[17]Despite the fact that t is negative, by definition $d\sigma^{(n)}/dt$ is positive. The variable of integration we have chosen to be $|t|$, and the lower limit of the integral is 0, while, for example, the upper limit will be $(s-4m^2)$ if the reaction is of type (54).

[18]A. Białas and K. Zalewski, Nucl. Phys. B42, 325 (1972).

[19]E. L. Berger and A. K. Krzywicki, Phys. Lett. 36B, 380 (1971).

[20]See, for example, H. A. Kastrup, in *Particles, Currents, and Symmetries*, edited by P. Urban (Springer, New York, 1968).

[21]For the elastic scattering ($n=0$) in small $|t|$ region, this is simply the "shrinkage of the diffraction peak." See, for example, D. Horn and F. Zachariasen, *Hadron Physics at Very High Energies* (Benjamin, Reading, Mass., 1973).

[22]Within the framework of the ordinary bremsstrahlung model (describing only the gross features of high-energy collisions) we can easily see how the violation of inequality (C11) can develop by analyzing the elastic differential cross section [which is obtained from (10) by setting $n=0$]. In fitting experimental data one usually writes

$$d\sigma^{el}/dt=(d\sigma^{el}/dt)_{t=0}\exp[-a(s,t)],$$

where in a small $|t|$ region $a(s,t)=\alpha(s)|t|-\beta(s)|t|^2+O(|t|^3)$ with $\alpha(s)|t|\lesssim 10$ and $0\le\beta(s)/\alpha^2(s)<<1$. [See, for example, L. Van Hove, Rev. Mod. Phys. 36, 655 (1964).] On the other hand, from the ordinary bremsstrahlung model we have $d\sigma^{el}/dt=(d\sigma^{tot}/dt)W_0$ where, as mentioned in the text, $d\sigma^{tot}/dt$ has a smoother dependence on $|t|$ than W_0 has. Consequently, we approximate W_0 as $W_0(s,t)\approx\exp[-(1-\epsilon)a(s,t)]$, where the factor $1-\epsilon$ $(0<\epsilon<<1)$ takes into account the fact that $d\sigma^{tot}/dt$ also depends on $|t|$, although not as much as W_0. (That this form for W_0 is consistent with the ordinary bremsstrahlung model was shown in the second paper of Ref. 3.) So we have

$$\frac{W_0(s,-\langle|t|\rangle)}{\langle|t|\rangle^2 d^2W_0(s,-\langle|t|\rangle)/d\langle|t|\rangle^2}\approx\frac{1}{\langle|t|\rangle^2\left\{(1-\epsilon)^2\left[\dfrac{da(s,-\langle|t|\rangle)}{d\langle|t|\rangle}\right]^2+(\epsilon-1)\dfrac{d^2a(s,-\langle|t|\rangle)}{d\langle|t|\rangle^2}\right\}}.$$

In a low-s region (where $\langle|t|\rangle$ by virtue of kinematics is required to be small), $\langle|t|\rangle\,(da(s,-\langle|t|\rangle)/d\langle|t|\rangle)$ is a small number, and, since $d^2a(s,-\langle|t|\rangle)/d\langle|t|\rangle^2$ can be neglected, we see that (C11) can be satisfied. However, in a high-s region, neither can $d^2a(s,-\langle|t|\rangle)/d\langle|t|\rangle^2$ be necessarily neglected nor is $\langle|t|\rangle\,(da(s,-\langle|t|\rangle)/d\langle|t|\rangle)$ small, and we see that (C11) should eventually be violated.

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Bjorken scaling from bounded transverse momenta and pion multiplicity

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A model for deep-inelastic ep scattering is presented, where the virtual photon propagates as a vector-meson state, emitting pions until it loses most of its momentum transfer. It then interacts with the nucleon. The nucleon receives a finite-momentum-transfer kick that depends on the scaling variable ω, but not on q^2. If the average multiplicity of the emitted pions now grows essentially as $\ln s$, then Bjorken scaling results. Quarks forming a nucleon core will not emerge as free particles in this model, since the core is only elastically scattered.

Present SLAC experiments have established that the deep-inelastic electron-proton scattering exhibits Bjorken scaling.[1] This has been widely interpreted, following Feynman, as being due to the highly virtual photon interacting with pointlike constituents of the nucleon.[2,3] Such an interpretation in turn raises the question: Why are such pointlike constituents (partons) not observed? Recent investigation[4] indicates that even with final-state parton-parton interactions it is difficult to explain why quark-parton quantum numbers are not seen experimentally.[5] In this paper an alternative interpretation of the Bjorken scaling phenomenon is proposed where quarks can form a nucleon core and do not emerge as free particles.

Our model is shown in Fig. 1 where the photon first converts to a vector meson ρ^0 or ω (for simplicity we disregard the ϕ meson). The vector meson, say, ρ^0, emits a pion and becomes ω; ω then emits a pion and becomes ρ^0, and the process continues. Finally the vector meson interacts with the incoming nucleon and scatters it.[6] We work in the ep c.m. system and take the direction of the initial nucleon momentum $\vec{P}$ as the positive z axis. We assume that the transverse momenta of all the produced hadrons in the ep c.m. system are bounded.

Our interest is in the electron-proton c.m. energy going to infinity, and so we introduce the plus and minus components suitable for infinite-momentum frame discussions. The plus and minus components of a four-vector $a_\mu = (a_0, \vec{a}_\perp, a_3)$ are defined by $a_+ = \frac{1}{2}(a_0 + a_3)$, $a_- = a_0 - a_3$, and we obtain $a \cdot b = a_+ b_- + a_- b_+ - \vec{a}_\perp \cdot \vec{b}_\perp$. In the ep c.m. system, the initial proton brings in an infinite plus component P_+, and the initial electron brings in an infinite minus component p_-. We now examine the over-all kinematic description of the model and see how the initial infinite +, − components are conserved.

Our assumption of bounded transverse momenta implies $q_+ \simeq -P_+/\omega$,[7] where $\omega = -2P \cdot q/q^2$ is the scaling variable. In our model the $J = 1$ photon state that propagates along the chain in Fig. 1 carries a finite fraction $1/\omega$ of the infinite plus component P_+ of the incident nucleon and thus accounts for the relation $q_+ \simeq -P_+/\omega$. The rest of the plus component $P_+(1 - 1/\omega)$ of the incident nucleon, as we see below, is carried off by the final nucleon. Furthermore, since $p_+ = m_e^2/2p_- \to 0$ and $q_+ = (p - p')_+ \simeq -p'_+$, the final electron has infinite plus component $p'_+ \simeq P_+/\omega$, and moves along the positive z axis. Regarding the minus component p_-, we have $p_- = q_- + p'_- \simeq q_-$ $[p'_- = (p_\perp'^2 + m_e^2)/2p'_+ \to 0]$. Hence the infinite minus component of the incident electron is taken up by the photon. Each of the emitted pions then carries off a finite fraction of this infinite minus component,[8] so that $q_- \simeq \sum_{i=1}^{n} k_{i-}$. The pions are therefore photon fragments in the present model. Since each pion has a large minus component and the transverse momentum is limited, its plus component ($k_{i+} = m_{i\perp}^2/2k_{i-}$, $m_{i\perp}^2 = k_{i\perp}^2 + \mu^2$) is negligible. Therefore, K_+ ($K = \sum_i k_i$, total 4-momentum of the pions) is negligible, and we get from 4-momentum conservation $P' + K = P + q$ the result $P'_+ \simeq P_+ + q_+ \simeq P_+(1 - 1/\omega)$. From these considerations we also obtain

$$\begin{aligned}(P' - P)^2 &= 2M^2 - 2(P'_+ P_- + P'_- P_+) \\ &\simeq 2M^2 - (1 - 1/\omega)M^2 - \frac{P_\perp'^2 + M^2}{1 - 1/\omega} \\ &\simeq -\frac{P_\perp'^2 + M^2/\omega^2}{1 - 1/\omega}\,.\end{aligned}$$

This result shows that because of the limited

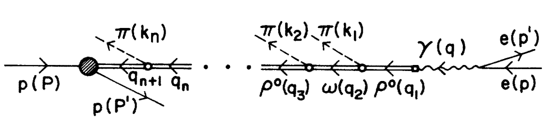

FIG. 1. Model of deep-inelastic ep scattering studied in this paper. The photon converts to a vector meson, say, ρ^0. ρ^0 emits a pion and becomes ω; ω then emits a pion and becomes ρ^0, and the process continues. Finally a vector meson interacts with the incoming nucleon.

transverse momenta, the momentum transfer $(P' - P)^2$ remains finite even when $q^2 \to -\infty$, and is a function only of $P_\perp'^2$ and the scaling variable ω.

As mentioned above, the energetic pions have vanishingly small plus components and so $q_{i+} \simeq q_+$ ($i = 1, 2, \ldots, n+1$). In other words, all plus components of the propagating photon state in Fig. 1 are equal. Because each q_{i-} (except $i = n+1$) is a finite fraction of q_- of the photon and $q_{i+} \simeq q_+$, we find that the momentum transfer $q_i^2 \simeq 2q_{i+} q_{i-}$ ($i = 1, \ldots, n$) for a propagator is a finite fraction of q^2 and so is infinitely large as $q^2 \to -\infty$. This shows that even though our model as described in Fig. 1 appears like a multiperipheral model, in reality it is not, since in multiperipheral models the momentum transfer along the chain is always bounded.[9] One may wonder at this point whether, if each q_i^2 tends to be infinitely large, then every time we introduce a vector propagator, the matrix element in Fig. 1 will be highly damped. However, as we shall see, this does not happen. The reason is that each $VV\pi$ vertex, being a derivative coupling, brings in two components of q in the numerator. The large +, − components in the denominators arising from the propagators are canceled by the large +, − components in the numerators coming from the vertices.

The diagram in Fig. 1 gives us the following matrix element of the current $J_\mu(x)$, where $J_\mu(x)$ is the hadronic part of the electromagnetic current:

$$\begin{aligned}&\langle k_n, \ldots, k_1; P' | J^\mu | P \rangle \\ &\quad = (-1)^{n+1} f_{\gamma V} f^n \frac{g^{\mu\rho_1}}{q_1^2 - m^2} \epsilon_{\rho_1\sigma_2\alpha_1\beta_2} q_1^{\alpha_1} q_2^{\beta_2} \frac{g^{\sigma_2\rho_2}}{q_2^2 - m^2} \\ &\qquad \times \cdots \frac{g^{\sigma_{n+1}\lambda}}{q_{n+1}^2 - m^2} \langle P' | J_\lambda^V | P \rangle \prod_{i=1}^{n} \frac{1}{(2k_{i0})^{1/2}}\,; \quad (1)\end{aligned}$$

here $\langle P' | J_\lambda^V | P \rangle$ is the vector-meson current between the nucleon states, and we have taken the $VV\pi$ vertex as a point coupling: $f \epsilon_{\rho\sigma\alpha\beta} \epsilon_1^\rho \epsilon_2^\sigma q_1^\alpha q_2^\beta$.[10] To evaluate Eq. (1) in the ep c.m. system, we keep only the +, − momentum components at each $VV\pi$ vertex, since only these are the dominant components in the asymptotic region we are interested in ($q_+ \to -\infty$, $q_- \to \infty$). This gives[11]

$$\begin{aligned}&\langle k_n, \ldots, k_1; P' | J^\mu | P \rangle \\ &\quad = (-1)^{n+1} f_{\gamma V} (\tfrac{1}{2} f)^n \frac{k_{1-} k_{2-} \cdots k_{n-}}{q_{1-} q_{2-} \cdots q_{n-}} \frac{1}{q_{n+1}^2 - m^2} \\ &\qquad \times T^\mu(P', P) \prod_{i=1}^{n} \frac{1}{(2k_{i0})^{1/2}}\,, \quad (2)\end{aligned}$$

where

$$T^\mu(P',P)=(-1)^{n/2}[g^{\mu 1}\langle P'|J_1^V|P\rangle + g^{\mu 2}\langle P'|J_2^V|P\rangle] \quad (n \text{ even}), \tag{3a}$$

$$=(-1)^{(n-1)/2}[g^{\mu 1}\langle P'|J_2^V|P\rangle - g^{\mu 2}\langle P'|J_1^V|P\rangle] \quad (n \text{ odd}), \tag{3b}$$

$$=0 \quad (\mu=0,3). \tag{3c}$$

Using the relation $q_{r-}=\sum_{i=r}^n k_{i-}$ and summing over all possible permutations of the emitted mesons, we obtain from (2)

$$\langle k_n,\ldots,k_1;P'|J^\mu|P\rangle = (-1)^{n+1} f_{\gamma V}\frac{1}{(P'-P)^2-m^2}T^\mu(P',P)\,(\tfrac{1}{2}f)^n \times \prod_{i=1}^n \frac{1}{(2k_{i0})^{1/2}}. \tag{4}$$

We find that all +, − components in the numerator and in the denominator have canceled out, and that each pion simply provides a multiplicative factor. The inelastic structure tensor $W^{\nu\mu}$ is now given by

$$W^{\nu\mu}=(2\pi)^3\frac{P_0}{M}\sum_n \delta(P+q-P'-K)\langle P|J^\nu|P';k_1,\ldots,k_n\rangle\langle k_n,\ldots,k_1;P'|J^\mu|P\rangle$$
$$=\frac{1}{2\pi}\frac{P_0}{M}\sum_n\int\frac{d^3P'}{(2\pi)^3}d^4x\,e^{i(P+q-P')\cdot x}f_{\gamma V}{}^2\frac{1}{[(P'-P)^2-m^2]^2}T^\nu(P',P)^*T^\mu(P',P)(\tfrac{1}{2}f)^{2n}\frac{1}{n!}\prod_{i=1}^n\int\frac{d^3k_i}{2k_{i0}(2\pi)^3}e^{-ik_i\cdot x}. \tag{5}$$

Equation (5) shows that the pions contribute only to phase space and the energy-momentum conservation. So the production of pions in this model is completely statistical.

It is convenient at this point to construct the invariants $A(\nu,q^2)=g_{\mu\nu}W^{\nu\mu}$ and $B(\nu,q^2)=P_\nu W^{\nu\mu}P_\mu/M^2$, in terms of which the usual inelastic structure functions $W_1(\nu,q^2)$ and $W_2(\nu,q^2)$ can be expressed.[12] Because of Eq. (3c), $W^{\nu\mu}$ given by Eq. (5) vanishes whenever $\nu,\mu=0,3$; in other words, only the transverse components of the virtual photon contribute. Since $P_\mu=(P_0,\vec{0},P_3)$, we get $B=0$. It then follows that $2MW_1=\omega\nu W_2$, i.e., $\sigma_L/\sigma_T=0$.

From Eq. (5) the invariant $A_n(\nu,q^2)$ corresponding to the production of n pions is

$$A_n=-\frac{1}{2\pi}f_{\gamma V}{}^2\int\frac{M}{P_0'}\frac{d^3P'}{(2\pi)^3}\mathfrak{F}(q'^2;P_\perp'^2)\frac{1}{n!}\times\int d^4x\,e^{i(P+q-P')\cdot x}\left(\frac{f^2}{16\pi^2}\right)^n\times\left[\frac{\mu K_1[\mu(-x^2+i\epsilon x_0)^{1/2}]}{(-x^2+i\epsilon x_0)^{1/2}}\right]^n, \tag{6}$$

where $q'^2\equiv(P'-P)^2$ and

$$\mathfrak{F}(q'^2;P_\perp'^2)=\frac{1}{2M^2}\left[-q'^2G_M{}^2-\frac{(G_M{}^2-G_E{}^2)P_\perp'^2}{2(1-q'^2/4M^2)}\right]\times\frac{1}{(q'^2-m^2)^2}; \tag{7}$$

$G_E=G_E(q'^2)$ and $G_M=G_M(q'^2)$ are the vector-meson nucleon form factors defined by

$$\langle P'|J_\mu^V|P\rangle=\left(\frac{M^2}{P_0P_0'}\right)^{1/2}\bar{u}(P')\times\left[G_M\gamma_\mu-\frac{1}{2M}(P+P')_\mu\frac{G_M-G_E}{1-q'^2/4M^2}\right]\times u(P). \tag{8}$$

To do the x integration in (6), we assume that the $x^2\simeq 0$ region dominates.[13] This gives

$$\left[\frac{\mu K_1[\mu(-x^2+i\epsilon x_0)^{1/2}]}{(-x^2+i\epsilon x_0)^{1/2}}\right]^n\simeq\frac{1}{(-x^2+i\epsilon x_-)^n}. \tag{9}$$

Introducing the Fourier transform of the right-hand side of (9), we get from Eq. (6)

$$A_n=-\frac{f_{\gamma V}{}^2}{16\pi^2}\int\frac{M}{P_0'}\frac{d^3P'}{(2\pi)^3}\mathfrak{F}(q'^2;P_\perp'^2)\,d^4K(2\pi)^4\times\delta(P+q-P'-K)\theta(K_-)\theta(K^2)\times\left(\frac{f^2}{16\pi^2}\right)^n\frac{1}{n!(n-1)!(n-2)!}\left(\frac{K^2}{4}\right)^{n-2}. \tag{10}$$

Writing $d^4K=dK_+dK_-\,d^2K_\perp$, we carry out the dK_- and $d^2K_\perp$ integrations which give $K_-=P_-+q_--P_-'\simeq q_-$ and $\vec{K}_\perp=\vec{q}_\perp-\vec{P}_\perp'$. However, for $P_+\to\infty$ and $K_+/P_+\to 0$,

$$\delta(P_++q_+-P_+'-K_+)\simeq\delta(1-(1/\omega)-\xi)/P_+,$$

where $\xi\equiv P_+'/P_+$. So this δ function does not determine K_+ in the above limit, and the integration over K_+ in Eq. (10) has to be carried out up to some cutoff value $K_{+\max}$. This cutoff value determines the largest invariant mass squared $K_{\max}{}^2$

of the produced pions allowed in this model, and in fact, as we shall see below, also determines the pion multiplicity.

Using the cutoff we obtain from (10)

$$A_n = -\frac{f_{\gamma V}{}^2}{16\pi^2} 2\pi M \int_0^\infty \frac{d\xi}{\xi} d^2P'_\perp \mathfrak{F}(q'^2;P'^2_\perp)\delta(1-(1/\omega)-\xi)\frac{1}{n!(n-1)!(n-2)!}\left(\frac{f^2}{16\pi^2}\right)^n \frac{1}{P_+}\int_{K_\perp{}^2/2K_-}^{K_{+\max}} dK_+(\tfrac{1}{4}K^2)^{n-2}. \tag{11}$$

Converting the K_+ integration to an integration over K^2, we get

$$A_n = -\frac{f_{\gamma V}{}^2}{16\pi^2} 8\pi M \int d^2P'_\perp \mathfrak{F}(q'^2;P'^2_\perp)\frac{f^2}{16\pi^2}\frac{1}{2P'_+K_-} \times \frac{1}{n!(n-1)!(n-1)!}\left(\frac{f^2K_{\max}{}^2}{64\pi^2}\right)^{n-1}; \tag{12}$$

q'^2 in Eq. (12) is given by

$$q'^2 \equiv (P'-P)^2 = -(P'^2_\perp + M^2/\omega^2)/(1-1/\omega). \tag{13}$$

In the above equation, $2P'_+K_- \simeq (P'+K)^2 = s$ and $K_{\max}{}^2 \simeq 2K_{+\max}K_- = (\sum_i m_{i\perp}{}^2/z_i)_{\max}$, where $z_i = k_{i-}/q_- \simeq P\cdot k_i/P\cdot q$. Since the pions carry off only finite fractions,[8] z_i's, of the minus component of the photon, $K_{\max}{}^2 < s^\epsilon$ for any positive number ϵ, however small. Thus Eq. (12) shows that for $s\to\infty$ and n fixed, $A_n(\nu,q^2)\sim 1/s$. Hence Bjorken scaling is violated when the number of pions emitted is fixed.

However, there still remains the possibility that if we sum over all n, the factor $s \simeq 2P'_+K_-$ in the denominator of Eq. (12) is canceled out, and Bjorken scaling is satisfied by the infinite sum. To this end, we asymptotically evaluate the series over n (Ref. 14):

$$\sum_{n\,(\text{even or odd})} \frac{\zeta^{n-1}}{n!(n-1)!(n-1)!} \simeq \frac{1}{4\pi\sqrt{3}}\frac{e^{3\zeta^{1/3}}}{\zeta^{2/3}}, \tag{14}$$

where $\zeta \equiv f^2K_{\max}{}^2/64\pi^2$. The reason for summing over either n even or n odd is that if we start with the ρ^0 meson and take n even, then the relevant form factors are ρNN form factors; on the other hand, if we take n odd, then the relevant form factors are ωNN form factors. Thus starting with the ρ^0 meson and summing over all n (even and odd), we get from (12) and (14)

$$A(\nu,q^2) = \sum_n A_n = -\frac{f_{\gamma\rho}{}^2}{16\pi^2} 2M \int d^2P'_\perp [\mathfrak{F}_{\rho NN}(q'^2;P'^2_\perp) + \mathfrak{F}_{\omega NN}(q'^2;P'^2_\perp)]\frac{f^2}{16\pi^2}\frac{1}{s}\frac{e^{3\zeta^{1/3}}}{\sqrt{3}\,\zeta^{2/3}}. \tag{15}$$

The quantity ζ is not known to us as it involves the undetermined cutoff mass squared $K_{\max}{}^2$. However, it can readily be expressed in terms of a physical quantity, namely, the average multiplicity:

$$\langle n\rangle = \sum_n nA_n/A = \zeta^{1/3}. \tag{16}$$

Equations (15) and (16) show that if $\langle n\rangle \sim \frac{1}{3}\ln s$, then the factor of s in the denominator of (15) cancels out. In fact, if we take

$$\langle n\rangle = \tfrac{1}{3}\ln\left(\frac{s}{s_0}\right) + \tfrac{2}{3}\ln\left(\ln\frac{s}{s_0}\right), \tag{17}$$

then $A(\nu,q^2)$ given by (15) is solely a function of ω in the limit $s\to\infty$, and Bjorken scaling is exactly obtained.[15] Thus in the present model we interpret Bjorken scaling as due to the above multiplicity growth, and obtain for the scaling function $A(\nu,q^2)$ the result

$$\lim_{\substack{s\to\infty\\ \omega\ \text{fixed}}} A(\nu,q^2) = A(\omega) = -\frac{f_{\gamma\rho}{}^2}{16\pi^2}\left(\frac{f^2}{16\pi^2}\right)\frac{18M}{\sqrt{3}\,s_0} \times \int d^2P'_\perp[\mathfrak{F}_{\rho NN}(q'^2;P'^2_\perp) + \mathfrak{F}_{\omega NN}(q'^2;P'^2_\perp)]. \tag{18}$$

To explore further the physical implications of the above interpretation, we examine now the single-pion inclusive distribution. From Eq. (6) we find

$$k_0\frac{dA_n}{d^3k} = -\frac{f_{\gamma V}{}^2}{2\pi}\int \frac{M}{P'_0}\frac{d^3P'}{(2\pi)^3}\mathfrak{F}(q'^2;P'^2_\perp)\frac{1}{(n-1)!} \times \int d^4x\, e^{i(P+q-P'-k)\cdot x}\frac{1}{4\pi}\left(\frac{f^2}{16\pi^2}\right)^n \times \left[\frac{\mu K_1[\mu(-x^2+i\epsilon x_0)^{1/2}]}{(-x^2+i\epsilon x_0)^{1/2}}\right]^{n-1}. \tag{19}$$

As before, we do the x integration assuming that the $x^2 \simeq 0$ region dominates. This leads to

$$k_0 \frac{dA_n}{d^3k} = -\frac{f\gamma^2}{16\pi^2} 8\pi M \int d^2P'_\perp \mathfrak{F}(q'^2; P'^2_\perp) \frac{1}{4\pi}\left(\frac{f^2}{16\pi^2}\right)^2 \frac{1}{2P'_+K'_-} \frac{1}{(n-1)!(n-2)!(n-2)!} \left(\frac{f^2 K'^2_{\max}}{64\pi^2}\right)^{n-2}, \tag{20}$$

where $K' = K - k$. Comparing the above equation with Eq. (12), we notice that apart from a multiplicative factor of $(1/4\pi)(f^2/16\pi^2)$, it is essentially the same as Eq. (12) with n replaced by $n-1$ and K replaced by K'; that is,

$$k_0 \frac{dA_n}{d^3k} = \frac{1}{4\pi}\frac{f^2}{16\pi^2} A_{n-1}(K \to K'). \tag{21}$$

Summing over all n and proceeding as earlier, we obtain

$$k_0 \frac{dA}{d^3k} = \frac{1}{4\pi}\frac{f^2}{16\pi^2} A. \tag{22}$$

The single-pion inclusive distribution is therefore given by

$$\frac{k_0}{\sigma}\frac{d\sigma}{d^3k} = \frac{1}{4\pi}\frac{f^2}{16\pi^2}. \tag{23}$$

This equation shows that the inclusive pion distribution is independent of s and thus satisfies Feynman scaling and limiting fragmentation.

We investigate now specific features of the pion rapidity distribution in this model. Integration of Eq. (23) over $k_\perp^2$ with a cutoff $k_{\perp\max}^2$ leads to

$$\frac{1}{\sigma}\frac{d\sigma}{dy} = \frac{f^2}{64\pi^2} k_{\perp\max}^2. \tag{24}$$

Hence the pions have flat rapidity distribution. To determine the width of the distribution in rapidity, we take note of two basic kinematic constraints in the model: (i) $k_- < q_-$, and (ii) $k_+ < K_{+\max}$. Since $k_- = m_\perp e^{-y}$, the first one gives

$$m_{\perp\max} e^{-y_{\min}} = q_-$$

or

$$y_{\min} = -\ln\left(\frac{q_-}{m_{\perp\max}}\right). \tag{25}$$

Similarly, from the second one using $2k_+ = m_\perp e^y$ and $2K_{+\max} q_- \simeq K_{\max}^2$, we get

$$y_{\max} = -\ln\left(\frac{q_-}{m_{\perp\max}}\right) + \ln\left(\frac{K_{\max}^2}{m_{\perp\max}^2}\right). \tag{26}$$

Hence,

$$\Delta y = y_{\max} - y_{\min} = \ln\left(\frac{K_{\max}^2}{m_{\perp\max}^2}\right). \tag{27}$$

This relation of course does not determine Δy, because $m_{\perp\max}^2 = k_{\perp\max}^2 + \mu^2$ is unknown. However, using (27) and $\langle n\rangle^3 = f^2 K_{\max}^2/64\pi^2$ [Eq. (16)], Eq. (24) can be written in the form

$$\frac{1}{\sigma}\frac{d\sigma}{dy} = \langle n\rangle^3 e^{-\Delta y}. \tag{28}$$

Since the rapidity distribution is flat, the left-hand side is equal to $\langle n\rangle/\Delta y$. We therefore obtain the following relation determining the width Δy in terms of $\langle n\rangle$:

$$\frac{e^{\Delta y}}{\Delta y} = \langle n\rangle^2. \tag{29}$$

As an approximation this gives $\Delta y \simeq 2\ln\langle n\rangle$, and correspondingly

$$\frac{1}{\sigma}\frac{d\sigma}{dy} \simeq \frac{\langle n\rangle}{2\ln\langle n\rangle}. \tag{30}$$

We next turn to the final nucleon. Its inclusive cross section using Eq. (11) and summing over all n is

$$\frac{1}{\sigma}\frac{d\sigma}{dy} = \left(1 - \frac{1}{\omega}\right)\delta(1 - (1/\omega) - \xi), \tag{31}$$

where $\xi = \bar{M}_\perp e^y/2P_+$, and $\bar{M}_\perp$ is an average transverse mass of the final nucleon.

The final-state hadron spectra as obtained in the present model is now shown in Fig. 2. We have a photon-fragmentation region of width $\simeq 2\ln\langle n\rangle$ and constant height $\simeq \langle n\rangle/2\ln\langle n\rangle$. Since $\langle n\rangle \sim \ln s$, the height of the photon-fragmentation region increases almost logarithmically with s. As for the target-fragmentation region, we simply have a δ function there, because the target nucleon does not really fragment in our model. Instead, it goes off with a

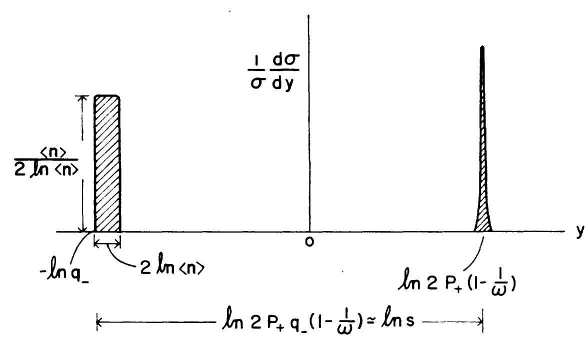

FIG. 2. Final-state hadron spectra in the present model. On the left-hand side is the photon-fragmentation region of width $2\ln\langle n\rangle$ and height $\langle n\rangle/2\ln\langle n\rangle$. On the right-hand side the δ function represents the nucleon-fragmentation region.

definite fraction $(1-1/\omega)$ of the plus component of the initial nucleon. This picture of final-state hadron spectra differs completely from the usual parton-model picture[16] where one has a photon-fragmentation region of length $\ln Q^2$ accompanied by a hadronic plateau of length $\ln(\omega-1)$ and a target-fragmentation region of length two units.

At this point it is important to observe that we are connecting the Bjorken scaling phenomenon with the pion multiplicity growth. Our interpretation of the scaling requires the pion multiplicity to depend solely on s and be independent of Q^2 [Eq. (17)]. In contrast, parton models generally predict a Q^2 dependence of the form $\langle n\rangle = C_{e^+e^-}\ln(Q^2/M^2) + C_h \ln(\omega-1)$.[17,18] Other models, such as the pulverization model,[19] predict $\langle n\rangle \sim s^{\alpha(\omega)}$, while field-theoretical[20] and multiperipheral[21] scaling models predict $\langle n\rangle \sim \ln\omega$ for large ω. In passing we note that the Cornell data[22] show no Q^2 dependence of the average charged-hadron multiplicity, and in fact the data are totally consistent with an $\langle n\rangle \sim \ln s$ behavior.

We would now like to collect a number of results which come out naturally in our model: (i) The ratio of the longitudinal to the transverse cross section $\sigma_L/\sigma_T = 0$; equivalently, $2MW_1 = \omega\nu W_2$ (Callan-Gross relation). This occurs because only the +, − (i.e., 0 and 3) components of the q_i's are important and the tensor $\epsilon_{\rho\sigma\alpha\beta}$ at each vertex then forces transverse polarizations for the vector mesons. (ii) For $\omega \to 1$,

$$W_1,\ \nu W_2 \sim \int d^2P_\perp' \frac{P_\perp'^2 + M^2/\omega^2}{(1-1/\omega)} \tilde{G}_M^{\,2}\left(-\frac{P_\perp'^2+M^2/\omega^2}{1-1/\omega}\right),$$

where $\tilde{G}_M(q'^2)$ is the nucleon electromagnetic form factor. This shows that the threshold behavior of the structure functions is going to be related with the asymptotic behavior of the elastic form factor (Drell-Yan-West relation). (iii) The pions contribute only to the phase space and the energy-momentum conservation, so that a statistical description for the pion production holds. (iv) Feynman scaling and limiting fragmentation for the pion inclusive distribution follow if Bjorken scaling is exactly satisfied. (v) Each channel defined by fixed n violates Bjorken scaling, while the sum over all channels satisfies it. (vi) The conventional vector dominance of ρ, ω, ϕ is quite compatible with Bjorken scaling, and one does not need the generalized vector dominance[23] of an infinite number of vector-meson states to explain scaling.

We draw two important physical conclusions from the present model. (1) Quarks in a nucleon can form a hadronic core. This core is surrounded by a cloud of vector gluons. When we attempt to probe the nucleon in deep-inelastic ep scattering, the highly virtual photon hits the gluon cloud and produces pions. *The nucleon core itself is only elastically scattered, and therefore the quarks do not emerge as free particles.*[24,25] (2) *Bjorken scaling phenomenon represents two dynamical mechanisms operating simultaneously*: (a) the dependence of the momentum transfer $(P'-P)^2$ on the scaling variable ω and not on q^2 due to the boundedness of the transverse momenta; (b) the logarithmic growth of pion multiplicity with s, the square of the virtual-photon-nucleon c.m. energy.

Added note: After completion of this work the author has noticed a report by Craigie and Rothe,[26] where the idea that the photon propagates as a vector-meson state emitting pions was used to discuss e^+e^- annihilation.

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[6]Our primary reason for considering only neutral ρ mesons is to keep the present formulation of the model simple. No essential changes have to be made in the model to include charged ρ mesons and correspondingly charged pion emission.

[7]Details of kinematic results and their derivation are given in the paper of M. M. Islam and J. R. Owens, Jr., Phys. Rev. D 7, 3784 (1973).

[8]By this we mean, when $s\to\infty$, $k_{i-}/q_- > s^{-\epsilon}$ for any positive number ϵ, no matter how small.

[9]W. R. Frazer *et al.*, Rev. Mod. Phys. 44, 284 (1972).

[10]In Eq. (1) the $q^\sigma q^\rho/m^2$ term in the vector-meson propagator does not contribute because of the antisymmetry of the tensor $\epsilon_{\rho\sigma\alpha\beta}$.

[11]Note that $\epsilon_{\rho\sigma\alpha\beta}q_1^\alpha q_2^\beta \simeq \epsilon_{\rho\sigma30}q_1^3q_2^0 + \epsilon_{\rho\sigma03}q_1^0q_2^3 \simeq \epsilon_{\rho\sigma30}q_+(q_{1-}-q_{2-})$ using $q_{1+}\simeq q_{2+}\simeq q_+$.

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[13]In other words, we are assuming light-cone dominance to evaluate the phase-space contributions of the pions. In a sense this is equivalent to taking the limit $\mu \to 0$ [see Eq. (9) below].

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[15]The coefficient of the ln$s$ term in Eq. (17) cannot be taken literally at this point, since inclusion of charged-pion production would modify it. However, it is already a reasonable number, and this makes the model reasonable.

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