectors that are obtained by repeated applications of the octonionic parafield, there will be states which are longitudinal (linear combinations of 1 and e_7). Those form a longitudinal subspace H_L of the octonionic Hilbert space H_ω. Now, since the ground field for the longitudinal states is associative, we expect H_L to be spanned by observable states. It turns out that states in H_L are singlets with respect to $SU_C(3)$ if the para-Fermi fields are taken as quark fields. Nonsinglet multiplets of $SU_C(3)$ occur in $H_T = H_\omega - H_L$ and are described by state vectors that have octonionic transverse components, hence they are unobservable by our criterion. The Green decomposition of a quark field can now be understood as a decomposition in the 3-dimensional C-spin space with the octonionic split units u_1, u_2, u_3 referring to three C-spin directions (or to the color directions in the color space of Gell-Mann). With the substitution of u_i for the Klein operators the C-spin space becomes identical with transverse octonion space which admits $SU_C(3)$ as a group of automorphism.[10,11]

II. OCTONIONIC QUARK FIELDS

From the octonion units $e_1, \ldots, e_7$ we form[10,11] the split octonionic units

$$\begin{aligned} u_0 &= \tfrac{1}{2}(1+ie_7), \\ u_n &= \tfrac{1}{2}(e_n + ie_{n+3}) \quad (n=1,2,3), \end{aligned} \tag{1}$$

which together with their complex conjugates (obtained by changing the sign of i) satisfy

$$u_0{}^2 = u_0, \quad u_0 u_0^* = 0, \tag{2a}$$

$$u_0 u_n = u_n u_0^* = u_n, \quad u_n u_0 = u_0^* u_n = 0, \tag{2b}$$

$$u_l u_m = \epsilon_{lmn} u_n^*, \quad u_m u_n^* = -\delta_{mn} u_0, \tag{2c}$$

and the complex-conjugate equations. Thus, the transverse elements of the octonion algebra can be regarded as three Fermi annihilation operators u_1, u_2, u_3 and creation operators u_1^*, u_2^*, u_3^* which cannot be represented by matrices as they are not associative. The vacuum state for this finite-dimensional system is the longitudinal idempotent octonion u_0. A general real octonion W can be decomposed into a longitudinal and transverse part according to

$$W = W_L + W_T, \tag{3a}$$

with

$$\begin{aligned} W_L &= w_0 u_0 + w_0^* u_0^*, \\ W_T &= \sum_{n=1}^{3} (w_n u_n + w_n^* u_n^*) \end{aligned} \tag{3b}$$

where w_α $(\alpha = 0, 1, 2, 3)$ are complex numbers. We have

$$u_0 W = w_\alpha u_\alpha = w, \quad u_0^* W = w_\alpha^* u_\alpha^* = w^*. \tag{4}$$

Note that with respect to $SU_C(3)$ which act on u_n, longitudinal octonions are C-spin singlets, while u_n and u_n^* belong respectively to the (3) and $(\bar{3})$ representations. We shall introduce field operators that act on the vacuum

$$\Omega = u_0 |0\rangle \tag{5}$$

represented by a longitudinal octonion, with $|0\rangle$ the ordinary vacuum acted upon by complex fields and u_0 the octonionic vacuum on which u^i operate. Now consider three Fermi fields q^1, q^2, q^3 associated with three C-spin quarks. The nine quarks will be represented by the complex spinors

$q_i^n(x)$, where the upper and lower indices refer to C-spin and quark indices, respectively ($q_1 = \mathcal{P}$, $q_2 = \mathfrak{N}$, $q_3 = \lambda$). We have, at equal times,

$$\{q_i^{\dagger(m)}(\vec{x}), q_j^{(n)}(\vec{y})\} = \delta^{mn}\delta_{ij}\delta(\vec{x}-\vec{y}), \quad \{q_i^{(m)}(\vec{x}), q_j^{(n)}(\vec{y})\} = 0 . \tag{6}$$

Construct the transverse octonionic fields

$$\Psi_i(x) = q_i^n(x)u_n + [q_i^n(x)]^C u_n^*, \quad [q_i^n(x)]^C = Cq_i^n(x)C^{-1} . \tag{7a}$$

Here C is a unitary operator that operates on the fields only and superscript C denotes charge conjugation which reduces to Hermitian conjugation in the Majorana representation of γ matrices. Ψ_i is then a real octonion with coefficients that are Hermitian fermion operators, so that

$$C\Omega = u_0 C|0\rangle = \Omega , \quad C\Psi_i C^{-1} = -e_1[e_2(e_3\Psi_i)] . \tag{7b}$$

Introduce the operators

$$\psi_i = u_0\Psi_i = q_i^n u_n , \quad \psi_i^C = u_0^*\Psi_i = q_i^{nC}u_n^* = -Ce_1[e_2(e_3\psi_i)]C^{-1} . \tag{8}$$

The quark field operators ψ_i are assumed to satisfy Dirac equations of the form

$$\gamma_\mu\partial_\mu\psi_i = m_i\psi_i - g\gamma_\mu(M_\mu^n\psi_i)u_n^* = m_i\psi_i - g\gamma_\mu B_\mu^{mn}(u_m\psi_i)u_n^* , \tag{9}$$

where M_μ^n are transverse octonion gauge vector-meson operators[19] related to the C-spin octet gauge mesons B_μ^{mn} by

$$M_\mu^n = B_\mu^{mn}u_m, \quad M_\mu^n u_n^* = 0 \quad (\text{or } B_\mu^{nn} = 0) . \tag{10}$$

The massless color mesons also obey a Yang-Mills–type equation. Note that we take B_μ^{mn} as boson operators that are singlets with respect to $SU_P(3)$. Then M_μ^n become parabosons of order 3. The model has the structure of a renormalizable gauge field theory and the relations (6) remain valid. The fields q_i^n are then fermions, while the fields ψ_i become parafermions of order 3 because Eq. (8) can be interpreted as a Green decomposition,[18]

$$\psi_i = \phi_i^1 + \phi_i^2 + \phi_i^3 , \quad \phi_i^n = q_i^n u_n \quad (n = 1, 2, 3) \tag{11}$$

where, because of Eqs. (2c) and (6), the fields ϕ_i^n commute among themselves at equal times. The same considerations apply to the three components of a given M_μ^n, namely $B_\mu^{1n}u_1$, $B_\mu^{2n}u_2$, $B_\mu^{3n}u_3$, which anticommute. Since q_i^n and B_μ^{mn} are normal fermion and boson operators, respectively, the octonion factors u_i play the role of Klein operators which convert fermions and bosons with normal commutation relations into anomalous ones.[5,16]

Now the Green decomposition of a field ψ_i makes sense only if the Green component fields ϕ_i^n ($n = 1, 2, 3$) carry the same (observable) quantum numbers such as spin, charge, hypercharge, etc. Since we want the spinor fields ψ_i ($i = 1, 2, 3$) to transform like a unitary-spin triplet, we shall assign them the respective quantum numbers of $\mathcal{P}$, $\mathfrak{N}$, and λ quarks of Gell-Mann. Therefore the Green component fields ϕ_i^n must have the same quantum numbers as ψ_i. However, the Green component fields ϕ_i^n have the form $\phi_i^n = q_i^n u_n$ ($n = 1, 2, 3$) and hence the generator of a symmetry transformation acting on ϕ_i^n will have two parts, one part acting on the complex component fields q_i^n and the other acting on the units u_n. Hence the generators of hypercharge and the third component of isospin acting on ψ_i (or ϕ_i^n) will decompose as

$$I_3^u = I_3 - I_3'' , \quad Y^u = Y - Y'' , \quad Q^u = I_3' + \tfrac{1}{2}Y' , \tag{12}$$

where I_3 and I_3'' act on the fields $q_i^n(x)$ and the units u_n, respectively. Note that since we want the automorphism group $SU_C(3)$ of our Hilbert space to be exact, the quantum-number assignment to the units u_n must be compatible with exact $SU_C(3)$. Since we want the fields ψ_1, ψ_2, and ψ_3 to have the same quantum numbers as $\mathcal{P}$, $\mathfrak{N}$, and λ quarks, $I_3' = \text{Diag}(\frac{1}{2}, -\frac{1}{2}, 0)$ and $Y' = \text{Diag}(\frac{1}{3}, \frac{1}{3}, -\frac{2}{3})$. However, for this choice of I_3' and Y' there are several choices for I_3, Y and I_3'', Y'' consistent with $SU_C(3)$:

(1) Choose $I_3' = I_3$, $Y_3' = Y_3$, i.e., $I_3'' = Y'' = 0$. In this choice the fields ϕ_i^n and q_i^n ($n = 1, 2, 3$) are assigned the same quantum numbers as the field ψ_i. Then one can interpret the indices n in $q_i^n u_n = \phi^n$ as the color indices of Gell-Mann, i.e., $SU_C(3)$ becomes equivalent to the color SU(3).

(2) Choose $I_3'' = I_3^C$ [$I_3^C = (\frac{1}{2}, -\frac{1}{2}, 0)$] and $Y_3'' = Y^C$ [$Y^C = (\frac{1}{3}, \frac{1}{3}, -\frac{2}{3})$]; then the fields q_i^n are assigned the same quantum numbers as Han-Nambu quarks, i.e.,

$$I_3 = I_3^u + I_3^C , \quad Y = Y^u + Y^C , \tag{13a}$$

$$Q = I_3 + \tfrac{1}{2}Y = Q' + Q'' = Q^u + Q^C , \tag{13b}$$

where operators with superscripts u and C act on the unitary-spin indices and C-spin indices, respectively. Therefore with this choice $SU_C(3)$ becomes equivalent to the SU(3)$''$ group of Han and Nambu.[4,5]

(3) Choose $I_3'' = I_3^C + \frac{3}{2}Y^C$, $Y'' = 0$. Then $Q = Q^u + L_3^C$ [$L_3^C = I_3^C + \frac{3}{2}Y^C = (1, 0, -1)$], and the charge assignment scheme becomes equivalent to the 3-triplet model with $SU(3)'\otimes SO(3)$ symmetry,[5,20] where the SO(3) group corresponds to an SO(3) subgroup of $SU_C(3)$ with $L_3 = L_3^C = I_3^C + \frac{3}{2}Y^C$.

Thus we see that there is a large degree of freedom in choosing the generators I_3'' and Y'' such that the observed unitary-spin multiplets have the right quantum numbers. However, if we impose the condition that the complex component fields $q_i^n(x)$ all have integer charges $(0, \pm 1)$ then the Han-Nambu scheme is uniquely selected.

III. MULTIQUARK STATES, OBSERVABLE SUBSPACE

Having discussed the possible quantum-number assignment schemes, let us now discuss the Fock-space properties of our transverse octonionic field operators ψ_i. The states $\psi_i \Omega$ and $\psi_i^C\Omega$ lie in the space H_T, as do the diquark states $(\psi_i\,\psi_i)\Omega$ and $(\psi_i^C\psi_i^C)\Omega$. Note that the operators $[\psi_i(x)]^2$ no longer vanish as they would for a normal fermion operator. On the other hand, the boson operators

$$V_{ij}(x) = \psi_i(x)\psi_j^C(x) \tag{14}$$

create quark-antiquark states that are longitudinal octonions, hence $SU_C(3)$ singlets, since we have in the Majorana representation

$$\begin{aligned} V_{ij}(x) &= \sum_n \sum_m q_i^n(x) q_j^{m*}(x) u_n u_m^* \\ &= -\sum_n q_i^n(x) q_j^{n*}(x) u_0 \end{aligned} \tag{15}$$

and

$$V_{ij}(x)\Omega = -\phi_{ij}(x)\Omega\,, \tag{16}$$

where

$$\phi_{ij}(x) = \sum_n q_i^n(x) q_j^{n*}(x) \tag{17}$$

are ordinary complex boson operators belonging to the singlet representation of $SU_C(3)$ and the (octet + singlet) representations of the physical $SU_u(3)$. If the suppressed quark spin indices are also added, the states (16) can be resolved into C-even, spin-zero states and C-odd, spin-one states in H_L.

We can also form fermion states in H_L by applying the paraquark field ψ (or ψ^*) three times on the vacuum at the same point. We find

$$[\psi_i(x)\psi_j(x)]\,\psi_k(x) = -\psi_{ijk}(x) u_0^* \tag{18}$$

and

$$\psi_i(x)[\psi_j(x)\psi_k(x)] = -\psi_{ijk}(x) u_0\,, \tag{19}$$

where the complex fields ψ_{ijk} are given by

$$\psi_{ijk}(x) = q_i^n(x) q_j^m(x) q_k^l(x) \epsilon_{nml}\,, \tag{20}$$

so that we obtain the nonzero longitudinal states

$$-\{\psi_i(x)[\psi_j(x)\psi_k(x)]\}\Omega = \psi_{ijk}(x)\Omega \tag{21a}$$

and

$$-\{[\psi_i^*(x)\psi_j^*(x)]\,\psi_k^*(x)\}\Omega = \psi_{ijk}^*(x)\Omega\,. \tag{21b}$$

The states (21) are $SU_C(3)$ singlets and contain octets and decuplets with respect to the physical $SU_u(3)$, since the q_i^n are normal Fermi operators corresponding to color quarks or Han-Nambu quarks depending on the charge assignment scheme and ψ_{ijk} is symmetric with respect to the combined quark and spin indices. Hence these states which lie in the observable H_L belong to the correct 56-dimensional representation of SU(6). Now any combination of ϕ_{ij} of Eq. (17) and ψ_{ijk}, ψ_{ijk}^* of Eq. (20) will create states in H_L, and hence will create other observable states, by acting on the vacuum Ω of Eq. (5). All such states will be $SU_C(3)$ singlets, while q, qq, and $qqqq$ types of states will all be in the unobservable H_T. We should also note that while in the paraquark scheme[5-7] it is possible to create, by repeated applications of the paraquark field operators, bosonic and fermionic states with noninteger charges, the observable fermionic or bosonic states in our scheme all carry integral charges for the various charge assignment schemes discussed above.

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$$
\begin{aligned}
B^{mn}_\mu R^*_{u_n} L_{u_m} w_L &= B^{mn}_\mu R^*_{u_n} L_{u_m}(w_0 u_0 + w^*_0 u^*_0)\\
&= B^{mn}_\mu w^*_0 (u_m u^*_0) u^*_n\\
&= -B^{mn}_\mu w^*_0 \delta_{mn} u_0\\
&= 0 .
\end{aligned}
$$

A systematic study of the octonionic field operators acting in an octonionic Hilbert space will be given elsewhere. In the rest of this paper we shall assume that all the field operators act from the left.

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Parton models and asymptotic freedom

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We generalize the scale-invariant parton model to describe fixed-point and asymptotically free gauge and nongauge theories. Q^2-dependent scaling laws for the moments of νW_2 are derived. In fixed-point theories the moments scale as powers of Q^2, while in asymptotically free theories they scale as powers of $\ln Q^2$. The behaviors of the elastic form factors, the ratio σ_S/σ_T, and the mean-squared transverse momentum of hadron secondaries are discussed in the various theories. The experimental study of these quantities should distinguish clearly between the conventional parton-model and asymptotically free theories of strong interactions.

I. INTRODUCTION

It has become clear that the light-cone behavior of renormalizable field theories must differ from the light-cone behavior of free fields. This is due to the infinite renormalizations necessary to define physical quantities in field theories with dimensionless coupling constants. Roughly speaking, the resolving power of an external probe can never be sufficient to uncover all the structure in the interacting fields. To study the short-distance character of interacting fields one defines a coupling