Ramifications of flux quantization

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The null results in the search for magnetic monopoles and the experimentation exhibiting dipolar flux quantization as obtained during the last two decades mildly invited a withdrawal from the Dirac-Schwinger symmetry hypothesis to the earlier position of an essentially asymmetric set of Maxwell equations. In this paper we attempt to account for the null result by a symmetry alternative that accommodates the persistence of this Maxwellian asymmetry. It is shown that a topological symmetry, more hierarchal in nature, can comply with the absence of magnetic monopoles. This alternative then places the law of flux conservation on the same fundamental footing as the law of conservation of electric charge. The ensuing law statements are now global in nature and correspondingly stronger in content than the traditional local statements. The physical implications of these global conservation statements are examined in relation to the existing observational evidence of dipolar flux quantization.

I. INTRODUCTION

The search for magnetic monopoles has received much attention in the past decade. Extensive experimental investigations have been made to put the existence of monopoles in evidence. Although these efforts had interesting instrumentational spin-offs, they were not rewarded with the discovery of what should be regarded as a new elementary particle with magnetic-monopole properties. Along with the theoretical publications elaborating the arguments supporting the existence of the magnetic monopole, there has been no lack of publications delineating conflict situations between existing theory and the magnetic-monopole hypothesis. Yet, the most eloquent contributions to this problem are undoubtedly the experimental studies. The efforts to generate magnetic monopoles in large accelerators have not yielded any positive results.[1] Likewise, cosmic-ray observations have failed to produce conclusive evidence that magnetic monopoles do exist.[2] Finally, the search for magnetic monopoles that could be trapped inside magnetic materials[3,4] only corroborates the mentioned earlier experiences. (The test samples that were used have ranged from ocean-bottom samples[3] to moon rocks.[4])

Summarizing the results of this extensive experimental search, it seems that nature conveys to us a message suggesting that no magnetic monopoles manifest themselves in that part of the universe that is available to the probing of our present detection gear.

So long as new disclosures to the contrary are not forthcoming, we may, for the purpose of this article, extrapolate the preliminary conclusions of these experimental investigations: Let us assume that magnetic monopoles do not constitute a realistic building block of nature. More specifically, let the nonexistence of magnetic monopoles be taken as a global truth that should be expressed by the appropriate mathematical criteria that are to be imposed on the relevant Maxwell fields.

The gist of this assumption is tacitly subscribed to by a large number of physicists, and at first sight there is hardly a dramatic element in this statement. In fact, does not this statement imply a return to earlier values that already failed us in the past by not yielding further insight? The main point of the present paper is to show that the answer to that question may well be "no." To substantiate the claim that the absence of magnetic monopoles was never fully exploited in the past, the monopole symmetry concept will be contrasted with some topology-oriented considerations supporting the older point of view.

In the sequel, different magnitudes of quantized magnetic-monopole strength and quantized flux units will be frequently mentioned and referred to. As general background information, we may recall that monopole strength g is measured in terms of total flux ϕ emanating through a closed surface that contains the monopole; it follows that g and ϕ are related according to $\phi=4\pi g$ (cgs) or $\phi=\mu_0 g$ (mks). Since the relations between hypothesized, predicted, and observed quantized data are replete with factors of 2, their mutual relations are listed in Table I for easy reference.

II. AN ALTERNATIVE TO THE DIRAC-SCHWINGER SYMMETRY ARGUMENT

Let the 2-form F represent the Maxwell field $\vec{E}, \vec{B}$ (electric field and magnetic induction), let the 2-form G represent the Maxwell field, $\vec{H}, \vec{D}$ (magnetic field and electric displacement), and

TABLE I. The table gives the magnitudes for the hypothesized Dirac and Schwinger magnetic monopoles, the predicted London flux unit, and the observed flux unit. The Dirac $\hbar$ equals the Planck h divided by 2π, c = the light velocity, e = electronic charge. For comparison the last column gives all values expressed in mks units of flux or equivalent flux, whereas the values in the other columns are expressed in cgs units.

	Flux	Flux in terms of equivalent pole strength	Pole strength	Pole strength in terms of equivalent flux	Equivalent flux
Dirac[5]	···	···	$\hbar c/2e$	hc/e	h/e
London[13]	hc/e	$\hbar c/2e$	···	···	h/e
Schwinger[6]	···	···	$\hbar c/e$	$2hc/e$	$2h/e$
Observation[14]	$hc/2e$	$\hbar c/4e$	···	···	$h/2e$

let the 3-form C be the four-current density. If the symbol d is taken to be the exterior derivative, the two sets of Maxwell equations can then be succinctly written as

$$dF=0\,,\qquad dG=C\,. \tag{1}$$

The Dirac[5]-Schwinger[6] argument addresses itself to the asymmetry of these equations. They tentatively complement these equations by assuming the existence of a magnetic four-current, say, C'. The equations then become

$$dF=C'\,,\qquad dG=C\,. \tag{2}$$

Since nature so far supports the equations (1) rather than the equations (2) and since we make this hypothesis the cornerstone of our present considerations, let us examine more closely the symmetry alternatives that conceivably could support the set (1).

Since $dd\equiv 0$, it follows from both (1) and (2) that $dC=0$: local conservation of electric charge. Conversely, the existence of $dC=0$ does not yet imply the existence of a field G such that $dG=C$. Yet, if all cyclic integrals of C vanish, one may infer according to de Rham's theorem[7] that indeed a field G exists such that $dG=C$.

The stronger statements[8,9] that all cyclic integrals of C vanish, $\oint C=0$, may be called the global conservation law for electric charge in contrast to the local conservation law, $dC=0$; C is now called exact.

Similarly, the local statement $dF=0$ in (1) may be taken as the local conservation law of flux. The fact that $dF=0$, everywhere at all time, does not yet imply the existence of a field A (say, vector potential) such that $dA=F$. However, if all cyclic integrals of F vanish ($\oint F=0$), one may infer, again according to de Rham's theorem, that indeed a field A exists such that $dA=F$; F is now called exact.

By making this mild but nontrivial distinction between local and global conservation, and by making the assumption that the conservation laws of flux and electric charge are indeed (strong) global conservation laws, one can uncover a symmetry underlying the equations (1) which is in sharp contrast with the local-symmetry assumption of the Dirac-Schwinger equations (2).

The new symmetry picture is one of an hierarchal nature as illustrated by the following diagram:

$$\begin{array}{ll} A=1\text{-form}, & G=2\text{-form},\\ F=2\text{-form(exact)}, & C=3\text{-form(exact)}. \end{array} \tag{3}$$

The exactness of F and C expresses the fundamental physical observations known as flux conservation (specifically global conservation as inferred from Faraday's induction law) and electric-charge conservation. Note that the field G shares with the field A that both are defined modulo an *exact* contribution, a remarkable feature that resolves considerable controversy associated with the interpretation of the fields $\vec{\mathrm{D}}$ and $\vec{\mathrm{H}}$. Hence, A and G exist by virtue of the special physical properties attributed to the fields F and C. (See Ref. 9.)

In addition to the presented distinctions between the 2-forms F and G, it should also be mentioned that the forms F and G have opposite parity.[10] It means that charge polarity can be related to enantiomorphism, either for electric or for magnetic charge, but not for both simultaneously. Time reversal presents other difficulties (see, e.g., Schiff, Ref. 11).

Finally, $dF=0$, everywhere and at all times, does imply $\oint F=0$ if the manifold over which F is defined is compact and simply connected. [Simple connectedness of space-time is meant to imply 1-connectedness (contractable circles), 2-connectedness (contractable spheres), and 3-connectedness (contractable 3-spheres).] An extrapolation of the given statements in a cosmological and microphysical sense thus implies far-reaching commitments concerning the topological nature of space-time and the fields it carries (see Secs. III, IV, and V). We, therefore, retain this nontrivial distinction between local (weak) and global (strong) conservation laws because mathematics permits us to be discerning for the useful purpose of classification. In fact, only the use of these distinctions enable us to make the otherwise hidden hierarchal symmetry (3) of the classical equations (1) clearly visible.

In the sequel, further topological features will be discussed comparing the physical relevance and merits of the motivation leading to (2) versus the motivation leading to (1) and (3). For the historical perspective it is important to note that the first suggestions of quantized magnetic monopoles were made before de Rham's theorem had emerged as a major milestone in modern mathematics.

III. GLOBAL ASPECTS OF QUANTIZATION

Since flux conservation in conjunction with (the observed phenomenon of) flux quantization is most important for many applications, a few words are in order concerning the nature of quantization.

The conditions of single-valuedness and square-integrability of wave functions are typically global conditions. The asymptotically equivalent rendering in terms of integrals over closed paths of integration has likewise very specifically global connotations. The procedure demands that the domains of integration are specified with adequate topological detail. In fact, wherever a field becomes, say, two-valued, one should branch the integration chain in such a way so as to make the process of integration well defined. A conspicuous example of this procedure is provided by the two-valued momentum field and the Bohr-Sommerfield conditions of quantization (see Ref. 9). The asymptotic rendering of quantum mechanics does not differ in this respect from the more encompassing renderings of Heisenberg and Schrödinger.

Let us now further specify the situation with respect to Sec. II in view of applications in the domain of elementary particles. We assume that fields exist over space-time; they are not part of space-time. This means that notions such as wormholes,[12] as a possible source of electric charge, are thus excluded from the present considerations. More specifically, in the microphysical domain, space-time will be assumed to be locally simply connected. Yet, fields, through the properties with which they are endowed, may well define multiply connected manifolds of integration embedded or immersed in the simply connected space-time.

Please note that this specification constitutes a choice over and above the stipulations made in Sec. II. Its rationale is based on the point of view that topology enters physics through the families of integration manifolds that are generated by physical fields believed to be physically relevant. Space-time is the arena in which these integration manifolds are embedded.

It was necessary to delineate this point of view because it is not the only choice one can make. The Misner-Wheeler wormhole[12] theory of electric charge is a specific example of a different choice; there space-time itself is endowed with topological structure relating to physics. The latter procedure affects the embedding possibilities of integration manifolds by virtue of the topological structure induced by a space-time so defined. Dirac's unidentified singularity line connecting magnetic monopoles of opposite polarity is another example of topologically structuring space-time itself for the purpose of physics (see Sec. IV).

IV. DELINEATION OF CONFLICT SITUATIONS THROUGH GLOBAL REQUIREMENTS

Most discussions of the magnetic-monopole hypothesis start from a working premise that traditional electromagnetic theory can live in reasonable coexistence with the new alien element of magnetic charge. As soon as the magnetic monopole is absent, the altered equations automatically reduce to the familiar and proven set of Maxwell equations. We buy this flexibility in treatment at the expense of a certain global validity of our conclusions. Let us further illustrate this point.

In physics, the vector potential is traditionally introduced as a local quantity; one infers that an A exists if $dF=0$. Globally, however, $dF=0$ is a necessary but not a sufficient condition to conclude on the existence of an A. The global condition for A to exist is the stronger requirement that *all* cyclic integrals of F vanish ($\oint F=0$). The two requirements become identical, for instance, if all integration domains are 1-, 2-, and 3- (i.e., simply) connected and compact.

For actual embeddings of physical structures in space-time one can neither guarantee compactness nor simple connectedness. The simple and elementary example of a multiple-loop solenoid vividly illustrates the practical need for multiply-connected integration domains. Conclusion: It seems we do well in retaining the distinction between the stronger ($\oint F=0$) and the weaker ($dF=0$) condition.

At first sight there is no dramatic distinction in actual practice. Although it is true that the Faraday-Maxwell law of nature is expressed by $\oint F=0$ rather than $dF=0$, the latter is an only slightly diluted version for convenience. Physicists have a predilection for thinking in terms of differential equations; the global problems usually return in the form of boundary conditions.

Now going a step further, it follows that whenever one involves A in a quantization procedure, one is committed to a globally defined A because the global notion of quantization so requires. Let us apply these notions to the phenomenon of flux quantization in superconducting rings. London (footnote, p. 152 of Ref. 13) predicted this effect by requiring that $\oint A$ equals a multiple of h/e. Many years later the effect was experimentally verified,[14] but the flux unit appeared to be $h/2e$ rather than h/e, a fact that Onsager[13] related to the pair-forming in superconductors.

In the vein of the present discussion, one should thus conclude that since the London-Onsager quantization condition requires a global A and since a global A exists only if $\oint F=0$, it follows that flux quantization is formally incompatible with the magnetic-monopole hypothesis.[15] Let us hang on to this as a preliminary conclusion and see how it compares to Dirac's procedure for the quantization of magnetic pole strength.

Dirac postulates the existence of magnetic monopoles, and since this hypothesis defies the statement $\oint F=0$, he permits a singularity line (not physically identified) to pierce through the surface enclosing a magnetic monopole. The "enclosing" surface is now no longer topologically equivalent to a (closed) sphere, but rather to an open sphere plus a point that is caused by the singularity line "hole" in space. The integral over the open sphere is taken as the magnetic pole strength and its magnitude is oppositely equal to the contribution of the point singularity. Wave-function uniqueness gives the quantization.

It thus follows that the Dirac procedure still honors the basic relation $\oint F=0$; therefore, a gobally defined A still exists, provided A is equipped to account for the assumed singularity of F (that can be done by filling up the singularity line with space and an intense flux). The loop integral of A taken over a small circle around the singularity line now equals the magnitude of the total flux emanating from the pole. An identification of the singularity line thus means that the $\oint A$ is physically the flux return through the singularity line. Note that according to Table I, the pole strength so quantized precisely corresponds to London's quantized flux, because the two procedures become identical after the Dirac singularity line is so identified. Of course, an important physical distinction remains: London's flux confinement can be physically substantiated (Meissner effect), whereas the flux confinement in Dirac's singularity line has to be postulated.

Hence, a Dirac magnetic-monopole pair may be interpreted as an anomolously shaped magnetic dipole. In this form, the hypothesis does not violate any of the traditional classical statements [Eqs. (1), (3)] as discussed in Sec. II. It follows that the symmetrized equations (2) of Sec. II are not even necessary to obtain this result. Some essentially equivalent remarks can be found in the literature (see, for instance, Goldhaber, Ref. 16, Sec. III C).

After the first magnetic-monopole experiments yielded a negative result, Schwinger[6] suggested a different quantization procedure leading to twice the Dirac pole strength. Schwinger's approach rests on the equations (2) of Sec. II. However, Peres[17] showed that the same result can be obtained with a double-singularity line. The surface enclosing a magnetic monopole now is pierced by two lines and there are two excepted points. In fact, by admitting more singularity lines, one can make magnetic monopoles arbitrarily strong; they then become gradually unobservable by virtue of their extremely high binding energy.

The essence of all this may be succinctly summarized as follows: The observed London flux quantization demands the existence of a globally defined A. The existence of a global A is formally incompatible with the existence of *isolated* magnetic monopoles. Through the artifact of singularity lines one may still introduce monopoles (with strings attached) that are compatible with a global A. A physical identification of the singularity line equates a Dirac monopole pair to an anomolously shaped London dipole. The Schwinger monopole, on the other hand, may depend on either the symmetrization (2) of Sec. II (thus questioning the fundamental significance of the London flux quantization), or in the sense of Peres[17] the enclosing surface needs to be pierced by two singularity lines so that the two singular points add up to twice the Dirac pole strength. One can thus, if one wants to, increase the Dirac pole strength by an arbitrary integral factor.

This conflict situation now precipitates a question of principle: To what extent may we proceed in physics using the somewhat arbitrary notion of physically unidentified singularity lines, and more seriously, can we permit such procedures to interfere with the presently rather firm body of knowledge relating to flux quantization and quantum interferometry?

This painful contrast is raised not merely for rhetoric. It involves a major point of current physical philosophy. *Unidentified* singularity lines relate to magnetic charge as wormholes[12] relate to electric charge; both notions make space-time itself multiply connected.

It is now instructive to compare corresponding policies in mathematics. Singularities are mathematical obscurities unless properly identified. The oldest examples are residues and Riemann surfaces in complex analysis. The natural generalization of these concepts for space-time fields are *periods* and Betti numbers of integration domains.

The residue relates to an integration domain of a given complex function; it is not an *a priori* topological feature of the complex plane. The Riemann surface, on the other hand, is an extension of the complex-plane concept to accommodate multivalued functions.

Physics is confronted with the additional complication that space-time has to serve a wider accommodation than the complex plane. It thus seems undesirable to prejudicially equip space-time with an *a priori* topological structure that may fit one physical feature while interfering with another. It is not unreasonable to suggest that the properties of space-time can only be known to us through what really exists in space-time. In the sequel, it will be attempted to honor this philosophy.

A viewpoint that is rather opposite to the philosophy that was just outlined has been very eloquently described in a (German) monograph by Wheeler.[18] In this booklet Wheeler examines the status of the alleged Einstein view to interpret much of basic physics in terms of geometry. Wheeler and his school have added topological considerations as a necessary ingredient to complement the older local-geometric investigations of the late twenties and the early thirties. This geometric approach when carried to the extreme is sometimes referred to as "geometric monism." The acceptance of singularity lines, which are really holes in space, is not normally categorized as a manifestation of geometric monism, yet it cannot be denied that there exists a close family tie. These methods do stand in sharp contrast to the more epistemologically oriented approach attempted in this paper.

However, reading Wheeler one need not fear of undue prejudice towards geometric monism. The two viewpoints are well delineated. In support of this opinion, let me cite Wheeler's own words where he characterizes the approach that seems closer to a strict epistemological procedure: "Space is to be regarded as the carrier, not as the structural substance of events. Fields in this less ambitious conception have their own degrees of freedom over and beyond those of geometry." (Translation from p. 64 of Ref. 18.)

Note that the adoption of this point of view shifts the topological emphasis from space itself to the fields that are assumed to exist in space. The latter feature constitutes a basic distinction of this presentation with respect to the Wheeler school.

V. ON THE NATURE OF FLUX CONSERVATION

The assumptions underlying the approach presented here resemble in many ways the well-established traditional point of view. There is only one, on the surface minor, distinction: The global aspects of the electromagnetic laws are taken far more seriously than is customary in traditional theory. These extensions, beyond the regular scope of the theory, demanded distinctions between local and global aspects of conservation and subsequent decisions as to whether or not the laws at hand should be regarded as local or global. Then it appeared that the substantiation of any such extension requires a specification of the topological features we normally attribute to space-time and the fields therein. The working premise adopted here was one of a locally simply connected space-time. The fields defined over this space-time generate, through their properties, topologically well-specified manifolds of integration. It was also noted that there exists a trend, as exemplified by the Dirac singularity lines and the Misner-Wheeler[12] wormholes, to endow space-time itself with an *a priori* multiple connectedness. The London procedure for flux quantization, however, may be categorized as one that subsumes a locally simply connected space-time. The Dirac approach becomes the London approach as soon as the singularity line ("hole") in space is filled up with space plus a flux return.

In the course of action adopted in this paper, we have biased our thinking towards the London-type procedure. As an upshot we find that the law of flux conservation, which always enjoyed a great *de facto* recognition, now suddenly emerges as a law of equal fundamental significance as the law of conservation of electric charge. Since this is a result of our perhaps subjective line of reasoning, let us examine more closely what conservation of flux means in the sense of the integral statement $\oint F=0$ for all cyclic space-time domains of integration.

First of all, it should be noted that $\oint F=0$ not only covers purely spatial domains of integration but it also covers space-time domains of integration, thus directly relating to the Faraday-Maxwell induction law.

For the purely spatial domains, Faraday envisioned for magnetic flux the ingenious model of tubes of force, which in the case of conservation are closed onto themselves. These tubes of force can be oriented in the sense of aligning their circulation directions. It follows that we can orient and assemble tubes of force by tying a "string" around them. Conservation of magnetic flux, in a more specific sense, now means that we cannot pull tubes of force out of the string, neither can we add new ones. The superconducting ring provides a physical realization of such a string by virtue of the Meissner effect.

Since flux enclosed by a superconducting ring is known to be quantized, one may further conclude for the given conditions that all tubes of force have equal magnitude of $|h/2e|$; similarly, all charges inside an enclosure are thought to have equal magnitude $|e|$.

It is seen that this somewhat extended Faraday picture covers rather nicely the case of static magnetic flux. The picture is also compatible for the case of a normally conducting solenoid with constant current, provided we take the complete current circuit plus source as the loop that confines tubes of force. Note that the current is now much more spatially dispersed than in the case of a superconductor; the feature of dramatically confining an integral number of tubes of force inside a single "string" is not now apparent.

Magnetic flux is just one, and perhaps the most frequently recognized, manifestation of flux. The cyclic integral $\oint F=0$ can also involve surface parts of a space-time nature that require an integration of the electric field component $\vec{E}$ of F. The Josephson[19] a.c. effect intriguingly suggests that the notion of a flux unit $h/2e$ retains a validity for the space-time domain, also. The expression

$$\int_0^{1/\nu} dt \int_1^2 \vec{E}\cdot d\vec{r} = h/2e\,,$$

with ν= frequency and $\int_1^2 \vec{E}\cdot d\vec{r}$= junction potential V, is an integral over an open space-time surface part, yielding the relation

$$\frac{2V}{\nu}=\frac{h}{e}\,,$$

which has provided very accurate h/e values.[20]

In other words, the Josephson junction may be seen as a device for slipping quantized tubes of force in and out of a superconducting ring so that the cyclic condition $\oint F=0$ is always met.

A normally conducting ring (unlike a superconducting ring without Josephson junction) permits a continuous change of tubes of force it encloses. The dynamics of the quantized tubes of force can now be seen as governed by the following conservation condition: The change in the number of quantized tubes of force enclosed by the ring equals the number of quantized tubes of force traveling through the ring confinement. This statement is an almost exact replica of conservation of charge, except for the nature of the objects being conserved. Since both statements relate to counting procedures, they may be expected to be topological in nature; it also means that they are metric-independent.[9]

In the more traditional mathematical language, the conservation of flux may be thus formulated: Let the vector density $\vec{n}$ denote the number of tubes of force per unit area, and let the motion of the tubes of force be given by the velocity vector field $\vec{u}$. The change in the number of tubes of force through the surface σ with boundary $\partial\sigma$ is

$$\frac{d}{dt}\int_\sigma \vec{n}\cdot d\vec{\sigma}\,.$$

The number of tubes of force traveling through the boundary $\partial\sigma$ is

$$\int_{\partial\sigma} \vec{n}\cdot\vec{u}\times d\vec{r} = \int_{\partial\sigma} \vec{n}\times\vec{u}\cdot d\vec{r}\,.$$

Conservation means that the sum of these expressions is zero. After multiplication with $h/2e$, the statement is simply Faraday's law of induction.

The presented arguments are meant to illustrate (not to prove) the universal connotation of flux conservation. In discussions with many physicists, I have found that one can think of situations which are not easily reduced to a universal statement $\oint F=0$. For instance, consider a collection of electrically charged plastic spheres. The spheres collide and exchange angular momentum. How is flux conserved?

The problem is one of identifying the electric field relating to changes in rotation resulting from angular momentum exchange. The changes in convection current are due to mechanical forces. Yet, in microscopic detail, electric forces, among other forces, do play a role in communicating an acceleration to the convected charges. Do all these forces add up so that the conditions of Faraday's law are met? This puzzle suggests a material-independent feature.

Let us leave this problem unresolved at this time; it is good to know that something can be done to improve our understanding of a law about which we thought we knew everything.

VI. CONCLUSION

The absence of magnetic monopoles generates intriguing physical perspectives, provided this absence is taken to be truly fundamental in nature.

This negation of a suspected property can be more fully appreciated as a constructive element in theory if in physics, as in geometry, the global viewpoint finds more general acceptance.

Since flux conservation emerges as a law of equal fundamental importance as charge conservation, one may now think of applications in the microphysical domain. The conservation of quantized flux in elementary-particle events conceivably could be related to the empirical conservation laws of lepton, muon, or baryon numbers. The process of β decay then requires an evaluation of the neutrino concept from a point of view of flux conservation. The existence of two neutrino types (muonic and electronic), with a same spin $\frac{1}{2}$, strongly suggests the need for an additional criterion to characterize their distinction. The separation of angular momentum and flux conservation in the microphysical domain will, however, require more detailed topological specifications of their associated fields.

It was judged wiser to refrain from these more speculative aspects so as to have flux conservation stand on its own as an independent fundamental law of nature.

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Since the subject matter treated in this paper is in many ways controversial, its content was discussed with a relatively large number of physicists representing different trends in thinking. Most of them gracefully complied by giving me the benefit of their views. The final result may be regarded as a common denominator compiled and colored by the opinions of the author. The following gives an account of roughly one year, of gathering background information.

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