Critical multiperipheral models*

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A critical meshing principle which fixes the density along the rapidity axis of produced particles is suggested in the context of multiperipheral models. Connections with duality and other implications are discussed.

I. INTRODUCTION

Dual models[1] satisfy various linear constraints (crossing, Regge behavior, duality). These constraints cannot, however, fix the over-all scale of scattering amplitudes, and additional nonlinear unitarity conditions have been utilized in various attempts to fix this scale. These include beside the earlier bootstrap calculations,[2] direct attempts to unitarize Veneziano amplitudes,[3,4] and other approaches based on a multiperipheral bootstrap.[5,6] In this latter case the object of interest is an effective coupling for particle emission g^2,[7] which determines up to some correlation corrections[8-10] the coefficient b in the expected logarithmic increase of the multiplicity with energy

$$n = a + b\ln(s), \tag{1}$$

and which in a certain class of models also determines $\alpha_R(0) - \alpha_P(0)$, the difference between the Pomeron and the other leading Regge singularities ($R=\rho$, ω, f^0, or A_2). An independent determination of $\alpha_R(0) - \alpha_P(0)$ could therefore yield g^2.

In this paper we would like to suggest a "criticality principle" which can fix b and thus effectively also g^2. It requires that b, the density [which in multiperipheral models (MPM's is roughly constant] of particles produced per unit rapidity, will be sufficiently big that pions coming from the decay of low-lying resonances neighboring in rapidity will, on the average, tend to resonate again. This could generate, from the original one-dimensional tree diagrams typical of simple MPM's, two-dimensional "honeycomb" diagrams[11] characteristic of certain approaches to dual models.[12-14] In such models the "partons" propagating inside big planar diagrams are believed to be different from the regular composite hadrons. Only by insisting that the big planar diagrams be regenerated again at the particle level do we inject an additional requirement fixing g^2.

The suggestion that an intricate overlapping-resonance situation may obtain in multiple particle production has been made before.[15,16] While no detailed specific scheme seems to work, the b predicted in Sec. II from the criticality principle agrees with experimental multiplicity. The criticality principle leads also to some additional theoretical possibilities discussed in Sec. III. Since MPM's are basic to our approach we briefly review in Sec. IV some recent evidence pertaining to their validity, where we also delve briefly into the distinction between "simple" and dual (or multi-Regge) MPM's. We close with some speculation about a possible deeper significance of the criticality principle.

II. CRITICALITY PRINCIPLE FOR MPM's

To illustrate our suggestion consider a simple (ABFST-like) MPM[17] where we produce, say, via π exchange, a string of low-energy resonances of mass m_R which decay into π pairs and we temporarily neglect the possible variety of resonance types and/or their internal and spin-parity quantum numbers. The model attains its simplest form in the weak resonance-$\pi\pi$ ($g_{R\pi\pi}$) coupling limit.[18,19] In this case interference terms of the type shown in in Figs. 1(a) and 1(b) are negligible.[19] This is so because the resonances are produced with a small density along the rapidity

$$y_R = \tfrac{1}{2}\ln\frac{E^R + P_\parallel^R}{E^R - P_\parallel^R}$$

or

$$y_R = \ln\frac{E^R + P_\parallel^R}{(m_R^2 + P_\perp^2)^{1/2}}$$

axis and are strongly ordered, and also pions which emerge from different resonances have on the average subenergies considerably in excess of m_R^2, so that they do not resonate.[20]

Retaining then only the graph of Fig. 1(c), one finds in the weak-coupling limit a Poisson distribution for the cross section for production of n_R resonances

$$\sigma_{n_R}(Y) \approx \frac{(g^2)^{n_R} Y^{n_R}}{n_R!}, \tag{2}$$

where

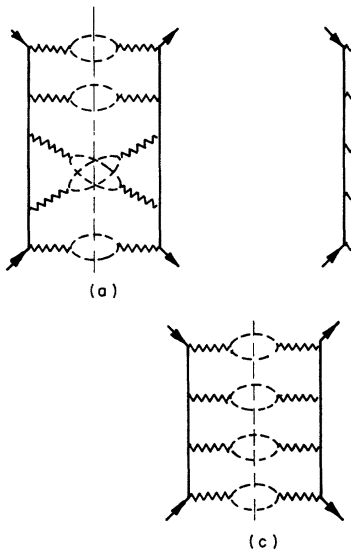

FIG. 1. (a), (b) Interference terms which are usually neglected in the MPM in the weak-coupling limit; (c) the "diagonal" term which is retained.

$$Y=\ln(s/m_R^2) \tag{3}$$

is the available rapidity interval. The average density per unit rapidity is thus

$$b_R=\langle n_R\rangle/Y=g^2 \ . \tag{4}$$

As we increase g^2 this picture becomes less and less applicable. The joint effect of increase density b_R and width Γ_R makes it more and more likely that two pions, each of which emerged originally from the decay of different neighboring (or next-to-nearest-neighboring) resonances on the rapidity axis, will resonate again. By continously increasing g^2 (and b_R) we can arrive at a "critical" situation where for every resonance originally produced a second resonance is created by reflections.

A particularly simple, though unrealistic, scheme which may illustrate the critical situation is shown in Fig. 2, where each pair of π's emitted by neighboring resonances resonate again.

Note that the sequence in Fig. 2(a) is suggestive of a particular way of approaching the critical situation, namely through a continuous deformation of the weak-coupling limit. Indeed the very description which we adopt for the purpose of our estimate as a sequential process, involving first a production of resonances and next their independent decay and reformation of resonances by reflections, in conceptually an extrapolation from the weak-coupling limit to the quite different situation of optimal meshing, strong correlations, etc. This is reminiscent of an attempt to extrapolate the state equation of a rare phase all the way down to a dense phase and may be a reasonable zero-order approximation. (See Fig. 3 for an illustration of the approximation involved.)

In order to optimize the probability of reflections into the resonance region the invariant subenergy $S_{n,n+1}$ of the nth and $(n+1)$th resonances has to equal $6m_R^2-8m_\pi^2$. In this case the average invariant mass squared of any of the six possible pairs formed from the 4 π's in R_n and R_{n+1} is m_R^2. Since

$$S_{n,n+1}=2m_R^2+2m_\perp^2\cosh(\delta y)-2p_{\perp R}^2\langle\cos\varphi_{n,n+1}\rangle ,$$

where $m_\perp^2=m_R^2+p_{\perp R}^2$ is the average transverse mass squared of the resonances and $\delta y=y_R^{n+1}-y_R^n$ is the average rapidity spacing between the resonances, we have (after neglecting the $\langle\cos\varphi_{n,n+1}\rangle$ term in $S_{n,n+1}$ which tends to increase n_R by 5–10%)

$$\cosh\delta y=\frac{4m_R^2-8m_\pi^2}{2(m_R^2+\langle p_{\perp R}^2\rangle)} \ . \tag{5}$$

Taking $m_R^2=0.5\ (\text{GeV}/c^2)^2\approx m_\rho^2, m_\sigma^2$, and $p_\perp^2=0.2\ (\text{GeV}/c)^2$, we obtain $\delta y=0.83$ and we can estimate the total number of resonances produced as $n_R=Y/\delta y$. Since each resonance decays into two pions, of which $\frac{2}{3}$, on the average, are charged, we expect the following logarithmic growth of charged prong multiplicity:

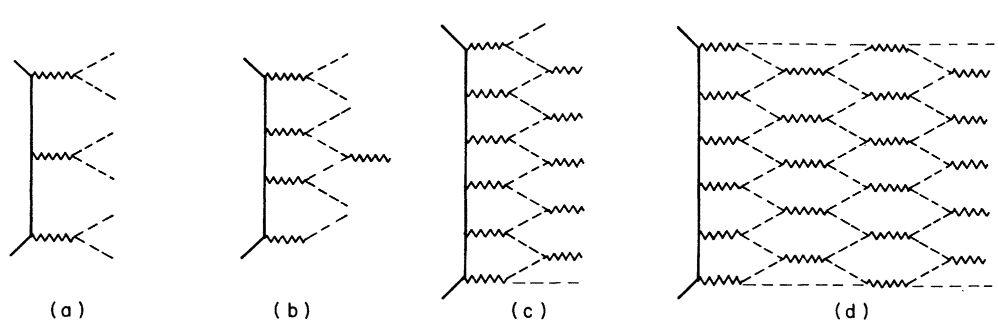

FIG. 2. The increased density through (a) the weak-coupling situation, (b) an intermediate coupling, and (c) the critical coupling. (d) The multiple rescattering which generates in the critical case "honycomb"-like diagram.

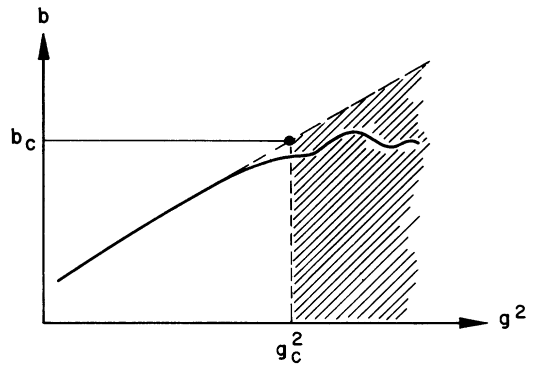

FIG. 3. A qualitative description of the extrapolation of the simple equations describing the weak-coupling model (rare phase) all the way down to the transition region to the honeycomb model (dense phase). As an illustration we consider the linear dependence of b on g^2.

$$\langle n_{\rm ch}\rangle \sim \text{const} + \frac{2\times\frac{2}{3}}{0.83}\ln\left(\frac{s}{s_0'}\right) = 1.6\ln\left(\frac{s}{s_0}\right) . \qquad (6)$$

The value 1.6 for the coefficient of the logarithmic increase which is "predicted" here agrees well with the value (1.65) extracted from high-energy proton-proton scattering[21,22] and the value (1.6) which we extract from the recent $\pi^- p$ measurement[23] at NAL with $n_c = 8$ at $P = 200$ GeV/c as compared with $\langle n_{\rm ch}\rangle = 5.8$ at $P = 50$ GeV/c (Serpukhov data[24]).

A larger value of $\langle n_c\rangle \approx 2\ln(s/s_0)$ is obtained if we assume that only about 80% of the cross section are governed by MPM's and the diffractive remainder corresponds to finite multiplicities only.[22]

Independently of any detailed model and value, the universality of the coefficient in pp and p collision[25] is a good omen for the general multiperipheral approach, the Feynman gas analog or the Mueller pionization diagrams. The choice $\langle P_{\perp R}^2\rangle = 0.2$ is estimated from the experimental π spectrum. Strictly speaking our picture also constrains $\langle P_{\perp R}^2\rangle$. If $\vec{K}$ is the 3-momentum of each of the two neighboring resonances in their c.m. system, then

$$6m_R^2 - 8m_\pi^2 = S_{n,n+1}$$
$$= 4m_R^2 + 4\vec{K}^2 ,$$

so that $\vec{K}^2 = \frac{1}{2}m_R^2 - 2m_\pi^2$. At such small K's we can expect only $l = 0, 1$ in the relative orbital excitation of the resonances,[26] so that spatial distribution of K may be roughly isotropic and

$$\langle P_{\perp R}^2\rangle \approx \tfrac{2}{3}\vec{K}^2 \approx \tfrac{1}{3}m_R^2 - \tfrac{4}{3}m_\pi^2 \quad (\approx 0.16 \text{ for } m_R^2 = \tfrac{1}{2}) .$$

Using this in Eq. (5) and neglecting m_π we get an estimate $\cosh\delta y = \frac{3}{2}$ (independently of m_R) which yields $n_{\rm ch} = 1.4\ln s$. In general a 20–30% variation of the multiplicity results from reasonable variations of the parameters in Eq. (5).

Considering the crudeness of our approach the agreement with the observed multiplicity (and the transverse momenta) seems to be quite fortuitious. It is nonetheless encouraging and suggests that the criticality conjecture may be viable. In order to prove this convincingly it is not enough to show that that average invariant masses are equal to the resonance energies. We also have to consider the widths of the mass distributions in comparison with the resonance widths. To include isospin (and exclude exotic, say, $\pi^+\pi^+$ resonances[27]), consider the possibilities of next-to-nearest-neighboring resonances as well as resonances which decay into three pions. Some preliminary attempts along this direction using a Monte Carlo program have been made and indicate that the criticality condition of one reflection resonance on the average per initial resonance can indeed be achieved, without necessarily adhering to a completely ordered rigid

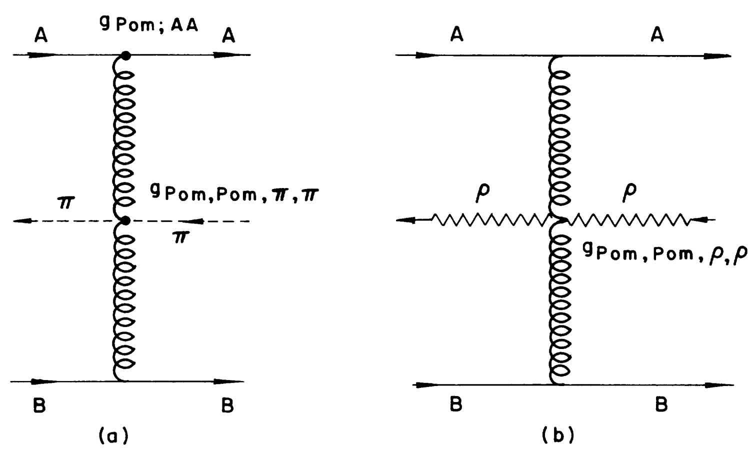

FIG. 4. (a) Mueller diagrams for pionization. (b) The corresponding Mueller diagram for production in the plateau region.

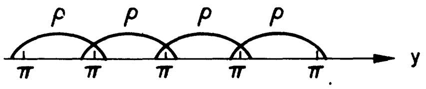

FIG. 5. Inclusive π and ρ count in an idealized critical situation.

structure which has not been revealed experimentally.[28]

III. ADDITIONAL COMMENTS ON CRITICAL MODELS

In terms of Mueller diagrams[29] it was conjectured that the inclusive cross section for pions in the pionization region is given by Fig. 4(a). For a factorizing Pomeron

$$\frac{d\sigma(\pi)}{dy(\pi)}\Big/\sigma_{\text{tot}}=g_{\pi\pi,\text{Pom Pom}}$$

the plateau height, which is proportional to the b coefficient in the multiplicity equation (1), is independent of the colliding particles (A,B).

A priori it seems quite unlikely that we will have the same plateau height for ρ production in the central region, though the relevant Mueller diagram 4(b) looks deceptively similar to 4(b) and $g_{\pi\pi,\text{Pom Pom}}\approx g_{\rho\rho-\text{Pom Pom}}$ seems a nice relation to have. However, even if all π's are produced via ρ's we have only half as many ρ's. This argument is, however, invalid. The inclusive cross section for ρ's counts all π pairs within the resonance region, original and reflection resonances alike. In the critical meshing situation we obtain therefore twice as many ρ' s, and $d\sigma_\rho/dy_\rho=d\sigma_\pi/dy_\pi$ becomes possible. (See Fig. 5.)

This could be extended beyond 2π resonances, i.e., grouping of pions in triads and quartets along the rapidity axis. However, unless we allow more and more nonplanar triplet, quartet, etc. configurations to resonate, the argument applies only to resonances with large masses $m\sim m_0e^{r\delta y}$ for resonances decaying into r mesons.

Note that no similar mechanism can correct for the inequality between inclusive π and K cross sections.

It may be worth pointing out that dual quark models strongly suggest optimal meshing. We view the production process as an excitation of a long $\bar{q}q$ string, an optimal uniform rapidity distribution of alternating q's and $\bar{q}$'s (Fig. 6). Again an overlapping resonance holds, but without restriction

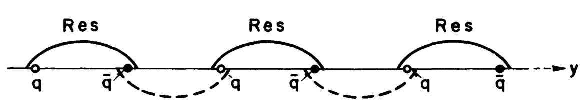

FIG. 6. A $\bar{q}q$ string illustrating the "meshed" resonance effects.

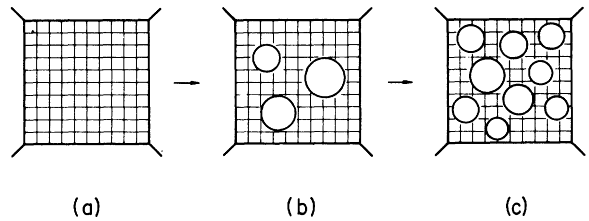

FIG. 7. Regeneration of big planar diagram propagating composite particles through unitarity corrections.

to specific particles π and ρ as in our earlier discussion.

At the level of dual models in general we can view our principle as a sort of "automodality"[30] demanding that real hadrons be as efficient in building large planar diagrams characteristic of certain approaches to duality as the basic partons themselves. This is roughly illustrated in Fig. 7. If in the original planar diagrams 7(a) we put more and more holes (indicative of unitarity corrections)[31] we wind up with a new structure [7(c)] similar to the original fishnet diagram except that its propagators are those of compound hadrons and not of partons. It is well known that viewed in the t channel the MP diagrams are ladders which may lead to bound states (Regge poles). Naively, in computing these bound states one has to add the potentials generated by exchanges of π's, ρ's, etc. In the weak-coupling-limit s-channel picture this corresponds to independent emissions of π's and ρ's in arbitrary sequence. This is necessarily modified in the critical situation since the same honeycomb diagram can be viewed either as a multiple π [Fig. 8(a)] or a multiple ρ [Fig. 8(b)] exchange. The obvious distinction between single ρ exchange and much-longer-range π exchange may thus tend to fade for multiparticle exchanges.[32]

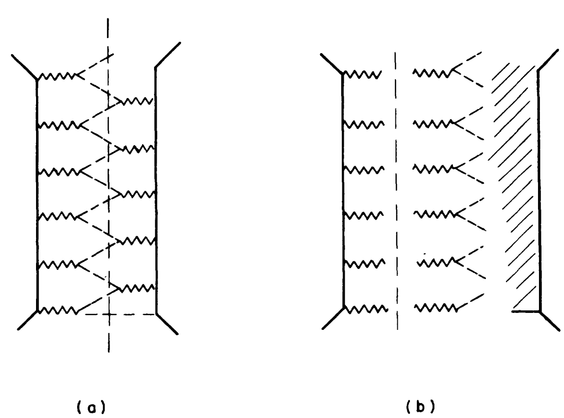

FIG. 8. Multiple π or multiple ρ exchange generated by different cuts of the same honeycomb diagram.

IV. EVIDENCE FOR AND SOME COMMENTS ON MULTIPERIPHERAL MODELS

Several authors[33–35] analyzing recent high-energy pp data find a dominant nondiffractive component with Poisson-like multiplicity distributions (i.e., indicative of short-range correlations). As mentioned above, the NAL $\pi^- p$ data seem to have very similar characteristics; in particular, the same b [of Eq. (1)] appears in pp and πp data. All these support the MPM which was the basis of our discussion. In the somewhat general context of MPM's the following comments may be relevant. If the Poisson distribution for n_R, the number of independently produced objects, applies for $n_R = 0$ then

$$\frac{\sigma^{n_R=0}(\text{nondiff.})}{\sigma_{\text{tot}}} = \frac{cRS^{2\alpha_R-2}}{\sigma_{\text{tot}}} = e^{-\langle n_R\rangle} \approx \left(\frac{s}{s_0}\right)^{-b_R}, \tag{7}$$

where $\langle n_R\rangle = b_R \ln(s/s_R)$ is the average multiplicity of these objects. $\sigma^{n_R=0}$(nondiff.) contains only $\rho'\rho'$-cut or charge-exchange contributions and not the largely diffractive completely elastic amplitude. $\alpha_R(0) \approx \frac{1}{2}$ implies $b_R \sim 1$, which is $(0.3-0.4)b$ being the coefficient of total multiplicity.[36] This is suggestive of independent emission of pion pairs or triplets[37] in qualitative agreement with our starting point of resonance production in the weak-coupling limit.

If the Poisson distribution applies also separately to the first (τ) component[38] (in the Harari-Freund sense) which is dominated by s-channel resonances, then

$$\frac{\sigma^{n=0}(\text{nondiff.})}{\sigma^{I}_{\text{tot}}} \approx \frac{s^{2\alpha_2-2}}{s^{\alpha_2-1}} = e^{-\langle n'\rangle_R}, \tag{7'}$$

where $\langle n'_R\rangle = b'_R \ln(s/s'_R)$ is now the number of objects emitted on the average from resonance decays. Equations (7) and (7′) yield $b'_R = \frac{1}{2}b_R$ (Ref. 39) or $b' = \frac{1}{2}b$; i.e., the logarithmic increase in π multiplicity from resonance decays is $1.2-1.5\ln s$, half as big as the corresponding multiplicity for the complete process. Indeed n_π(resonances) $\sim(1-1.3)\ln(s/s_0)$ fits the increase of multiplicity through the f^0 and g resonances.[40]

In general for any MPM any information such as the charge state of the incident particle A, its polarization, etc., fades exponentially, $\sim e^{-\lambda(y_A-y)}$, as we proceed along the multiperipheral chain, where $\lambda = \alpha(0) - 1$ and $\alpha(0)$ is the intercept of the trajectory associated with the particular information considered. In particular, I_3 information corresponding to

$$\frac{d\sigma_{\pi^-}}{dy} - \frac{d\sigma_{\pi^+}}{dy} \sim \frac{dQ}{dy}$$

should fade like $e^{[\alpha_\rho(0)-1]y} = e^{-y/2}$. This follows from Mueller diagrams with a factorizing Pomeron pole,[29] from the parton picture,[40] and directly in the MPM itself.[38] Preliminary data on dQ/dy (Ref. 41) in πp reactions confirm these expectations.[42] Recently a thorough analysis of such quantities has been made by Benecke *et al.*[43] In that analysis it was also found that ΔQ, the difference in charge between an incident particle and all final-state particles moving in the same hemisphere in the c.m. system, is strongly limited to $\Delta Q = 0$, (Ref. 44), consistent with a MPM and nonexotic exchanges. Also the rough equality of $\Delta Q = 0$ and $\Delta Q = 1$ is suggestive of the dominance of the MPM over a simple diffractive picture.

Let us consider two particles emitted in the mth and lth locations in a simple MPM. A correlation among the transverse momenta involves, roughly, speaking, a flow of momenta in the $\Delta\pi$ propagators separating the two particles (Fig. 9), and therefore drops like

$$\left[\frac{1}{\langle t_{\text{av}}\rangle + (\Delta Q)^2}\right]^{\Delta n} \sim e^{-\lambda\Delta n}.$$

Indeed, for all quantities of interest we have primarily an exponential decay with the number of intervening particles on the chain. Since, on the average, $\Delta n \sim b\Delta y$, the correlation behaves like $e^{-\lambda'\Delta y}$.

That result would always be true (not only in an average sense) if we have a multi-Regge model in which, because of the shrinkage effect, the product of the propagators is always proportional to $e^{(\alpha-1)\Delta y}$. In principle, using completely identified multiparticle production events, one could check which (Δn or Δy) is more relevant.

Theoretically, the simple MPM tends to predict (in the simple weak-coupling limit) larger slopes $[\alpha'(0)]$ for the trajectories with larger intercepts.[45] Alternatively this can be seen from the discussion above—the lower the intercept, the smaller is $b'_R \ln s$, the increase of the multiplicity "associated" with this trajectory, and hence by the well-known random-walk argument it corresponds in impact space to a more slowly ($\sim b'_r \ln s$) increasing disk, i.e., smaller $\alpha'(0)$.

This difficulty can be avoided in a multi-Regge model in which the individual step sizes $\langle b_i^2\rangle$ in-

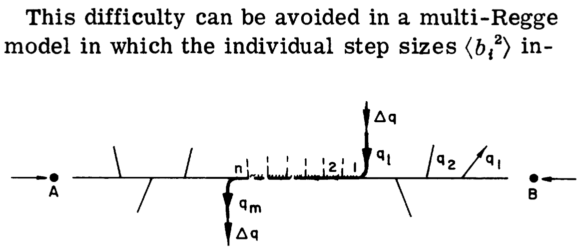

FIG. 9. The transverse momentum flow required in order to have a correlation between q_l and q_m.

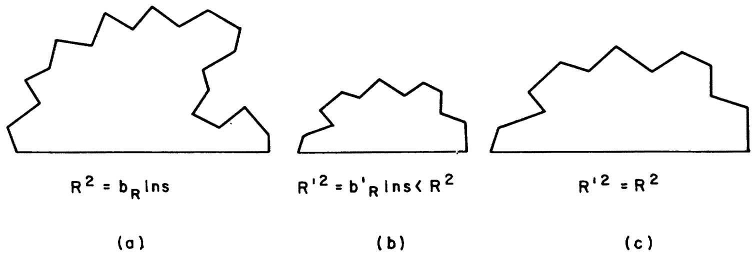

FIG. 10. The comparison between (a) the standard random-walk length, (b) the walk length which results from a random walk with fewer steps ($b'_R \ln s$) but the same step length (the usual MPM), and (c) the random walk with fewer steps but larger step length (the multi-Regge model).

crease with the corresponding Δy_i's (Fig. 10) and a consistent universal slope is reproduced.[46]

V. SUMMARY AND CONCLUSIONS

We have presented above a conjecture on a criticality condition and have seen that the expected multiplicities (and transverse momenta) are consistent with the experimentally observed values.

The criticality principle can in no way be compared with the really fundamental principles of particle physics, such as causality and unitarity. It is admittedly quite vague and at best can be a helpful guideline in a limited range of physical phenomena. Nonetheless there is the rather intriguing and speculative possibility that the criticality principle and the "ordered (or partially ordered) structure" which results for a sufficiently large coupling can serve to define strong interactions.

Also, in turn, it may well be that the strength of the strong interactions derives from the ordered structure and the possibility of multiple rescattering which enhances the amplitude. Insofar as there is no reason to believe that this situation will persist over much smaller scale lengths, hadronic matter in the small (e.g., when explored in very deep-inelastic lepton-hadron processes) may not exhibit any of this ordered structure and for that matter may be indistinguishable from purely weakly interacting matter.

All this is very speculative but could, we hope, encourage more thorough investigations of the critical-coupling possibility.

ACKNOWLEDGMENTS

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[36]The connection between multiplicity and $\alpha_{P'} - \alpha_P$ is originally due to Chew and Pignotti (Ref. 7).
[37]Effects due to correlations have been considered in Refs. 8–10. Indeed, a good result for α_R with only weak correlation was found in Ref. 33 when the number of negatively charged tracks is considered (consistent with earlier claims of C. P. Wang that a Poisson distribution works very well for the number of charged pairs produced). This has nothing to do with the obvious fact that the formalism of Ref. 33 is self-consistent in the sense that if instead of the number of negative tracks we considered the total number of charged (or all) pions produced, the assumption of independent π^- emissions induces just the correct amount of correlations so as to reproduce the same α_R. (I would like to thank H. Harari for an enlightening discussion on this point.)
[38]One can motivate this by noting that the first component can be treated by selecting $I_t = 1$ processes for which a separate multiperipheral model can be set up [see, e. g., S. Nussinov, Phys. Rev. D 5, 2221 (1972)].
[39]In Ref. 10 and also in the similar work of Huan Lee [Phys. Rev. Lett. 30, 719 (1973)] independent theoretical considerations yield $b'_R = \frac{1}{2} b_R$, which then implies $\alpha_P(0) = 1$, a result implicitly assumed above.
[40]The smaller growth of total multiplicity at lower energy could by due in part to the importance of the resonance component. Alternatively we could consider the distribution $\sigma(\pi^- p \to n \text{ prongs}) - \sigma(\pi^+ p \to n \text{ prongs})$. In spite of the large errors in these quantities, the preliminary analysis of Ref. 38 is consistent with an average multiplicity considerably smaller than that of $\sigma(\pi p \to n \text{ prongs})$.
[41]L. Susskind, Phys. Rev. D 6, 894 (1972); H. D. I. Abarbanel, Phys. Lett. 34B, 69 (1971).
[42]A. Casher, S. Susskind, and S. Nussinov, 1972 (unpublished).
[43]J. Benecke, Lecture at Minerva Symposium at Weizmann Institute, 1973 (unpublished).
[44]The variable ΔQ (together with several other multiparticle variables) has been considered by M. Foster *et al.* [Phys. Rev. D 8, 3848 (1973)]. Unfortunately the data considered there were at lower energies and smaller statistics than those of J. Benecke *et al.*
[45]This can be read off directly directly from $\alpha(t) = \alpha(g^2 = 0) + g^2 k(t)$. See, e.g., R. Eden, P. Landshoff, D. Olive, and J. Polkinghorne, *The Analytic S Matrix* (Cambridge Univ. Press, New York, 1966), with $k(0) > 0$ and $k'(0) > 0$.
[46]Obviously $\alpha'_{\text{Pom}}(0) \approx \frac{1}{2} \alpha'_{\text{Regg}}(0)$ calls for special explanation.