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Right-left multiplicities and the Pomeranchuk singularity*

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The left-right multiplicity cross sections for producing n_L particles in the left c.m. hemisphere and n_R particles in the right c.m. hemisphere are discussed. It is found that these distributions provide a set of quantitative tests for the existence of a nonfactorizable part of the Pomeranchuk singularity. It is also found that the multiplicity distribution in a single hemisphere is a more sensitive test of the Wilson two-component idea than is the total multiplicity distribution. New NAL data show evidence for such a two-component structure.

Important questions are currently being raised about the nature of the Pomeranchuk singularity.[1,2] Among the new experiments pertinent to these questions are those designed to measure high-energy inelastic processes. We would like to discuss here the right-left multiplicity distribution, the dependence of the cross section on the number of particles going to the right and the number of particles going to the left in the c.m. system. This distribution can provide a set of quantitative tests for the existence of a nonfactorizable part of the Pomeranchuk singularity. In addition, features of the distribution can be used to provide information about the fraction of inelastic events which can be labelled diffractive fragmentation.[3]

From the inelastic cross sections, $\sigma(n_L, n_R)$, for producing n_L charged prongs in the left c.m. hemisphere and n_R in the right, we define the generating function

$$\sum_{n_L, n_R=0}^{\infty} \sigma(n_L, n_R) z_L{}^{n_L} z_R{}^{n_R} \equiv Q(z_L, z_R)\,. \tag{1a}$$

For compactness the sums extend from zero to infinity even though $\sigma(n_L, n_R)$ vanishes identically if $n_L + n_R <$ initial charge and $n_L + n_R > N_{\max}$, where $N_{\max}$ is the maximum allowed number of particles at the given energy. We can write

$$Q(z_L, z_R) = (s/s_0)^{p(z_L, z_R)}, \tag{1b}$$

where $\sqrt{s}$ is the c.m. energy and s_0 defines a scale. The "effective pressure" $p(z_L, z_R)$ is an implicit function of energy which asymptotically approaches a constant plus correction terms of order, for example, $1/\ln(s/s_0)$.

The cross sections can be reconstructed from the generating function in the usual way.[4] As examples we write

$$\sigma_{\text{inel}} = (s/s_0)^{p(1,1)}, \tag{2a}$$

$$\sigma(1,1) = \frac{\partial^2}{\partial z_L \partial z_R} \left(\frac{s}{s_0}\right)^{p(z_L, z_R)} \bigg|_{z_L = z_R = 0}. \tag{2b}$$

We define the total N_R-prong cross section by

$$\sigma(\text{tot}, N_R) = \sum_{n_L=0}^{\infty} \sigma(n_L, N_R) = \frac{1}{N_R!} \left(\frac{\partial}{\partial z_R}\right)^{N_R} \left(\frac{s}{s_0}\right)^{p(1, z_R)} \bigg|_{z_R=0}. \tag{3}$$

The multiplicity moments are given by

$$\left[\ln\left(\frac{s}{s_0}\right)\right]^{-1} \langle n_R \rangle = \frac{\partial p}{\partial z_R}(1, z_R)\bigg|_{z_R=1}, \tag{4a}$$

$$\left[\ln\left(\frac{s}{s_0}\right)\right]^{-1} [\langle n_R(n_R-1)\rangle - \langle n_R\rangle^2] = \frac{\partial^2 p}{\partial z_R{}^2}(1, z_R)\bigg|_{z_R=1}, \tag{4b}$$

$$\left[\ln\left(\frac{s}{s_0}\right)\right]^{-1}[\langle n_R n_L\rangle-\langle n_R\rangle\langle n_L\rangle] = \frac{\partial^2 p}{\partial z_R \partial z_L}(z_L,z_R)\bigg|_{z_L=z_R=1}, \quad (4c)$$

etc. From inspection of (2)–(4) it is evident that a study of right-left correlations can be facilitated by studying $p(z_L, z_R)$ directly. To do this we expand p in a power series around z_L, $z_R=1$,

$$p(z_L, z_R)=\sum_{\lambda,\rho=0}^{\infty} p_{\lambda\rho}\frac{(z_L-1)^\lambda}{\lambda!}\frac{(z_R-1)^\rho}{\rho!}. \quad (5)$$

To proceed further we note the connection between the expansion parameters $p_{\lambda\rho}$ and the inclusive correlation functions $C_{\lambda\rho}$:

$$p_{\lambda\rho}=[\ln(s/s_0)]^{-1}\int_{(y_1\cdots y_\rho)\in[-Y/2,0]} dy_1\cdots dy_\rho \int_{(y_1'\cdots y_\lambda')\in[0,\frac{1}{2}Y]} dy_1'\cdots dy_\lambda' C_{\lambda\rho}(y_1\cdots y_\rho, y_1'\cdots y_\lambda'). \quad (6)$$

Under quite general assumptions we see that the terms which mix left and right, $\lambda,\rho\geq 1$, behave differently at high energy than those which do not. Assuming that $C_{\lambda\rho}$ is asymptotically a function only of the rapidity differences y_i-y_{i+1} we see that if the correlations are short-range, the integral is finite as $s\to\infty$ unless $\lambda=0$ or $\rho=0$ in which case it can grow like $Y=\ln(s/s_0)$. We therefore have

$$p_{0\rho}=O(1), \quad p_{\lambda 0}=O(1), \quad (7a)$$

$$p_{\lambda\rho}=O([\ln(s/s_0)]^{-1}), \quad \lambda,\rho\geq 1. \quad (7b)$$

If we then write

$$\begin{aligned} p(z_L, z_R) &= p_{00}+\sum_{\rho=1}^{\infty} p_{0\rho}\frac{(z_R-1)^\rho}{\rho!} \\ &+\sum_{\lambda=1}^{\infty} p_{\lambda 0}\frac{(z_L-1)^\lambda}{\lambda!} \\ &+\sum_{\lambda,\rho=1}^{\infty} p_{\lambda\rho}\frac{(z_R-1)^\rho}{\rho!}\frac{(z_L-1)^\lambda}{\lambda!} \\ &\equiv p_0+p_L(z_L)+p_R(z_R)+p_c(z_L,z_R), \end{aligned} \quad (8)$$

so that the function $p_c(z_L, z_R)$ correlates right and left multiplicities, we see that in the absence of long-range forces

$$p_c(z_L, z_R)\to 0. \quad (9)$$

The conditions under which (9) is expected to hold can be made more precise in terms of the Feynman-Wilson gas analog.[5] The mathematical isomorphism between models for production processes and classical gases has been investigated by a number of authors.[6] We here only quote the result that for models with exclusive cluster decomposition in rapidities, the function $p(z_L, z_R)$ is the analog pressure and the coefficients p_{10} and p_{01} are densities. All formal relations which hold between quantities in a gas can be claimed to hold for the analog quantities. In particular (9) should hold in the absence of phase transitions as $(\ln s)\to\infty$.

However the argument is phrased, the long-range rapidity space correlations which are necessary for (9) to break down correspond, in Regge language, *to a leading J=1 nonfactorizable singularity* in the Mueller-Regge description of the inclusive distribution. The experimental evidence at the CERN ISR[7] and NAL[8] that $\langle n_R\rangle$ is approximately independent of n_L in pp collisions gives an indication that at these energies

$$p_{\lambda 1}, p_{1\rho}\cong 0. \quad (10)$$

The important fact to be noticed is that this kind of measurement cannot be considered a complete test of factorization. There are actually an infinite set of measurements

$$\begin{aligned} &\langle n_R(n_R-1)\rangle \text{ vs } n_L, \\ &\langle n_R(n_R-1)(n_R-2)\rangle \text{ vs } n_L, \\ &\text{etc.} \end{aligned} \quad (11)$$

on right-left multiplicities which can be made to test factorization. Since it is not possible in practice to make all these tests and since with finite statistics the higher moments will cease to be decisive, we note that there is another set of tests implied by the vanishing of the $p_{\lambda\rho}$. Using (8) we see that

$$\begin{aligned} p(1,0)+p(0,1) &= 2p_0+p_L(1)+p_L(0)+p_R(1)+p_R(0) \\ &= p(0,0)+p(1,1)-p_c(0,0) \end{aligned} \quad (12)$$

so if $p_c(0,0)\to 0$ we have an equality between logarithms of measurable cross sections. In pp collisions $p(0,1)=p(1,0)$ so we have

$$p(1,0)=\tfrac{1}{2}(p(0,0)+p(1,1)):$$

The energy dependence of the cross sections $\sigma(N,\text{ tot})$ is the average of the energy dependences of the exclusive and the total inelastic cross sections. For example, if (ignoring logarithms)

$$\sigma(\text{tot}) \sim (s/s_0)^0, \quad \sigma(1,1) \sim (s/s_0)^{-1}, \tag{13}$$

we then have

$$\sigma(1,\text{tot}) \sim (s/s_0)^{-1/2} \tag{14}$$

in the absence of right-left correlations. With $p_c(0,0) \to 0$ and for pp reactions, Eq. (12) implies the series of tests

$$R(N) = \frac{2\ln\sigma(N,\text{tot})}{\ln\sigma(N,N) + \ln\sigma(\text{tot})} = 1 + O(1/\ln s). \tag{15}$$

These tests are weaker than the complete set of tests (11) but they are independent of any finite subset. Another set of tests for factorization involving only exclusive cross sections is

$$\frac{\sigma(N,M)\sigma(M,N)}{\sigma(N,N)\sigma(M,M)} \sim 1. \tag{16}$$

To see how sensitive the tests on right-left multiplicities are, we have compared the data at NAL and ISR on $\langle n_L\rangle_R$ vs n_R with a model which has an explicitly nonfactorizable Pomeron, a version of the critical-point gas model of Arnold and Thomas.[9] The model gives the formula

$$\sigma(n_L, n_R, n_0) = A\frac{g_L^{\,n_L}}{n_L!}\frac{g_R^{\,n_R}}{n_R!}\frac{g_0^{\,n_0}}{n_0!} \times (Y - nb)^n e^{an^2/Y}, \quad n < Y/b;$$

$Y = \ln s - 1.5$, $A = 31$ mb, $n = n_L + n_R + n_0$, $a = \frac{27}{64}$, $b = \frac{1}{8}$, $g = g_L + g_R + g_0 = 4\exp(-\frac{7}{4})$. Here n_0 represents the number of unseen events, including neutrals:

$$\sigma(n_L, n_R) = \sum_{n_0} \sigma(n_L, n_R, n_0).$$

This type of model and the choice of parameters is discussed in detail in Refs. 9 and 10. Since this formula gives asymptotically nonzero left-right correlations for all choices of Van der Waals parameters (except $a = b = 0$), and hence automatically violates the cluster decomposition assumption [such as Eq. (9)] required of a noncritical analog gas, strictly speaking it is not an analog critical point model for left-right correlations. Nevertheless it does correctly show the trend one might expect for such a model.

The ratios $g_L : g_R : g_0$ are as follows. In Fig. 1(a), the ratios are 2 : 2 : 5, based on the assumptions that on the average (1) charges are twice as numerous as neutrals, and (2) there are equally as many charges detected going left, as detected

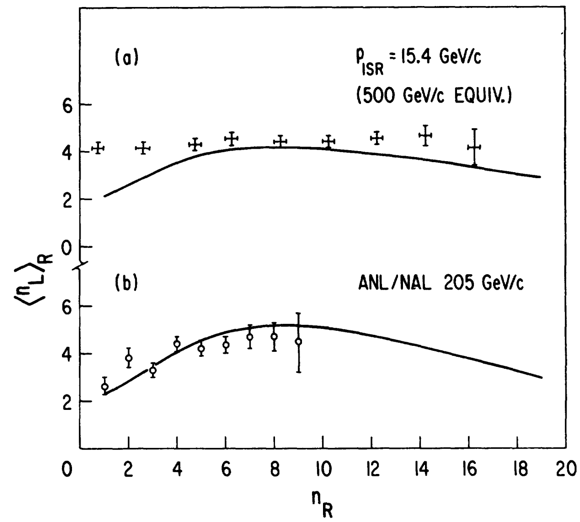

FIG. 1. Data represent $\langle n_L\rangle_R$ vs n_R at ISR [(a) Ref. 7] and at NAL [(b) Ref. 8]. The curves described in the text describe left-right correlations arising from a nonfactorizable model.

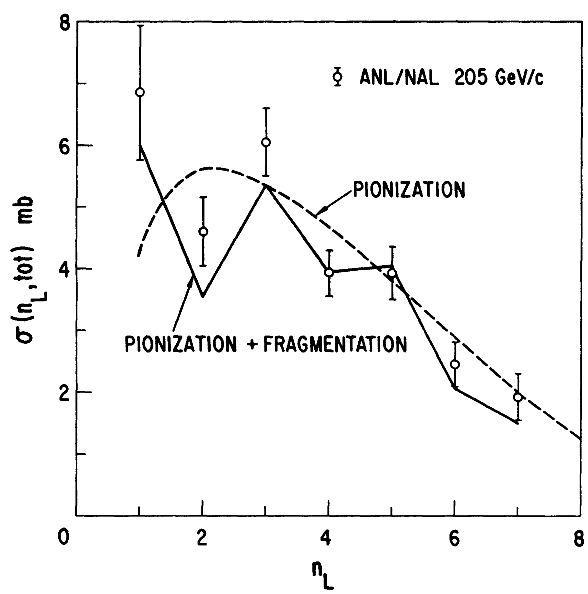

FIG. 2. The data represent the left-prong cross section $\sigma(n_L, \text{tot})$ vs n_L (Ref. 8) taken at NAL at 205 GeV/c. The solid curve is a factorizable two-component model

$$\sigma(n_L, n_R) = \left(\frac{1}{n_L^{\,2}} + 6e^{-4}\frac{4^{n_L}}{n_L!}\right)\left(\frac{1}{n_R^{\,2}} + 6e^{-4}\frac{4^{n_R}}{n_R!}\right),$$

having a rapidly dropping fragmentation component and a pionization component chosen to be a simple Poisson distribution. For comparison a representative pionization model for $\sigma(n_L, \text{tot})$ can be computed using the same formula as in Fig. 1(b). This is shown by the dashed line. The main characteristic expected here for any pionization model is the drop of $\sigma(n_L, \text{tot})$ for $n_L \lesssim 2$.

going right as being undetected. (This is to approximate the cut made in the ISR data. They see only events in the region $4° < \theta_{c.m.} < 31°$.) In Fig. 1(b), the ratios are 1 : 1 : 1 based on the above assumptions, but with no charges going undetected. For reference, the model (b) gives the value $R(N) = 1.16$ [see Eq. (15)] at infinite energy.

This model has strong long-range correlations and asymptotically gives $\langle n_L \rangle_R \propto n_R$ but finite-energy phase-space effects tend to make the curve flatter. The data at ISR are taken only in a fraction of the available volume, and unseen events tend to wash out any correlations which might be present. The comparison is shown in Fig. 1. We see that the data have the capability of ruling out this model.

For the tests, Eq. (15), the NAL 205-GeV/c data give the values

$$R(1) = 0.81 \pm 0.10, \quad R(2) = 0.74 \pm 0.06, \quad R(3) = 0.85 \pm 0.06. \tag{17}$$

Comparing these with values at other energies and with specific models can also provide a sensitive test of factorization. We will discuss the use of these tests to distinguish between models in more detail elsewhere.

As a related point we note that the distribution of $\sigma(n_L, \text{tot})$ vs n_L observed in the NAL data shows a dip at $n_L = 2$. This can be interpreted as evidence for a Wilson 2-component structure of the inelastic processes.[11] These data are shown in Fig. 2 compared with a factorizable 2-component model. We note that the evidence, if it persists, decides only in favor of the existence of fragmentation, not necessarily diffraction. For the latter it is still necessary to study the energy dependence [e.g., $\sigma(1, \text{tot})$ vs s].

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