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Comments and Addenda

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Leading logarithmic behavior of electromagnetic form factors in bound-state models*†

George F. Sterman
Center for Theoretical Physics, Department of Physics and Astronomy, University of Maryland, College Park, Maryland 20742
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A detailed calculation of the asymptotic behavior of electromagnetic form factors of the Bethe-Salpeter ladder model of the S-wave fermion-scalar bound state for large spacelike momentum transfer has been carried out. In contrast to earlier results, it is found that the magnetic form factor behaves as $(q^2)^{-2}\ln^5(-q^2)$ when the fermion constituent is charged. Implications of this high power of the logarithm are briefly discussed.

The Bethe-Salpeter (BS) ladder model for the fermion-scalar bound state has become a subject of renewed interest in recent years as a simple theoretical approach to the properties of composite nucleons. A major aim of studies of this model has been the calculation of matrix elements of operators between bound states, in particular, matrix elements of the electromagnetic current. It is well known that, with a scalar gluon, the electromagnetic form factors which result from giving an elementary coupling to one of the constituents have dipole falloff in momentum transfer squared, to within logarithms.[1-6]

Although previous calculations based on this model have all resulted in the same asymptotic power behavior for the form factors, the situation with respect to logarithms is less clear. With the electromagnetic vertex expressed as $\gamma_\mu F_1(q^2) + i\sigma_{\mu\nu}q^\nu F_2(q^2)$, we find both the dependence $(q^2)^{-2}\ln^2(-q^2)$ and $(q^2)^{-2}\ln(-q^2)$ quoted for $F_1(q^2)$, as well as both $(q^2)^{-2}\ln(-q^2)$ and $(q^2)^{-2}$ for $F_2(q^2)$.[1,2,4,5] We may note that these differences do not correspond simply to choices of which constituent (spin-0 or spin-$\frac{1}{2}$) is taken as charged. In any case, the low powers of the logarithm which occur seem basically consistent with experimental data at large $-q^2$, so that the calculation of form factors has been considered an indication that such models, although oversimplified, might be useful in the analysis of other reactions such as electroproduction. It is the purpose of this note to reconsider the question of the logarithmic behavior of the form factors. We find, among other things, that, with the fermion constituent charged, the magnetic form factor of the bound state behaves asymptotically as $(q^2)^{-2}\ln^5(-q^2)$, which differs appreciably from previous results with respect to the power of the logarithm.

We can give a rough argument for the existence of a $(q^2)^{-2}\ln^5(-q^2)$ contribution to $F_1(q^2)$ when the fermion constituent is charged, as follows. The form factors are calculated from the Bethe-Salpeter wave function of the bound state by means of the expression[7]

$$\langle P_2|J_\mu(0)|P_1\rangle = e\int d^4k\,\overline{\Phi}_{P_2}(k)L_\mu\Phi_{P_1}(k)\,. \qquad (1)$$

L_μ is taken as $(k^2-\mu^2)\gamma_\mu$ or $(-\not{k}-m)(2k+P_1+P_2)_\mu$ for charged fermion or scalar constituent, respectively, and $\overline{\Phi}_P(k)$ is defined as $-\Phi_P^\ddagger(\vec{k},k_0^*)$.[8] Here, and below, m is the fermion and μ is the scalar, constituent mass. Figure 1 illustrates the situation. We shall see below that when $-k$ denotes the scalar momentum, the wave function $\Phi_P(k)$ behaves as $(k\cdot P)^{-2}\ln^2(-k\cdot P)$ for $k\cdot P\to-\infty$ with k^2 fixed and less than μ^2.[9] Since q is a spacelike

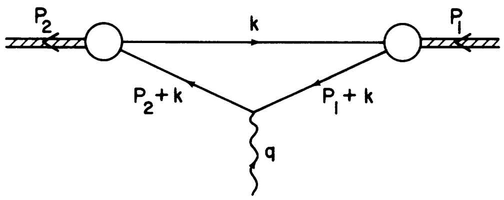

FIG. 1. Electromagnetic form factor in the ladder model.

momentum, we may choose the Breit frame to evaluate (1). With $q=(0;0,0,Q)$ we have, with $k_\pm=(k_0\pm k_3)$, $P_2\cdot k=(k_-Q)+O(k_\pm/Q)$ and $P_1\cdot k=(k_+Q)+O(k_\pm/Q)$. Now consider the region of the k integration in (1) given by

$$M_1^2\le k_+k_-\le M_2^2\,,$$
$$M_2^2-\mu^2\le k_1^2+k_2^2\le M_3^2-\mu^2\,,$$
$$M_0^{2-\epsilon}Q^{\epsilon-1}\le -k_+\le M_0^\epsilon Q^{1-\epsilon}\,.$$

ϵ is a small fixed positive number, while the M_i are fixed masses with $M_1^2<M_2^2<M_3^2$. The volume of this domain in Minkowski space is easily seen to be proportional to $\ln(Q^2)$. On the other hand, the integrand of (1) is already of order $Q^{-4}\ln^4(Q^2)$ throughout the region. The fifth power of the logarithm will then be present in the asymptotic behavior in the absence of cancellation.

Of course, the above argument is completely heuristic. To evaluate (1) more carefully we shall use a DGSI (Deser-Gilbert-Sudarshan-Ida) spectral representation for the wave function.[10] Consider the wave function for an S-wave bound state of a fermion and scalar particle, which may be written as

$$\Phi_P(k)=[A(k^2,P\cdot k)+(\not k+M)B(k^2,P\cdot k)]u_M(P)\,. \tag{2}$$

$u_M(P)$ is a free Dirac spinor of mass M. The appropriate Bethe-Salpeter equation with a pointlike scalar ladder of mass κ is

$$\Gamma_P(k)=\lambda\int d^4k'\frac{\Phi_P(k')}{(k-k')^2-\kappa^2}, \tag{3}$$

$$\Gamma_P(k)=[\Gamma_1(k^2,P\cdot k)+(\not k+M)\Gamma_2(k^2,P\cdot k)]u_M(P)$$
$$=[(\not P+\not k)-m](k^2-\mu^2)\Phi_P(k)\,.$$

The invariant functions A and B are assumed to possess DGSI spectral representations:

$$A(k^2,P\cdot k)=\int_0^\infty d\sigma\int_{-1}^0 d\beta\frac{a(\beta,\sigma)}{[(k+\beta P)^2-\sigma-s(\beta)]^3}, \tag{4}$$
$$B(k^2,P\cdot k)=\int_0^\infty d\sigma\int_{-1}^0 d\beta\frac{b(\beta,\sigma)}{[(k+\beta P)^2-\sigma-s(\beta)]^3}.$$

Here $s(\beta)=M^2(\beta+\beta^2)-\beta m^2+(1+\beta)\mu^2$. By substituting the forms (4) into (3) and using some standard manipulations,[11] we may transform the BS equation into two two-dimensional integral equations for the weight functions $a(\beta,\sigma)$ and $b(\beta,\sigma)$. These equations have been investigated elsewhere in some detail,[12] and certain asymptotic properties of the weight functions have been derived. The results include the following, which are valid with either a massive or a massless gluon:

$$\begin{aligned}
&a(\beta,\sigma)\underset{\sigma\to\infty}{\sim}a(\beta)\sigma^{-1},\\
&a(\beta)\underset{\beta\to-1}{\sim}\tfrac{1}{2}\pi^2\lambda c_1(1+\beta)^2\\
&\qquad\underset{\beta\to0}{\sim}\pi^2\lambda c_2|\beta|\ln|\beta|,\\
&\int_0^{M'^2}a(\beta,\sigma)d\sigma\underset{\beta\to-1}{\sim}mc_1(1+\beta)\\
&\qquad\underset{\beta\to0}{\sim}\pi^2\lambda c_2|\beta|\ln^2|\beta|,\\
&\int_0^{M'^2}b(\beta,\sigma)d\sigma\underset{\beta\to-1}{\sim}c_1(1+\beta)\\
&\qquad\underset{\beta\to0}{\sim}c_2|\beta|.
\end{aligned} \tag{5}$$

The constants c_1 and c_2 are expressible in terms of integrals involving $a(\beta,\sigma)$ and $b(\beta,\sigma)$; we shall not need their explicit forms here. M' is any finite mass.

The significance of the asymptotic behavior of the weight functions for the momentum-space behavior of the wave function may be seen from Eqs. (4). With $-k$ corresponding to the four-momentum of the virtual scalar, the limit $\beta\to-1(0)$ in the spectral functions controls the leading momentum-space contribution to the wave function when the scalar (fermion) leg goes far off-shell, while the fermion (scalar) leg retains a fixed invariant mass. Define $k^2=s_1$, $(P+k)^2=s_2$. Then Eqs. (3), (4), and (5) lead to the following. For $s_1\to-\infty$, s_2 fixed and less than m^2,

$$\begin{aligned}
&A(s_1,\tfrac{1}{2}(s_2-M^2-s_1))\sim mB(s_1,\tfrac{1}{2}(s_2-M^2-s_1))\sim s_1^{-2},\\
&A-mB\sim s_1^{-3}\ln^2|s_1|,\\
&\Gamma_1(s_1,\tfrac{1}{2}(s_2-M^2-s_1))\sim s_1^{-1},\\
&\Gamma_2(s_1,\tfrac{1}{2}(s_2-M^2-s_1))\sim s_1^{-2}\ln^2|s_1|;
\end{aligned} \tag{6a}$$

while for $s_2\to-\infty$, s_1 fixed and less than μ^2,

$$\begin{aligned}
&A\sim s_2^{-2}\ln^2|s_2|,\quad B\sim s_2^{-2},\\
&\Gamma_1\sim s_2^{-1},\quad \Gamma_2\sim s_2^{-2}\ln^2|s_2|.
\end{aligned} \tag{6b}$$

Finally, for $s_1\to-\infty$, s_1/s_2 fixed and positive,

$$A \sim s_1^{-3}\ln|s_1|,\quad B\sim s_1^{-3},$$
$$\Gamma_1\sim s_1^{-1},\quad \Gamma_2\sim s_1^{-2}\ln|s_1|. \tag{6c}$$

The leading behavior of Γ_P given in (6) may also be derived from a direct examination of the Bethe-Salpeter equation based on contour integrals in momentum space.[3] We note that (6c) differs from the behavior found by Ciafaloni and Menotti[1] from a similar analysis in the zero-energy case: $A\sim s_1^{-3}$, $B\sim s_1^{-4}$. The latter solution to the equations they give, however, is a nonleading one. As may be explicitly verified, (6c) is also a solution and dominates the asymptotic behavior in the zero-energy system as well.[4] The limits of (6a) and (6b), of course, cannot be investigated in the $P=0$ case. Note further that although the dependence of Γ_1 on the large variable is the same in each of the three limits of Eqs. (6), for Γ_2 this is not the case.

We can now employ the spectral representations (4) to do the momentum integration in (1). The asymptotic behavior of the form factors for large $-q^2$ is determined by the resulting integrals over the weight variables using (5). The results are found to depend on which constituent is chosen charged. For charged scalar we find

$$F_1(q^2)\sim (q^2)^{-2}\ln^2(-q^2),$$
$$F_2(q^2)\sim (q^2)^{-2}\ln(-q^2), \tag{7a}$$

while for charged fermion

$$F_1(q^2)\sim (q^2)^{-2}\ln^5(-q^2),$$
$$F_2(q^2)\sim (q^2)^{-2}. \tag{7b}$$

The high power of $\ln(-q^2)$ in $F_1(q^2)$ in (7b) comes from the dependence on β of the weight function $a(\beta,\sigma)$ for $\beta\to 0$. This is the same dependence which is responsible for the asymptotic behavior of A and Γ_2 given in Eq. (6b), and it is precisely this leading behavior of A (Ref. 1) and Γ_2 (Ref. 4) which has been absent in previous detailed estimates of the form factors with charged fermion constituent.

Another approach to the calculation of form factors is provided by a direct examination of the behavior of diagrams in perturbation theory.[13,14] In view of their close connection to the ladder Bethe-Salpeter equation, we might expect that diagrams like Fig. 2 contain asymptotic behavior similar to that above for the form factors. Straightforward calculations show that this is in fact the case. As an illustration, let us trace the origin of the $(q^2)^{-2}\ln^5(-q^2)$ contribution to F_1 with charged fermion. First, consider Fig. 2 with m and n greater than one. The Greek letters labeling the lines represent the usual Feynman parameters

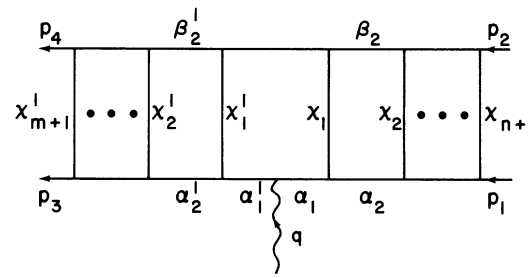

FIG. 2. Five-point ladder diagram related to the form factor.

over which integrals remain after the loop momenta have been eliminated. We want to consider the behavior of this diagram for large values of $-q^2$ with other invariants fixed.[15] If all the lines in Fig. 2 are taken as scalar it is easily seen that the leading asymptotic behavior is $(q^2)^{-2}\ln(-q^2)$. In the terminology associated with parametric integrals, there are two "d paths": (α_1, x_1) and (α_1', x_1').

When the line connecting p_3 and p_1 is a fermion, however, the situation changes and the effects of momentum factors in the numerator must be considered.[16] The presence of these factors results in the introduction of both Feynman parameters and explicit factors of the external momenta into the numerator of the parametric integrals. The extra parameters in the numerator will tend to suppress asymptotic behavior, while the momentum factors will, of course, enhance it. Keeping in mind that eventually we expect to sandwich this diagram between two free Dirac spinors of the bound-state mass and momentum (p_1+p_2) and (p_3+p_4), suppose we evaluate the effect of the product of the four numerator momenta corresponding to the parameters α_2', α_1', α_1, and α_2. We find, among other contributions, a factor like

$$(\not{p}_1+\not{p}_2)(\not{p}_3+\not{p}_4)\gamma_\mu(\not{p}_1+\not{p}_2)(\not{p}_3+\not{p}_4),$$

which effectively becomes $(q^2)^2\gamma_\mu+O(q^2)$ because of the Dirac equation. In the corresponding parametric integral we now find a number of new features. The whole integral is asymptotically of order $(q^2)^{-4}$ so that we get $(q^2)^{-2}$ over all. We find end-point contributions not only from the d paths (α_1, x_1) and (α_1', x_1'), but also from the longer paths (α_1,α_2,x_2) and $(\alpha_1',\alpha_2',x_2')$. These four paths account for three powers of the logarithm of q^2. In addition, the sets of lines $(\alpha_1,\alpha_2,x_2,\beta_2,x_1)$ and $(\alpha_1',\alpha_2',x_2',\beta_2',x_1')$ become "singular configurations" relative to this integral, raising the power of the logarithm of q^2 from three to five. Thus we have found the leading $(q^2)^{-2}\ln^5(-q^2)$ asymptotic behavior in the ladder contributions to the five-point function. The other entries of Eq. (7a) and (7b) may be identified in a similar way. Standard techniques may be used to verify that these contributions to the five-point function survive when poles are produced in the momenta (p_1+p_2) and (p_3+p_4) by summing over infinite sets of graphs.[5,13,14]

The most striking result of this study is the asymptotic behavior of F_1 in the case of a charged-fermion constituent. The proton magnetic form factor, $G_M=F_1+F_2$, is found experimentally to fall by an order of magnitude when $-q^2$ ranges from 7 to 25 GeV^2,[17] while a function like $(q^2)^{-2}\ln^5(-q^2/m_\rho^2)$ $(m_\rho^2\approx 0.71\text{ GeV}^2)$ only decreases by about 30% over this range. Unless the momentum transfers thus far confronted are not truly asymptotic, the agreement between predictions of such models and the data is lost. On the other hand, a recent analysis of the available data suggests that inclusion of a suitable combination of logarithms up to $\ln^4(-q^2)$ actually improves agreement with the data.[18] Future experiments determining the precise power of the logarithm of q^2 in the asymptotic region will be of great interest.

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A simple extension of the Eilam-Gell-Margolis-Meggs statistical picture

J. Schlesinger*

Faculté des Sciences, Université de l'Etat, 7000 Mons, Belgium

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The statistical picture proposed by Eilam, Gell, Margolis, and Meggs for differential cross sections near $\theta_{\text{c.m.}}=90°$ has been extended to the reaction $\bar{p}p\to\pi^+\pi^-$ at all angles. The heuristic value of the model in the case of $\bar{p}p$ interactions is pointed out.

In a recent publication, Eilam, Gell, Margolis, and Meggs[1] (hereafter referred to as EGMM) have applied a simple statistical model to differential cross sections near $\theta_{\text{c.m.}}=90°$. Their results are restricted to $\pi^\pm p$, K^-p, and $\bar{p}p$ elastic scattering though the formalism also applies to two-body inelastic processes. In this note, we propose to extend the EGMM formalism to some $\bar{p}p$ interactions, $\bar{p}p\to\pi^+\pi^-$ being given as an example.

Here is the EGMM picture presented in a nutshell. Assuming the scattering near 90° to be statistical in nature, EGMM describe the differential cross section in this region by an incoherent superposition of resonances. With a Hagedorn-