Joint rapidity and multiplicity dependences of correlation functions in high-energy collisions*

Richard C. Arnold and Gerald H. Thomas

High Energy Physics Division, Argonne National Laboratory, Argonne, Illinois 60439

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Using language of statistical mechanics, we discuss the agreement between two-particle correlations at fixed multiplicity (canonical ensemble) and correlations defined inclusively (grand ensemble), with particular emphasis on the corrections to asymptotic equality expected for these observables. It is furthermore shown that multiperipheral models with predominantly positive correlations imply asymptotically the increase of two-particle semi-inclusive correlation functions with increasing n near $\langle n \rangle$. Recent data at 205 GeV/c do not confirm these expectations, and alternative interpretations are discussed.

I. INTRODUCTION

Striking new data from the 205-GeV/c 30-in. ANL–NAL–Stony-Brook collaboration bubble chamber exposure have recently been found[1] on semi-inclusive rapidity correlations. As a function of the number of charged particles n, and the c.m. rapidities y of each charged particle, the functions

$$C_e(y_1, y_2, n) = \frac{1}{\sigma_n}\frac{d^2\sigma_n}{dy_1\,dy_2} - \left(\frac{1}{\sigma_n}\frac{d\sigma_n}{dy_1}\right)\left(\frac{1}{\sigma_n}\frac{d\sigma_n}{dy_2}\right) \tag{1}$$

have been obtained. As defined here, $C_e(y_1, y_2, n)$ is the difference between the two-particle semi-inclusive normalized rate (σ_n is the prong cross section) and the uncorrelated product of normalized single-particle semi-inclusive rates. We focus on the empirical observation[1] that $C_e(0,0,n)$ is near zero and almost independent of n for $n \gtrsim 4$ charged prongs. The new observation is made here that asymptotically, for an important class of short-range-order (SRO) models, semi-inclusive correlations *increase in strength for increasing multiplicity* for n near $\langle n\rangle$. The data at 205 GeV/c indicate an opposite trend. If confirmed at higher energies, this trend would be compelling evidence against all such models.

First we review the experimental and theoretical situation prior to these new data. Observation of the *fully inclusive* correlation[2,3]

$$C_i(y_1, y_2) = \frac{1}{\sigma_{\text{in}}}\frac{d^2\sigma}{dy_1\,dy_2} - \left(\frac{1}{\sigma_{\text{in}}}\frac{d\sigma}{dy_1}\right)\left(\frac{1}{\sigma_{\text{in}}}\frac{d\sigma}{dy_2}\right) \tag{2}$$

indicated a strong positive value for $C_i(0,0)$, translation invariance, namely

$$C_i(y_1, y_2) = C_i(|y_1 - y_2|),$$

and a possible exponential falloff of

$$C_i(|y_1 - y_2|) \sim \exp(-|y_1 - y_2|/L)$$

with a correlation length $L \simeq 2$. The naive interpretation of these facts[4,5] has been some form of a Regge-pole dominance of the Mueller $n - n$ amplitudes.[6] Such interpretation is usually coupled with simple assumptions about factorization, which imply that the inclusive n-particle rate is calculable from the couplings used in the $(n-1)$-particle rate. Taken together, these interpretations and assumptions amount to a claim of support for the multiperipheral description of the data,[7] at least for particles produced in the central region. The multiperipheral description can be framed either in inclusive or exclusive language[8]; a combination of these languages will be useful for discussing the significance of the new semi-inclusive data.

Exclusive multiperipheral models (EMPM) which can correctly account for data on $C_i(y_1, y_2)$ are constrained by data on multiplicity distributions. We know that C_i integrates to a positive value, since it is known that the second moment

$$f_2 = \int C_i\,dy_1\,dy_2 = \langle n(n-1)\rangle - \langle n\rangle^2$$

is positive. Such positive correlations must arise in an EMPM through the assumption of a predominantly positive two-particle correlation in the exclusive kernel. Resonances are a common way of introducing such a correlation, though positive exclusive pair correlations is a more general concept. We now give arguments that models having positive exclusive pair correlations needed to describe inclusive data necessarily have, at asymptotic energies, $C_e(0,0,n)$ positive in (1), and increasing as n increases, near $n = \langle n\rangle$. Note that

$$\int C_e(y_1, y_2, n)\, dy_1\, dy_2 = n(n-1) - n^2 = -n$$

is negative, so that the average value of $C_e(n)$ does not suggest the above behavior.

II. RELATION BETWEEN INCLUSIVE AND SEMI-INCLUSIVE CORRELATIONS

In simple models it is not hard to verify explicitly the increase of C_e with n. To see the result more generally, it is convenient to use a different measure of the exclusive correlations which can be more readily compared with the known positive inclusive correlation. In statistical mechanics, from which our intuition of correlation functions has been derived, there is a natural unified representation of both exclusive and inclusive correlations.[9] All inclusive and exclusive information about cross sections can be exhibited in a generating (grand ensemble partition) function

$$\Omega(s, z) = \sum_n \sigma_n(s)\, z^n . \qquad (3)$$

All correlation functions can then be defined as in the inclusive case, but with these partition functions replacing everywhere the usual inelastic or total cross sections. Thus we write

$$\rho_1(y, z) = \frac{1}{\Omega}\frac{d\Omega}{dy} = \frac{1}{\Omega}\sum \frac{d\sigma_n}{dy} z^n , \qquad (4)$$

$$\rho_2(y_1, y_2, z) = \frac{1}{\Omega}\frac{d^2\Omega}{dy_1\, dy_2} = \frac{1}{\Omega}\sum_n \frac{d^2\sigma_n}{dy_1\, dy_2} z^n , \qquad (5)$$

$$C(y_1, y_2, z) = \frac{1}{\Omega}\frac{d^2\Omega}{dy_1\, dy_2} - \left(\frac{1}{\Omega}\frac{d\Omega}{dy_1}\right)\left(\frac{1}{\Omega}\frac{d\Omega}{dy_2}\right) . \qquad (6)$$

A dependence on $Y = \ln s$ is understood to be present, although these functions should approach a (scaling) limit as $Y \to \infty$. We also define integrated z-dependent moments:

$$f_1(z) = \langle n \rangle_z , \quad f_2(z) = \langle n(n-1)\rangle_z - \langle n \rangle_z^2 ,$$

etc. [cf. Eq. (8) below]. Whenever $z = 1$, the quantities (3)–(6) are the total cross section, single-particle and two-particle inclusive densities, and inclusive two-particle correlation $C_i(y_1, y_2)$, respectively. Intuitively we see that as z passes from 0 to 1, the interpretation of the above quantities changes from being exclusive to totally inclusive. For $z > 1$, these "cross sections" emphasize multiplicities greater than the mean.

The quantities (3)–(6) can be easily computed in models and from data; particularly, we note that $C(y_1, y_2, z)$ is far easier to obtain in MPM's than are the C_e's. They should, therefore, be very useful quantities for comparing data with models in more detail than the two-particle total inclusive ($z = 1$) correlations allow. We note that we could also define a normalized correlation

$$R(y_1, y_2, z) = C(y_1, y_2, z)/\rho_1(y_1, z)\rho_1(y_2, z) ,$$

in analogy to the correlations presented in experimental publications. We prefer not to discuss R here since the characteristic z dependence of EMPM shows up most clearly in $C(y_1, y_2, z)$.

An exact relation between C_e and $C(z)$ can now be derived, based only on the definitions above. If $C_e(y_1, y_2, n)$ is given for all n (these are the canonical ensemble correlation functions), then

$$\begin{aligned} C(y_1, y_2, z) = {} & \langle C_e(y_1, y_2, n)\rangle_z \\ & + \langle \rho_1(y_1, n)\rho_1(y_2, n)\rangle_z \\ & - \langle \rho_1(y_1, n)\rangle_z \langle \rho_1(y_2, n)\rangle_z , \end{aligned} \qquad (7)$$

where we define, for any function $f(n)$ of n,

$$\langle f(n)\rangle_z \equiv \sum_{n=0}^{\infty} z^n f(n)\, \sigma_n / \Omega(z) , \qquad (8)$$

and where $\rho_1(y, n)$ is the semi-inclusive single-particle rapidity density for multiplicity n. Conversely, if $C(z)$, the grand ensemble correlation function, is given as a function of z, then

$$\begin{aligned} C_e(y_1, y_2, n) = {} & D_n[C(y_1, y_2, z)] \\ & + D_n[\rho_1(y_1, z)\rho_1(y_2, z)] \\ & - D_n[\rho_1(y_1, z)]\, D_n[\rho_1(y_2, z)] , \end{aligned} \qquad (9)$$

where we define a linear operator D_n acting on any function $F(z)$ by

$$D_n[F(z)] \equiv \frac{\partial^n}{\partial z^n}[F(z)\Omega(z)] \Big/ \frac{\partial^n \Omega(z)}{\partial z^n}\bigg|_{z=0} . \qquad (10)$$

We note for later discussion that if, as expected at asymptotic energies, the single-particle canonical densities $\rho_1(y, n)$ for y near zero are linear in n, i.e.,

$$\rho_1(y, n) = a + nb/Y$$

in the central region, then we find the grand ensemble density

$$\rho_1(y, z) = a + bf_1(z)/Y$$

and a simple relation between canonical and grand ensemble correlations:

$$C_e[y_1, y_2, f_1(z)] \cong C(y_1, y_2, z) - \frac{b^2}{Y^2}[f_1(z) + f_2(z)] , \qquad (11)$$

when $f_1(z)$ is large (we keep only the dominant correction in a steepest-descent computation of $\langle C_e\rangle_z$). In MPM's, we expect that $b \to 1$, $a \to 0$, and f_1/Y and f_2/Y approach constants, as $Y \to \infty$ for fixed z.

This implies asymptotic equality between $C(y_1, y_2, z)$ and $C_e[y_1, y_2, f_1(z)]$.

In the language of statistical mechanics, this is the statement that *grand and canonical ensembles must yield identical observables in the bulk limit* $f_1(z) = n \to \infty$, (n/Y) *fixed*. In more general models retaining cluster decomposition (11) continues to hold (with $b \to 1$, $a \to 0$ as $Y \to \infty$), although the correction term may decrease with Y more slowly than Y^{-1}. For $z = 1$, (11) implies asymptotic equality of inclusive correlations with correlations at the mean multiplicity. We note that $(f_1 + f_2) > 0$; thus $C_e[f_1(z)]$ must approach $C(z)$ from below. Also note that for $C(z) = 0$, i.e., strict Poisson distribution for σ_n's (as would hold in a strong-ordering limit of a Chew-Pignotti multiperipheral model),[10] we would obtain

$$C_e(y_1, y_2, n) = -n/Y^2\,, \tag{12}$$

independent of y_1 and y_2.

III. ASYMPTOTIC BEHAVIOR OF $C(z)$ IN MPM

The precise statement we make about EMPM having positive correlations is that, at asymptotic energies, $C(0, 0, z)$ will *increase* as z increases from unity and *decrease* as z decreases from unity. Our task is to explain in general terms how this comes about in such models, and illustrate z dependence and associated n dependence with a simple algebraic calculation. Numerical results of a typical simple MPM are shown for comparison in Fig. 1.

We argue as follows. At sufficiently high energies, we expect Feynman scaling in the central region implying translation invariance in rapidity, both for the inclusive single-particle density $(1/\sigma)\,d\sigma/dy$, and for the two-particle density $(1/\sigma)\,d^2\sigma/dy_1\,dy_2$; the latter is then only a function of $\Delta y = y_1 - y_2$. Moreover, in MPM's with SRO, this scaling should hold not only for the corresponding partition functions at $z = 1$ (by definition) but also for a range of z's around unity. This can be explicitly verified in most models discussed in the literature. In this range of z we expect the gross z dependence of $C(\Delta y, z)$ for $\Delta y = \epsilon \ll L(z)$ to be

$$C(\epsilon, z) \sim \tfrac{1}{2} f_2(z)/[L(z)\,Y]\,, \tag{13}$$

where $L(z)$ is the z-dependent correlation length. The precise functional form for $C(y)$ will not, of course, be a pure exponential. However, for large $\Delta y \gg L$ we expect C to fall exponentially with slope $L(z)^{-1}$ so that (13) integrates approximately to $f_2(z)$, the second moment $\langle n(n-1)\rangle_z - \langle n\rangle_z^2$ of the partition function $\Omega(s, z)$. The z dependence of C should therefore be qualitatively determined by $f_2(z)$ and $L(z)^{-1}$, even though (13)

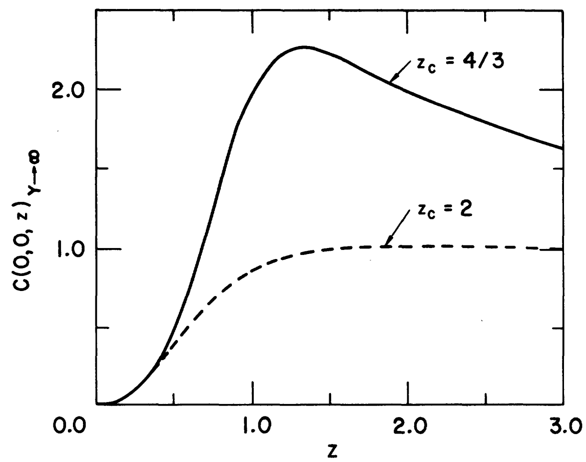

FIG. 1. Two examples of the asymptotic correlation $C(0,0,z)$, Eq. (15), are given such that the asymptotic density,

$$\langle n\rangle/Y = \tfrac{1}{2}z(a+b) + \tfrac{1}{2}zL(z)[(l_1 - l_2)(a-b) + z((a-b)^2 + 4ab)]\,,$$

is approximately 2 at $z = 1$. The solid curve has $a = \frac{3}{8}$, $b = \frac{21}{8}$, and $l_1 - l_2 = 3$; the dashed curve has $a = \frac{3}{4}$, $b = \frac{5}{4}$, $l_1 - l_2 = 2$. The values were chosen to give, respectively, for the solid and dashed curves a maximum C of about 2 (or 1) and a value of $1/z_c = \lambda$ which, in the δ-function limit (see text), corresponds to 2 (or 4) particles per resonance decaying on the average. Because the emphasis is on understanding C at $\Delta y = 0$, we do not interpret the correlation length in (15) to represent the large-Δy behavior; with more poles included, we would expect to get both the small- and large-Δy behavior reliably.

is not an accurate estimate of the numerical value of $C(\epsilon, z)$.

In EMPM's $L(z)^{-1}$ is the spacing of the leading and next leading (secondary) Regge singularities. Moreover, in such models z acts like a coupling strength for the exclusive kernel. If there is positive exclusive pair correlation in the kernel, then any increase in the strength z will cause $f_2(z)$ and $L(z)^{-1}$ to increase; any decrease of z towards zero will reduce the model to one with only weak correlations and $f_2(z)\,L(z)^{-1}$ should go to zero. EMPM calculations of $C(\Delta y, z)$ at asymptotic energies as well as Monte Carlo simulation studies[11] using multiperipheral-like models at NAL energies confirm this behavior.

It is instructive to illustrate the above general remarks with a concrete example. We choose the exclusive kernel (in the j-plane representation)

$$K(j) = \frac{a}{j - l_1} + \frac{b}{j - l_2} \tag{14}$$

to have two poles such that $l_1 = 2\alpha_1 - 1$ approximates the leading (cut) singularities and $l_2 = 2\alpha_2 - 1$

approximates the lower nonleading singularities. Asymptotically positive cross sections require $a>0$; positive inclusive correlations require $b>0$. Indeed it is a straightforward calculation to show that for $s\to\infty$ the rapidity correlation is

$$C(\Delta y, z)=z^2ab\left[\frac{L(z)}{L(0)}\right]^2\exp\left[\frac{-\Delta y}{L(z)}\right], \qquad (15)$$

where the correlation length $L(z)$ is given by the z-dependent spacing of the two output trajectories

$$L(z)^{-1}=\{(l_1-l_2)^2+2(l_1-l_2)(a-b)z + z^2[(a-b)^2+4ab]\}^{1/2}. \qquad (16)$$

The maximum over z of the correlation function is

$$C(0,z_c)=\tfrac{1}{4}(l_1-l_2)^2 \qquad (17)$$

and this maximum occurs for $z=z_c=(l_1-l_2)/(b-a)$ which in magnitude is bigger than the magnitude of z_L, the point at which $L(z)$ has a maximum: $z_c/z_L=1+4ab/(a-b)^2$.

We can now prove that the observed trend of $(2f_2+f_3)$ positive implies for our simple model

$$\frac{\partial C}{\partial z}(\epsilon,z)\bigg|_{z=1}>0. \qquad (18)$$

Suppose the opposite, $(\partial C/\partial z)_1<0$; this means the maximum z_c has $0<z_c<1$. Since $z_L<z_c$, then $(\partial L/\partial z)_1<0$ and thus from $C(0,1)$ and $L(1)>0$ we have $[\partial(CL)/\partial z]_1<0$. But twice the latter quantity is

$$\frac{1}{Y}\left(\frac{\partial f_2}{\partial z}\right)_1=\frac{\partial}{\partial z}\left(z^2\frac{\partial^2}{\partial z^2}\frac{\ln\Omega(s,z)}{Y}\right)_{z=1} =(2f_2+f_3)/Y,$$

which we infer therefore to be negative. This is contrary to our assumption and so we cannot have $(\partial C/\partial z)_1<0$; hence (18) holds.

Equation (18) is our main observation about MPM's. We can ask: Is this truly what one should expect *generally* in EMPM's with positive correlations SRO? We note first that strong correlations imply, through (17) that the effective nonleading trajectory must be fairly low-lying; otherwise $C(\epsilon,1)$ will be very small. It has been previously shown[12] that the limiting case of $l_2\to-\infty$, with $\lambda=-b/l_2$ fixed, gives a prototype multiperipheral-resonance model (with zero Q value); the probability of a resonance decaying into m particles is λ^{m-1}. We may reasonably expect therefore that sophisticated cluster emission models[13] will give (18) asymptotically, and generally conform to the qualitative behavior of (15).

The form (14) for $K(j)$ is a special case of a two-channel model; the results [(15)–(18)] remain true with the product ab replaced by an off-diagonal coupling c^2. In such models the total correlation must be positive. In general N-channel models, the correlation for all produced particles $C(\Delta y, z)$ is positive, approaches zero like z^2 as $z\to0$, and goes to a constant as $z\to\infty$. The possibility for maxima of C and L exists; for specific models an analysis similar to the one above could be performed. However, we believe that except for pathological cases, the two-pole model serves as an approximation to these models and (18) should still be valid. We note that when charge is included, the charge correlations can be positive or negative, and one can show that for specific charge correlations, as $z\to\infty$, $C(0,z)$ grows like z^2.

IV. PHENOMENOLOGICAL DISCUSSION OF DATA

We can now exhibit the behavior of $C_e(y_1,y_2,n)$ following from the above considerations of $C(y_1,y_2,z)$, compare with 205-GeV/c data, and discuss possible interpretations.

First, we can check internal consistency of data on inclusive and exclusive correlations by testing relation (11). We find that (11) holds within quoted experimental errors.

Since data show that the $C_e(0,0,n)$ are very small in magnitude, we can ignore C_e in Eq. (7) and compute $C(z)$ using data at 205 GeV/c on $\rho_1(y,n)$. At $y=0$ we see

$$\rho_1(n)\cong-0.8+0.3n, \qquad (19)$$

which yields, at 205 GeV/c, the numerical result

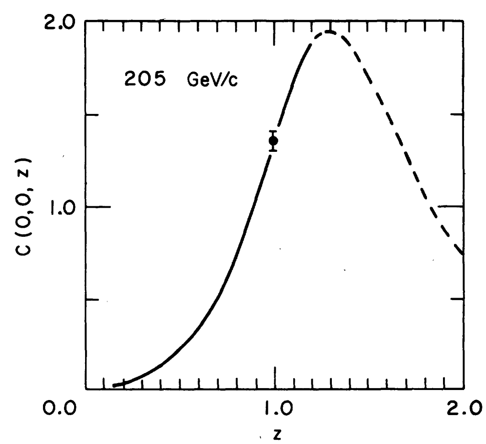

FIG. 2. The experimental values (Refs. 1, 2, and 14) for $C(0,0,z)$ at 205 GeV/c. These are obtained from the semi-inclusive data using (7) and the accurate approximation (19). We ignore $C_e(0,0,n)$ ($\approx -0.002n^2$) because it is observed to be small at this energy. The error indicated at $z=1$ is somewhat larger than one obtains for $z<1$, and smaller than one obtains for $z>1$. The curve becomes dashed where we find the errors so large as to make $C(0,0,z)$ unreliable.

$$C(0,0,z)\cong 0.09[f_1(z)+f_2(z)]. \qquad (20)$$

From multiplicity data[14] we numerically compute f_1 and f_2, which yields the curve shown in Fig. 2.

If asymptotic energies have been reached (for $y_1=y_2=0$), we can ask: Do $C(z)$ and $C_e(n)$ increase as expected in SRO models with positive pair correlation?

We see that $C(z)$ shows a strong increase near $z=1$ expected in an asymptotic MPM, but $C_e(0,0,n)$ does not increase with n. The nonscaling correction term in (11) is comparable to C itself.

There are then two possible viewpoints, reflecting two alternative hypotheses on the proximity to asymptotic behavior: (a) for fixed n (canonical ensemble), or (b) for fixed z (grand ensemble).

(a) If the $C_e(n)$ are near their asymptotic limits, the $C(z)$ must grow progressively negative with increasing energy. This is difficult to reconcile with CERN ISR data on inclusive ($z=1$) correlations. On the contrary, the inclusive correlations in the central region near $\Delta y=0$ are very similar for the 205-GeV/c and ISR data, and the exclusive ISR correlations[3] seem to be increasingly positive. This suggests the opposite hypothesis.

(b) If $C(z)$ is near its asymptotic limit at 205 GeV/c, then the $C_e(n)$ must increase and approach closely the curve shown in Fig. 2 [with n replaced by $f_1(z)$]. This increase should be enough to be seen clearly between beam momenta of 100 and 400 GeV/c.

Hypothesis (b) would be implied by a more general conjecture: that typical exclusive correlation ranges in rapidity are much longer than typical inclusive correlation lengths. Such circumstances can be arranged explicitly in some simple models. We have suggested, in connection with the analog equation of state,[15] that the grand ensemble observables are a better guide to asymptotic behavior than the canonical. The latter was used by Bander[16] in his discussion of the analog p-v diagram, which gave no indication of critical behavior; the former, we have argued, indicates critical behavior when NAL multiplicity data are used.[17] Thus, hypothesis (b) would be favored from our previous comments. Note that the behavior of correlation functions near $\Delta y=0$ are in no way directly related to a presence or absence of critical behavior.

In spite of possible subtleties of individual models, if the 205-GeV/c results are confirmed at higher energies (e.g., NAL and ISR) then an important class of EMPM with positive pair correlations can be rejected. We emphasize that these models are currently considered attractive by many people, and are able to explain a good deal of existing data. It is extremely important to have additional measurements at other energies. It is also important to check our conjecture about sophisticated MPM Monte Carlo calculations and confirm that they cannot give the observed trend of the semi-inclusive data, at asymptotic energies. Note, however, that according to (11), any model which at any given energy correctly fits all the canonical densities $\rho_1(y,n)$, and simultaneously fits $C(z)$, will give a qualitatively correct set of values for $C_e(n)$.

V. NON-SRO MODELS

We conclude with some remarks about more general models than the short-range-order models (finite correlation length models) discussed above. For this purpose we consider (I) a simple long-range non-nearest-neighbor model previously discussed, and (II) two-component hybrid models.

In case (I), the KUH model,[18] individual $C_e(0,0,n)$ are small or negative, the argument being that except for weak attractive long-range rapidity correlations $[C\sim 1/(\Delta y)^{1+\delta}]$ and threshold effects at small subenergies, the particles are produced independently. The net effect, in a mean field approximation, is to multiply the independent emission cross section by a weight measuring the cumulative strength of the long-range forces. However, since the long-range forces act between *every* pair of particles, the cumulative weight can be quite large. In the KUH model, the weight leads to a factor $\exp(an^2/Y)$ in the n-particle cross section where $Y=\ln s$ and a is the strength of the long-range force between one pair. For this model the correlation $C(\Delta y,z)$ for small Δy has been worked out in the literature,[18] and is found to have the same behavior as in a system having zero long-range strength: $C(\Delta y,z)$ for small Δy is that for particles emitted independently, subject only to the energy momentum or multiparticle rapidity thresholds. Nevertheless for large Δy, $C(\Delta y,z)$ can behave as $1/(\Delta y)^{1+\delta}$ (where δ is a critical exponent), and the integral of C can very well be positive. The scale of Δy over which the long-range behavior shows up depends on the density $\rho(z)$, and could be of order unity for $z\sim 1$; this could simulate a finite L for experimentally accessible Y values. We find in all such models

$$C_e(y_1,y_2,f_1(z))-C(y_1,y_2,z)\to 0$$

asymptotically as $Y\to\infty$, but not necessarily as fast as Y^{-1}, if critical phenomena are present at $z=1$.[17]

In case (II), two-component models,[19] one finds *for* $z>1$ that the behavior of correlation functions $C(z)$ is qualitatively identical to that of MPM's. Our previous remarks on $C(z)$ for $z>1$ apply in the asymptotic energy regime. For $z<1$, all fixed-z densities approach *zero* asymptotically [in particular $C(z)\to 0$], and $C_e(y_1, y_2, n)$ in the bulk limit remains a well-defined function of n/Y. The appropriate expressions for densities $\rho_1(y,n)$ and $\rho_2(y_1,y_2,n)$ are those for a two-phase region, as discussed in KUH [cf. their Eq. (56)]. If $C_i(y_1,y_2)$ is the asymptotic inclusive correlation in the "multiperipheral" component, we find for $Y\to\infty$ and n/Y fixed the surprisingly simple result for all $n \leq \langle n\rangle$

$$C_e(y_1, y_2, n) = \frac{n}{\langle n\rangle} C_i(y_1, y_2) + n(\langle n\rangle - n)/Y^2,$$

independent of any detailed property of the "diffractive" component (except the vanishing of all "diffractive" densities ρ_k as $Y\to\infty$). Just at $z=1$, we find that $C(y_1, y_2, z)$ is not well defined asymptotically.

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[19] K. Wilson, Cornell University Report No. CLNS-131, 1970 (unpublished). Note that we interpret the two-component concept somewhat differently than is usual, since we expect no dip structure in σ_n to result. We assume (1) that the leading singularities in exclusive cross sections factorize, even if this singularity is not a pure pole; this leads to exclusive rapidity cluster decomposition (ERCD). We assume (2) that inclusive densities $(1/\sigma)d^n\sigma/dy_1\cdots dy_n$ are bounded for $(y_1,\ldots,y_n)$ in the central region as $Y\to\infty$; this is generalized Feynman scaling (GFS). The two-component possibility then is interpreted to mean that for $n/Y\to 0$, σ_n is purely diffractive; for n/Y greater than $\langle n\rangle/Y$, σ_n is purely multiperipheral; and for $0<n/Y<\langle n\rangle/Y$ both components survive. The main difference between these statements and Wilson's idea is that here σ_n cannot develop a dip due to the assumptions of ERCD and GFS (Huang, Ref. 9). This has important phenomenological consequences which we are currently investigating. We note that our conception of the two-component picture agrees with various theoretical schemes which incorporate diffractive and nondiffractive contributions. See, e.g., J. S. Ball and F. Zachariasen, Phys. Lett. 40B, 411 (1972); 41B, 525 (1972); W. R. Frazer, D. R. Snider, and C.-I Tan, Phys. Rev. D 8, 3180 (1973). These authors find no dip in σ_n due to the two components.