Energy spectrum of H^- in a strong magnetic field

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The binding energy of the hydrogen negative ion is calculated by a variational method for magnetic fields in the range $0-3\times10^9$ gauss.

Strong magnetic fields $\approx10^6-10^7$ G are believed to exist in certain white dwarf stars[1,2] and it is generally accepted that even stronger fields exist in pulsars. This motivated the study of the effect of such fields on the energy levels of atomic hydrogen.[3] This work is also of relevance to certain problems in solid-state physics,[4] such as the energy levels of excitons and excitonic molecules. We turn now to a study of H^- in a magnetic field. We extend the variational techniques used in the study of hydrogen[3] to the case of two electrons.

The Hamiltonian for a hydrogen negative ion in a magnetic field B oriented along the z direction may be written

$$H=H_0+H_1+H_2\,, \tag{1}$$

where the unperturbed Hamiltonian in the absence of a magnetic field is

$$H_0=\frac{1}{2\mu}(p_1^2+p_2^2)-\frac{e^2}{r_1}-\frac{e^2}{r_2}+\frac{e^2}{|\vec{r}_1-\vec{r}_2|}\,, \tag{2}$$

with $\vec{p}_i$ and $\vec{r}_i$ the momentum and spatial coordinate of the ith electron. The perturbation due to the magnetic field gives rise to the additional terms

$$H_1=\omega_L(L_z+2S_z)\,, \tag{3}$$

where the Larmor frequency is

$$\omega_L=(eB/2\mu c) \tag{4}$$

and

$$H_2=\tfrac{1}{2}\mu\,\omega_L^2(r_1^2\sin^2\theta_1+r_2^2\sin^2\theta_2)\,,$$

where θ_i is the angle between $\vec{r}_i$ and the field vector taken to define the z axis. In Eq. (3), L_z and S_z represent operators for the projection along the z direction of the total orbital and spin angular momenta, L and S, respectively.

Due to the magnitude of the field under consideration, the H_1 term, which represents the linear Zeeman effect, gives rise to a complete Paschen-Back effect. We therefore choose the angular and spin parts of our basis functions to be the set

$$\zeta_\Gamma(\hat{r}_1\vec{s}_1\hat{r}_2\vec{s}_2)=\sum_{m_1m_2m_{s_1}m_{s_2}} C(l_1l_2L;m_1m_2M_L)C(\tfrac{1}{2}\tfrac{1}{2}S;m_{s_1}m_{s_2}M_S)\,Y_{l_1m_1}(\hat{r}_1)Y_{l_2m_2}(\hat{r}_2)X_{m(s_1)}(\vec{s}_1)X_{m(s_2)}(\vec{s}_2)\,, \tag{5}$$

where C is a Clebsch-Gordan coefficient and Y and X are spherical harmonics and spin functions, respectively. The composite index $\Gamma\equiv n_1l_1,n_2l_2,L$. In this basis, the matrix elements of H_1 are proportional to (M_L+2M_S), where M_L and M_S are the eigenvalues of L_z and S_z, respectively. Further, the matrix elements of H_2 are diagonal in M_L, M_S, and S.

Since the total Hamiltonian is invariant under rotations about the z axis and under inversion, the eigenstates can be labeled by M_L, M_S, S, and parity π. Thus, a general form of the trial solution may be written

$$\psi(\gamma,\vec{r}_1\vec{s}_1,\vec{r}_2\vec{s}_2)=A\sum_\Gamma (r_1r_2)^{-1}F_\Gamma(r_1)G_\Gamma(r_2)\times\zeta_\Gamma(\hat{r}_1\vec{s}_1,\hat{r}_2\vec{s}_2)\,, \tag{6}$$

where $\gamma\equiv(M_LM_SS\pi)$, and the reduced radial part of the trial function (F or G) is expanded in a linear combination of Slater orbitals of the form $a_ir^{n_i+1}e^{-\alpha_ir}$. A is the antisymmetrization operator.

At zero magnetic field, the only bound state of H^- is apparently the ^1S state[5] and the ion can be considered as two H atoms in ^1S states plus the Coulomb repulsion of the two electrons. We calculate this ground-state term to be -1.0537 Ry, compared with the excellent calculation of Pekeris[5] who obtained -1.0555 Ry. We consider the agreement to be satisfactory since we have not explicitly included correlation terms, i.e., terms involving $|\hat{r}_1-\hat{r}_2|$, in the expansion (6). It is clear however that correlation terms are *implicitly* included in the wave function chosen.

In the sum over Γ, 18 terms are included in the

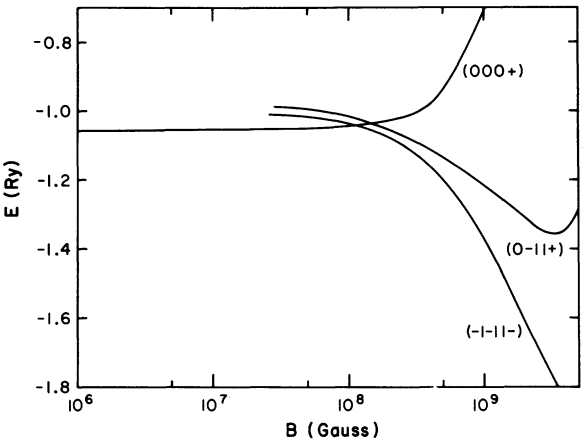

FIG. 1. Energy levels of the hydrogen negative ion as a function of the magnetic field strength B.

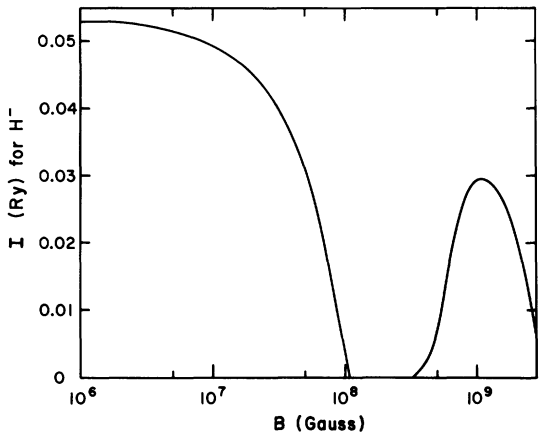

FIG. 2. Electron affinity of the hydrogen negative ion as a function of the magnetic field strength B.

calculation of the ground-state energy at zero magnetic field. For the largest magnetic fields ($\sim 10^9$ G) reported here, we use up to 86 terms, with a maximum L of nine.

Figure 1 gives the energies of some of the lowest states of H^- as a function of magnetic field strength. As the magnetic field is increased in strength, the linear term of Eq. (3) first becomes important. The lowest-energy state goes from the singlet, even-parity state $\gamma = (000+)$ to the triplet, odd-parity state $\gamma = (-1\,-1\,1-)$. The crossover occurs at approximately $B = 1.2\times 10^8$ G.

The matrix elements of H_2 depend only on $|M_L|$, whereas those of H_1 depend on the sign of $(M_L + 2M_S)$. Consequently, the values of B, at which the energies of the components with $(M_L + 2M_S)$ negative reach their minima, indicate approximately the magnetic field strength at which the quadratic effect becomes dominant for each component. In Fig. 1, for example, this occurs at $B = 3.3\times 10^9$ G for the triplet, even-parity state $\gamma = (0\,-1\,1\,+)$.

Figure 2 gives the non-negative values of the binding energy for H^- as a function of magnetic field strength B. The electron affinity (binding energy) of H^- is given by

$$I(H^-) = -E(H^-) - I(H), \tag{7}$$

where $E(H^-)$ is the energy of H^- and $I(H)$ is the binding energy for the hydrogen atom in the ground state. The quantities $I(H)$ are given in Ref. 3. If $I(H) > |E(H^-)|$ we have a "continuum" state, in which one electron is at infinity, unaffected by the Coulomb field but still under the influence of the magnetic field.

For $B < 1.2\times 10^8$ G, the only state with a positive electron affinity is the singlet state $\gamma = (000+)$. In the intermediate range $1.2\times 10^8 < B < 3.3\times 10^8$ G, there are no bound states of the system. In this range of B the "continuum" state with the lowest energy corresponds to one bound electron with spin down and one unbound electron with spin down in the lowest Landau level. At a B value of 3.3×10^8 G the electrons become bound again. In the range $3.3\times 10^8 < B < 3.3\times 10^9$ G, the bound state is the triplet state $\gamma = (-1\,-1\,1\,-)$. In this range the binding energy initially increases with increasing magnetic field. However, it exhibits a maximum at 1.2×10^9 G. It might have been conjectured, by analogy with atomic hydrogen, that the binding energy of the lowest triplet state would increase monotonically with increasing magnetic field strength. However, the Coulomb repulsion between the two electrons plays an important role. As the magnetic field strength is increased, the electrons move in orbits of decreasing radii and decreasing interorbit distances. In other words, the electrons are being forced closer together as B is increased. This tends to decrease the binding energy. Eventually this effect starts to dominate, which explains the turnover in the binding energy curve. As a check on this interpretation, we repeated the calculation with the electron-electron interaction term omitted and found that the curve no longer turned over.

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Ghost neutrinos in general relativity

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An exact solution to the Einstein-Dirac equations is presented for a static, plane-symmetric spacetime generated by neutrinos. We find the neutrino field to be nonzero and correspond to a neutrino current along the symmetry axis of the space. The neutrinos yield a zero energy-momentum tensor and therefore the gravitational field is exactly the same as for the vacuum case. A comparison with other solutions is presented along with a discussion of the possible physical significance of this "ghost neutrino" field.

I. INTRODUCTION

There have been many discussions of neutrinos in general relativity since the classic paper by Brill and Wheeler on the subject.[1] In this paper we present an exact solution to the Einstein-Dirac equations for the case of neutrinos. The field equations we wish to solve are

$$G_{jk} = R_{jk} - \tfrac{1}{2} g_{jk} R = (8\pi\kappa/c^4) T_{jk} , \tag{1.1}$$

where T_{jk} is the energy-momentum tensor for the Dirac field Ψ:

$$T_{jk} = -\tfrac{1}{4}\hbar c(\Psi^\dagger \gamma_j \Psi_{;k} - \Psi^\dagger_{;k}\gamma_j\Psi + \Psi^\dagger\gamma_k\Psi_{;j} - \Psi^\dagger_{;j}\gamma_k\Psi) . \tag{1.2}$$

Ψ satisfies the zero-mass Dirac equation

$$\gamma^j \Psi_{;j} = 0 . \tag{1.3}$$

Here the semicolon stands for covariant differentiation. Since for zero-mass Dirac particles the trace of the energy-momentum tensor vanishes, it follows from Eqs. (1.1) that the scalar curvature R also vanishes. Hence, for zero-mass Dirac particles the Einstein equations reduce (just as for the electrovac case) to

$$R_{jk} = (8\pi\kappa/c^4) T_{jk} . \tag{1.4}$$

In this paper we shall study solutions of Eqs. (1.3) and (1.4) in spacetimes of plane symmetry where the metric is defined by[2]

$$ds^2 = e^{2v}(dy^2 + dz^2) + e^{2u}(dx^2 - dt^2) , \tag{1.5}$$

where u and v are functions of (x, t). As is clear from Eq. (1.5), the x axis is the symmetry axis. In this paper we shall restrict our discussion to the static case. Hence, all functions u, v, Ψ, etc. depend on only the one space coordinate x. We shall carry out the calculations in the Cartan orthonormal frame defined by

$$\omega^1 = e^u dx, \quad \omega^2 = e^v dy , \quad \omega^3 = e^v dz , \quad \omega^4 = e^u dt . \tag{1.6}$$

The formulation of the Dirac equation in Cartan frames has been discussed by Brill and Cohen.[3] The essential point of this approach is to identify the Bargmann *vierbein* frame discussed by Brill and Wheeler in Ref. 1 with the Cartan orthonormal frame used to describe the geometry. The tangent vectors ω_i dual to the one forms ω^i in Eqs. (1.6) are given by

$$\omega_1 = e^{-u} e_1, \quad \omega_2 = e^{-v} e_2 , \quad \omega_3 = e^{-v} e_3 , \quad \omega_4 = e^{-u} e_4 , \tag{1.7}$$

where

$$e_i = \partial/\partial x^i .$$

In the orthornormal frame the covariant derivatives of the Dirac spinor are given by

$$\Psi_{;j} = \omega_j[\Psi] - \Gamma_j \Psi , \tag{1.8}$$