dicted from one-pion exchange; in practice, the corrections from this latter proviso hardly make any difference).

[14]For each mass bin, Estabrooks *et al*. find one solution (Solution 1) with $|\cos\theta_{10}|$ of Eq. (4) always close to unity. They prefer this alternative because it agrees better with $\pi^0\pi^0$ data (cf. Ref. 6).

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Fit to the pion electromagnetic form factor

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(Received 7 January 1974)

The pion electromagnetic form factor is fitted with a simple unitarized Veneziano ansatz. Comparison with experimental data is made in the region $-2.0 < t < +6.0$ $(\text{GeV}/c)^2$. The agreement is good in the spacelike region and reasonable in the timelike region.

Veneziano-type models in zero-width (or partially unitarized) versions have proved to be successful in fitting the data for the pion and proton electromagnetic form factors in the spacelike region up to $t=-0.4$ $(\text{GeV}/c)^2$ and $t=-25$ $(\text{GeV}/c)^2$, respectively.[1] It would be more satisfactory to have a unitarized expression for the form factors (i.e., built up by resonances with finite widths) which would behave properly in both spacelike and timelike regions.

To start with, let us consider the nonunitarized ansatz[2] for the pion form factor

$$F_\pi^0(t)=C\frac{\Gamma(1-\alpha(t))}{\Gamma(r_\pi+1-\alpha(t))}=2Cm_\rho^2\sum_{n=0}^{\infty}\frac{(-1)^n}{n!\,\Gamma(r_\pi-n)}\frac{1}{m_n^2-t}\,, \tag{1}$$

where

$$\alpha(t)=\frac{1}{2}+\frac{t}{2m_\rho^2}$$

is the ρ trajectory, $m_n^2=(2n+1)m_\rho^2$ $(n=0,1,2,\ldots)$ is the Veneziano spectrum, and r_π is a free parameter related to the asymptotic behavior of the form factor. From the dispersion-relation point of view we can say that

$$\text{Im}F_\pi^0(t)=\theta(t-4m_\pi^2)2Cm_\rho^2\pi\times\sum_{n=0}^{\infty}\frac{(-1)^n}{n!\,\Gamma(r_\pi-n)}\delta(m_n^2-t) \tag{2}$$

in the sense that if we calculate

$$\frac{1}{\pi}\,\text{P}\int_{-\infty}^{\infty}\frac{\text{Im}F_\pi^0(t')dt'}{t'-t}$$

we obtain the expression (1). A natural way to unitarize (2) is to replace the δ function by the corresponding Breit-Wigner expression

$$\delta(m_n^2-t)\to\frac{1}{\pi}\frac{\Gamma_n m_n}{(m_n^2-t)^2+\Gamma_n^2m_n^2}\,,$$

where Γ_n is the finite width associated with each resonance. Moreover, we have to satisfy the threshold condition

$$\text{Im}F_\pi|_{t=4m_\pi^2}=0.$$

In our calculation the mass scale is set up by m_ρ so that we can consider $t=0$ at threshold as a good approximation. With all these requirements in mind we make the following ansatz:

$$\text{Im}F_\pi(t)=\theta(t)2Cm_\rho^2\times\sum_{n=0}^{\infty}\frac{(-1)^n}{n!\,\Gamma(r_\pi-n)}\frac{\Gamma_n m_n}{(m_n^2-t)^2+\Gamma_n^2m_n^2}\frac{t}{m_n^2}\,, \tag{3}$$

which obviously reproduces (2) in the zero-width limit. In order to proceed further we must make an assumption on the n dependence of the widths Γ_n. Following Greco,[3] we assume $\Gamma_n=\gamma m_n$, where the constant γ is fixed from the known ρ parameters. Then we find (model 1) for the form factor, which we now call $F_\pi^{(1)}$,

$$\text{Im}F_\pi^{(1)}=A\theta(t)\,\text{Im}\left[\frac{\Gamma(\frac{1}{2}-z)}{\Gamma(r_\pi+\frac{1}{2}-z)}\right]\,, \tag{4}$$

$$\text{Re}F_\pi^{(1)}\equiv\frac{1}{\pi}\,\text{P}\int_{-\infty}^{\infty}\frac{\text{Im}F_\pi^{(1)}(t')dt'}{t'-t}=A\left\{\left(1-\frac{\arctan\gamma}{\pi}\right)\text{Re}\left[\frac{\Gamma(\frac{1}{2}-z)}{\Gamma(r_\pi+\frac{1}{2}-z)}\right]+\frac{\gamma y}{\pi}S(y,\gamma,r\pi)\right\}\,, \tag{5}$$

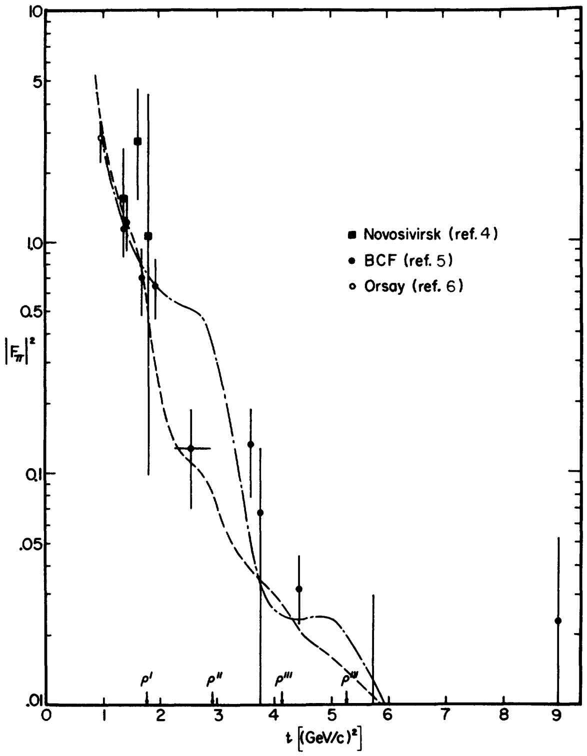

FIG. 1. Pion form factor in the region $1.0 < t < 6.0$ $(\mathrm{GeV}/c)^2$ for model 1 (dashed line) and model 2 (dash-dotted line). The positions of the resonances are indicated by arrows.

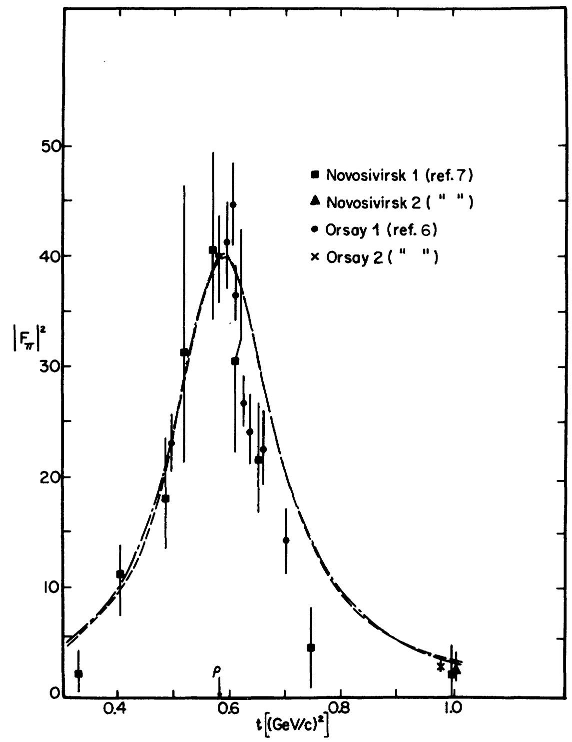

FIG. 2. Pion form factor in the ρ-meson region. Model 1 (dashed line) and model 2 (dash-dotted line) are almost coincident here.

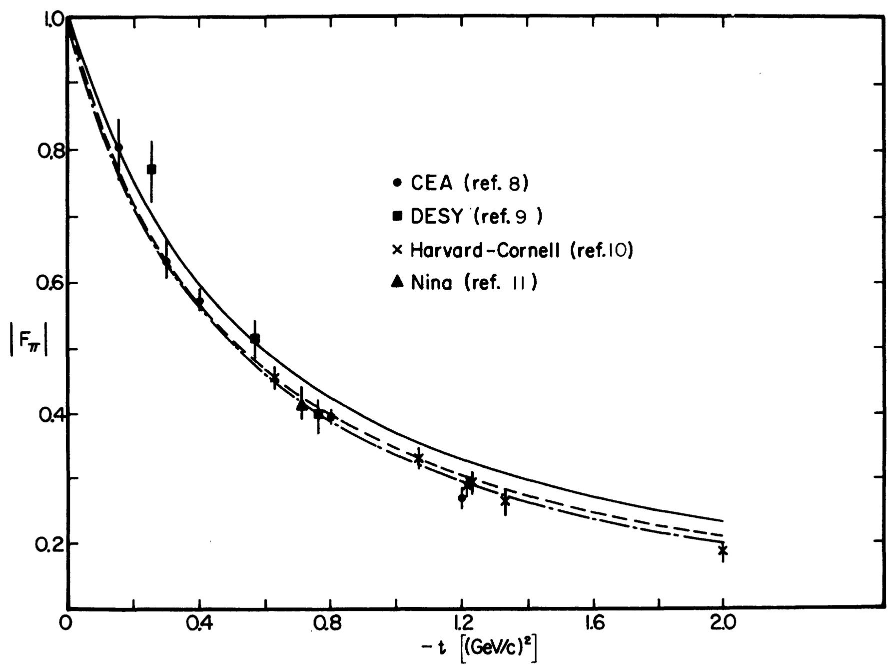

FIG. 3. Pion form factor in the spacelike region for model 1 (dashed line) and model 2 (dash-dotted line). The reference curve (solid line) corresponds to zero-width ρ dominance.

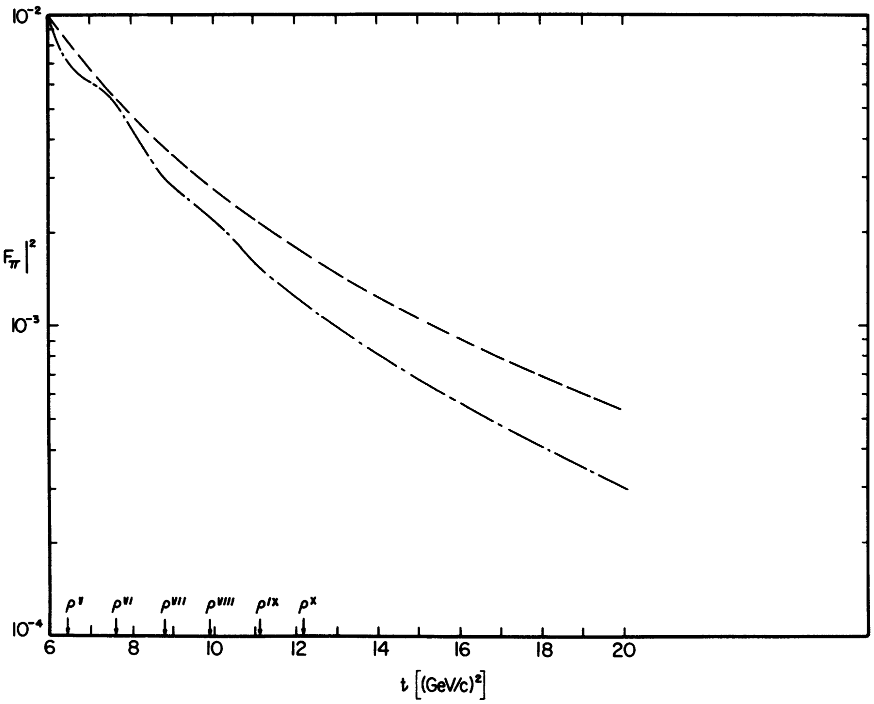

FIG. 4. Predicted values for the pion form factor in the region $6.0 < t < 20.0$ $(\text{GeV}/c)^2$. The positions of the resonances are indicated by arrows.

where

$$z = \frac{t}{2m_\rho^2} \frac{1+i\gamma}{1+\gamma^2},$$

$$y = \frac{t}{m_\rho^2},$$

$$S(y,\gamma,r_\pi) = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!\,\Gamma(r_\pi - n)} \frac{\ln[(2n+1)^2(1+\gamma^2)/y^2]}{(2n+1-y)^2+\gamma^2(2n+1)^2},$$

and the normalization constant A, such that $F_\pi^{(1)}(0) = 1$, is

$$A = \frac{1}{1-(\arctan\gamma)/\pi} \frac{\Gamma(r_\pi + \frac{1}{2})}{\Gamma(\frac{1}{2})}.$$

Another closely related model along the same lines is obtained by skipping every odd daughter in the Veneziano spectrum, i.e., considering only the resonances $m_n^2 = (1+4n)m_\rho^2$ $(n = 0, 1, 2, \ldots)$. For this case we obtain (model 2)

$$\operatorname{Im} F_\pi^{(2)} = A'\,\theta(t)\,\operatorname{Im}\left[\frac{\Gamma(\frac{1}{4}-z')}{\Gamma(r_\pi + \frac{1}{4} - z')}\right], \qquad (6)$$

$$\operatorname{Re} F_\pi^{(2)} = A'\left\{\left(1 - \frac{\arctan\gamma}{\pi}\right)\operatorname{Re}\left[\frac{\Gamma(\frac{1}{4}-z')}{\Gamma(r_\pi + \frac{1}{4} - z')}\right] + \frac{2\gamma y}{\pi} S'(y,\gamma,r_\pi)\right\}, \qquad (7)$$

where

$$z' = \frac{t}{4m_\rho^2} \frac{1+i\gamma}{1+\gamma^2},$$

$$S'(y,\gamma,r_\pi) = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!\,\Gamma(r_\pi - n)} \times \frac{\ln[(4n+1)^2(1+\gamma^2)/y^2]}{(4n+1-y)^2+\gamma^2(4n+1)^2},$$

$$A' = \frac{1}{1-(\arctan\gamma)/\pi} \frac{\Gamma(r_\pi + \frac{1}{4})}{\Gamma(\frac{1}{4})}.$$

To make a comparison with the experimental data[4-11] we have plotted our predictions in Figs. 1–4 for the following set of parameters: $\gamma = 0.190$, $r_\pi = 1.20$ for model 1, and $\gamma = 0.190$, $r_\pi = 1.385$ for model 2. Given the value of γ, r_π is chosen so that $|F_\pi| \sim \sqrt{40}$ at $t/m_\rho^2 = 1$ (ρ peak). This peak value for $|F_\pi|$ is what we would expect if ρ-ω interference were absent. We take $m_\rho = 765$ MeV, which implies $\Gamma_\rho = 145$ MeV in both cases. This value is within the experimental error.

Both models are essentially equivalent in the spacelike and ρ-meson regions, but differ appreciably in the region $1.0 < t < 6.0$ $(\text{GeV}/c)^2$ plotted in Fig. 1. There the experimental data are not sufficient to reject one model in favor of the other. As $t \to \infty$ we have $F_\pi^{(1)} \sim t^{-1.20}$ and $F_\pi^{(2)} \sim t^{-1.39}$. It is worth noting that the peaks corresponding to the higher vector mesons do not appear so prominent when $|F_\pi|^2$ is plotted.

The fact that we do not have a perfect agreement in the region $0.3 < t < 1.0$ (GeV/c)^2, shown in Fig. 2, can be partly understood because (i) we do not have the right threshold behavior [one should expect $\text{Im}F_\pi(t) \sim t^{3/2}$ at threshold according to the Gounaris-Sakurai formula[12]], and (ii) we are not considering ρ-ω interference. We expect that when these things are taken into account, appreciable modifications will only arise in the vicinity of the relevant regions.

In the spacelike region plotted in Fig. 3, the agreement is very good and we get for the pion radius the value 0.67 F for both models. The ρ-dominance comparison curve corresponds to $F_\pi = m_\rho^2/(m_\rho^2 - t)$.

In Fig. 4 we plot our predictions for the region $6 < t < 20$ (GeV/c)^2.

I would like to thank Professor J. J. Sakurai for suggesting the problem and for many useful discussions. Thanks are also due to J. Frez for valuable help in writing the computer program for the calculations.

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Errata

Erratum: Unified model of current-hadronic interactions [Phys. Rev. D 8, 2152 (1973)]

J. W. Moffat and A. C. D. Wright

The left-hand side of Eq. (3.12) should read $A(s, t, q_1^2, q_2^2)$ instead of $A_{33}(s, t, q_1^2, q_2^2)$, and on p. 2159, first column, twelfth line from the bottom, A_{33} should read A.

Erratum: Measurement of the branching ratios of $K^+_{\mu 2}$, $K^+_{\pi 2}$, K^+_{e3}, and $K^+_{\mu 3}$ [Phys. Rev. 155, 1505 (1967)]

L. B. Auerbach, J. MacG. Dobbs, A. K. Mann, W. K. McFarlane, D. H. White, R. Cester, P. T. Eschstruth, G. K. O'Neill, and D. E. Yount

A mistake was made in the computation of the $K^+_{\mu 3}$ branching ratio. This error, which was pointed out by Gaillard and Chounet,[1] also affects the other branching ratios, $K^+_{\mu 2}$, $K^+_{\pi 2}$, and K^+_{e3}, presented in the paper. Although the experiment was performed and published some time ago, these results are still used in data compilations and contribute substantially to the degree of disagreement among the experiments on K^+ decay. It is, therefore, useful to recalculate all of the results of the