[5]W. C. Carithers *et al.*, Phys. Rev. Lett. 30, 1336 (1973).

[6]The contribution of the 2γ intermediate state to the decay rate is taken to be 6×10^{-9}. See Ref. 10.

[7]S. Glashow, J. Iliopoulos, and L. Maiani, Phys. Rev. D 2, 1285 (1970).

[8]Other graphs exist to this order, but under the assumptions (a) or (b), the one shown in Fig. 2 is dominant. Whereas the graph for $\mathfrak{N}\overline{\lambda} \to 2p \to 2\gamma$ is proportional to $\ln(m_{\wp'}^2/m_{\wp}^2)$, the graphs for $\mathfrak{N}\overline{\lambda} \to 2W \to 2\gamma$ are proportional to $m_{\wp'}^2/M_W^2$, which is assumed to be much smaller.

[9]K_L has a small $CP=+1$ component which is assumed not to have an anomalously large CP-violating decay rate into 2μ.

[10]L. M. Sehgal, Phys. Rev. 183, 1511 (1969).

[11]F. J. Hasert *et al.*, Phys. Lett. 46B, 138 (1973).

[12]Because of the functional form of Eq. (10), $m_{\wp'}$ changes less than 0.2 GeV within the range in question.

[13]M. K. Gaillard and B. W. Lee, Phys. Rev. D (to be published).

PHYSICAL REVIEW D VOLUME 9, NUMBER 11 1 JUNE 1974

Charge symmetry and electromagnetic corrections to $\Sigma^\pm \to \Lambda e^\pm \nu$ decays*

David Eimerl†

Department of Physics, University of California, San Diego, La Jolla, California 92037

(Received 5 October 1973; revised manuscript received 24 January 1974)

Electromagnetic corrections to $\Sigma^\pm \to \Lambda e^\pm \nu$ decays are investigated with a view to the detection of a possible nonelectromagnetic charge asymmetry. A single renormalization of the Fermi constant renders both rates finite. The electromagnetic asymmetry is due mostly to electromagnetic $\Sigma\Lambda$ isospin mixing, and may be as much as 10%. The remaining electromagnetic asymmetry depends on the hypercharge densities of the hyperons, and approximate bounds on it are estimated.

I. CHARGE SYMMETRY AND THE Σ DECAYS

The semileptonic decays of the Σ hyperons to the Λ hyperon[1] provide a possible experimental test for charge asymmetry in the weak interaction. There are several reasons for considering these decays for such a test. The absence of any nuclear structure corrections and the isospin selection rule on the weak current ($\Delta I=1$) are perhaps the most persuasive, while the large energy release (~80 MeV) opens up the possibility of measuring the weak magnetism and pseudotensor components of the current matrix element. The subsequent decay of the Λ hyperon to a proton and a pion serves to measure the helicity of the Λ, allowing an almost complete reconstruction of the Σ decay. (The electron helicity is fixed by the $V-A$ form of the lepton current.) Also, these matrix elements of the current are interesting in that they are the only accessible examples of the matrix element of the current between states of zero hypercharge, owing to the instability of the neutral Σ hyperon.

The semileptonic Hamiltonian density is

$$\mathcal{H}_W = \frac{G}{\sqrt{2}}(J_W^\mu l_\mu^\dagger + \text{H.c.}), \tag{1}$$

where the weak hadronic current responsible for the Σ decays to Λ carries isospin 1 and may be written as the sum of a first-class and a second-class[2-4] piece:

$$J_W^\mu \equiv J^{\mu(+)} = J_1^{\mu(+)} + J_2^{\mu(+)} . \tag{2}$$

These satisfy

$$\begin{aligned} J_{1,2}^{\mu(-)} &= e^{i\pi I_2} J_{1,2}^{\mu(+)} e^{-i\pi I_2} \\ &= \pm J_{1,2}^{\mu(+)\dagger} , \end{aligned} \tag{3}$$

where the negative sign refers to J_2, the second-class piece. The weak current is defined to be charge-symmetric if it contains no second-class term, in which case

$$\begin{aligned} \langle\Lambda | J_W^\mu |\Sigma^-\rangle &= \langle\Lambda | J_1^{\mu(+)} |\Sigma^-\rangle \\ &= \langle\Lambda | J_W^{\mu\dagger} |\Sigma^+\rangle . \end{aligned} \tag{4}$$

The decay amplitudes are

$$T^{(+)} = \bar{u}(\nu)\gamma_\mu(1-\gamma_5)v(\bar{e})\langle\Lambda | J_W^{\mu\dagger}|\Sigma^+\rangle \tag{5}$$

and

$$T^{(-)} = \bar{u}(e)\gamma_\mu(1-\gamma_5)v(\bar{\nu})\langle\Lambda | J_W^{\mu}|\Sigma^-\rangle . \tag{6}$$

On forming the transition probabilities, and carrying out all hadron spin summations, one obtains

$$\overline{\sum} |T^{(\pm)}|^2 = \tfrac{1}{2} H_{\mu\nu}^{(\pm)} L^{\mu\nu(\pm)} , \tag{7}$$

where

$$\begin{aligned} L_{\alpha\beta}^{(\pm)} = 4[&g_{\alpha\sigma}\nu_\beta + g_{\beta\sigma}\nu_\alpha - g_{\alpha\beta}\nu_\sigma \pm i\epsilon_{\alpha\beta\sigma\lambda}\nu^\lambda] \\ &\times (e \mp m_e s)^\sigma , \end{aligned} \tag{8}$$

and s is the lepton helicity vector. If time-reversal invariance holds, as we shall assume, then the antisymmetric part of $H_{\mu\nu}^{(\pm)}$ is also odd under parity, and comes from the VA interference term. Then, for a charge-symmetric current, the rates for the mirror transitions are equal only in the case of a pure Fermi or Gamow-Teller transition. In the Cabibbo theory[5] these decays are pure Gamow-Teller, and the hadron current is symmetric. Thus in the Cabibbo theory the two transition probabilities are equal (up to kinematic effects due the mass difference of the charged Σ hyperons).

The hadron current matrix element may be parametrized by a set of spinor functions and form factors,

$$\langle\Lambda|J_W^\mu|\Sigma^-\rangle = \bar{u}(\Lambda)\Gamma^\mu_{(-)}u(\Sigma^-) \tag{9}$$

and

$$\langle\Lambda|J_W^{\mu\dagger}|\Sigma^+\rangle = \bar{u}(\Lambda)\Gamma^\mu_{(+)}u(\Sigma^+), \tag{10}$$

where

$$\Gamma^\mu_{(\pm)} = f_1^{(\pm)}\gamma^\mu + f_2^{(\pm)}\frac{i\sigma^{\mu\nu}}{2m}q_\nu + f_3^{(\pm)}\frac{iq^\mu}{2m} + g_1^{(\pm)}\gamma_5\gamma^\mu + g_2^{(\pm)} - \gamma_5\frac{\sigma^{\mu\nu}}{2m}q_\nu + g_3^{(\pm)}\frac{\gamma_5 q^\mu}{2m}, \tag{11}$$

$$2m = m_\Lambda + m_\Sigma . \tag{12}$$

Time-reversal invariance requires that f_1, f_2, g_1, and g_3 be real, and that f_3 and g_2 be pure imaginary. If the current is charge-symmetric then all form factors satisfy

$$F_j^{(+)} = F_j^{(-)} . \tag{13}$$

In this case, SU(2) symmetry requires f_1 to vanish, by CVC (conservation of vector current), and relates f_2 to the Σ^0 lifetime, while PCAC (partial conservation of axial-vector current) implies

$$2mg_1 = f_\pi g_{\Sigma^+\Lambda\pi^+} = (m_\pi{}^2 - q^2)g_3/2m, \tag{14}$$

g_2 remaining unconstrained. The Wigner-Eckart theorem of SU(3), however, gives further conditions. In this case f_3 and g_2 represent second-class terms in the weak current, and for a charge-symmetric weak current SU(3) symmetry predicts

$$f_2 = -\tfrac{1}{2}\sqrt{3}\,\mu(n), \tag{15}$$

$$g_1{}^2 \simeq (f_\pi/2m)^2\tfrac{8}{27}g_{\pi N}{}^2, \tag{16}$$

where Eq. (13) has been used, and $\mu(n)$ is the anomalous magnetic moment of the neutron, in particle magnetons.

Any deviation from Eq. (13) indicates a charge asymmetry. The origin of such an asymmetry may be in a genuine nonelectromagnetic second-class interaction or in the electromagnetic corrections. Here we examine the electromagnetic effects, with a view to the possible detection of a nonelectromagnetic second-class interaction. In addition to the vertex corrections and the inner bremsstrahlung, there are contributions from electromagnetic mixing of the neutral hyperons (see Fig. 1) and from kinematic and dynamic effects of mass splitting in isomultiplets.

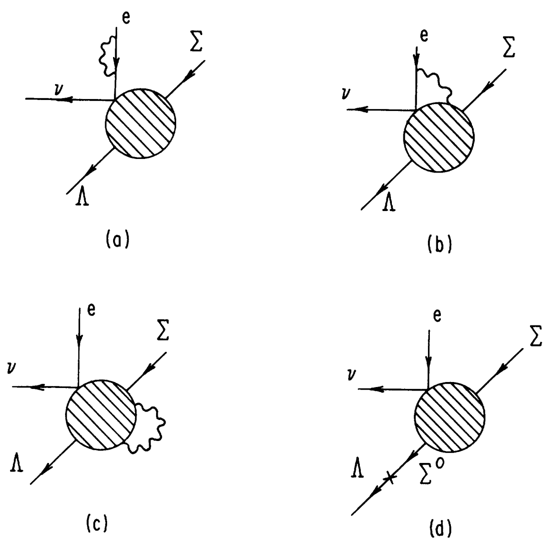

FIG. 1. (a)–(c) represent the usual radiative vertex corrections; (d) is the mixing diagram.

In Sec. II, the conventional corrections are discussed; in Sec. III, the effects of the strong interactions. The effects of mixing are examined in Sec. IV and the results presented in Sec. V. The appendix contains a description of the basic formulas in the text.

II. ELECTROMAGNETIC CORRECTIONS

In order to discuss the electromagnetic corrections, we assume a renormalizable theory of hadronic matter. The electromagnetic interaction is given by the density $\mathcal{L}_{\rm em}(x)$:

$$\mathcal{L}_{\rm em} = eA_\mu J^\mu_{\rm em} + \delta g, \tag{17}$$

where the additive counterterm δg is inserted to implement the renormalization of the masses and coupling constants in the theory of hadrons. To second order in e, where we work, this is always possible. δg is a local polynomial in the fields of the theory of the strongly interacting particles. The electromagnetic current is the sum of leptonic and hadronic parts,

$$J^\mu_{\rm em} = J^\mu + j^\mu, \tag{18}$$

the lepton current is bilinear in the lepton fields, and the hadronic current is a vector functional of the strong interactions,

$$J^\mu(x) = \mathfrak{D}^\mu(x)\{\mathcal{L}_s\} . \tag{19}$$

The S matrix, to lowest order in G and all orders in e, is the sum of two pieces:

$$S = S^{(+)} + S^{(-)}, \tag{20}$$

where

$$S^{(+)} = \frac{iG}{\sqrt{2}} \int dx\, T^* l_\mu(x) J_W^\mu(x) \exp\left[i \int dy\, \mathcal{H}_{\rm em}(y)\right], \tag{21}$$

$$S^{(-)} = \frac{iG}{\sqrt{2}} \int dx\, T^* l_\mu^\dagger(x) J_W^\mu(x) \exp\left[i \int dy\, \mathcal{H}_{\rm em}(y)\right], \tag{22}$$

where T^* denotes the covariant-ordered product. The generalizations of Eqs. (5) and (6) are

$$iT^{(+)}(2\pi)^4\delta^4(P_f - P_i) = \langle \Lambda\bar{e}\nu | S^{(+)} | \Sigma^+ \rangle, \tag{23}$$

$$iT^{(-)}(2\pi)^4\delta^4(P_f - P_i) = \langle \Lambda e\bar{\nu} | S^{(-)} | \Sigma^- \rangle. \tag{24}$$

Under the discrete transformation

$$U = CPTG, \tag{25}$$

where G is the G-parity operator (which commutes with all functions of the lepton fields), we obtain[6]

$$U|\Sigma^+\rangle = |\Sigma^-\rangle, \tag{26}$$

$$U|\Lambda e\bar{\nu}\rangle = |\Lambda\bar{e}\nu\rangle. \tag{27}$$

Thus

$$\langle \Lambda\bar{e}\nu | S^{(+)} | \Sigma^+\rangle = \langle \Lambda e\bar{\nu} | U S^{(+)} U^{-1} | \Sigma^-\rangle. \tag{28}$$

The condition

$$U S^{(+)} U^{-1} \sim S^{(-)} \tag{29}$$

requires that the weak current have definite G parity and that the isoscalar part of $\mathcal{H}_{\rm em}$ not contribute to the S matrix. Under these conditions the two Σ decay amplitudes are equal to all orders in electromagnetism.

These conditions are satisfied in the case of radiative corrections to the decay of bare hyperons, and where the bare weak vertex in J_W is pure axial-vector, as in the Cabibbo theory. In this case, which is one of the very few where detailed dynamical calculations are possible, the two decay amplitudes are the same to all orders in α. Thus the electromagnetic asymmetry derives wholly from the details of the strong interactions, and cannot be evaluated according to the conventional methods in the literature, which are based largely on the dynamics of bare hadrons.[7]

The S matrix contains both infrared and ultraviolet divergences. The infrared problem is handled systematically[8] by introducing a photon mass λ. The numerator of the photon propagator is then replaced by $-g_{\mu\nu} + k_\mu k_\nu/\lambda^2$. The second part does not contribute to the S matrix because of gauge invariance; thus the only effect of the photon mass is to replace k^2 by $k^2 - \lambda^2$ in the denominator. The divergence appearing as $\lambda \to 0$ is, of course, canceled by corresponding terms in the soft-photon emission rate.

The ultraviolet divergence is handled in a gauge-invariant manner by multiplying the photon propagator by $-\Lambda^2(k^2 - \Lambda^2)^{-1}$. This method is, of course, no more physical than any other, and the finite part of the result, as well as the explicit dependence on Λ, depends upon this choice.

This may be summarized by writing the photon propagator as

$$D_{\mu\nu}(k) = -g_{\mu\nu} D(k), \tag{30}$$

where

$$D(k) = -\Lambda^2 (k^2 - \Lambda^2)^{-1} (k^2 - \lambda^2)^{-1}. \tag{31}$$

III. EFFECTS OF THE STRONG INTERACTIONS

In order to expand the S matrix to order α, it is necessary to assume that all contact terms in $\mathcal{H}_{\rm em}$ are canceled by counterterms or Schwinger terms. Then the Hamiltonian density is simply $-eJ\cdot A$ and the vertex correction for Σ^- decay may be written

$$T^{(-)} = \frac{G}{\sqrt{2}} \bar{u}(e)\gamma_\sigma(1-\gamma_5)v(\bar{\nu}) \times \left[Z_f t^\sigma - \frac{ie^2}{2(2\pi)^4} \int dk\, D(k)(E_1^\sigma - 2E_2^\sigma + E_3^\sigma) + \delta t^\sigma \right], \tag{32}$$

where

$$Z_f = 1 + \frac{2ie^2}{(2\pi)^4} \int dk\, D(k)(k^2 - 2e\cdot k)^{-1} \left[\frac{e\cdot k(e^2 + e\cdot k)}{e^2 k^2} - 1 \right], \tag{33}$$

$$E_1^\sigma = k^2 \frac{\partial}{\partial k^\sigma}(g_{\alpha\beta} T^{\alpha\beta} k^{-2}) + 2\epsilon^{\alpha\beta\rho\sigma} T_{\alpha\beta} k_\rho k^{-2} - q^\lambda \frac{\partial}{\partial q^\sigma}(g_{\alpha\beta} T^{\alpha\beta\lambda}), \tag{34}$$

$$E_2^\sigma = (k^2 - 2e\cdot k)^{-1} \times T_{\alpha\beta}[g^{\beta\sigma}(q + 2e)^\alpha + 2e\cdot k k^{-2}(k^\sigma g^{\alpha\beta} - i\epsilon^{\alpha\beta\rho\sigma}k_\rho)], \tag{35}$$

$$E_3^\sigma = \frac{\partial}{\partial q^\sigma}(g_{\alpha\beta} M^{\alpha\beta}) + 2(k^2 - 2e\cdot k)^{-1} M^\sigma. \tag{36}$$

A derivation of this and the corresponding expression for $T^{(+)}$ is given in the Appendix, together with definitions of the hadron tensors. The vector q is the momentum release:

$$-q = e + \nu. \tag{37}$$

The methods of the first perturbation calculations [9–12] of the corrections to hadron β decay are equivalent to saturating the hadron tensors with only the initial and final hadron states. No form factors are introduced and all terms proportional to q are neglected. Such a technique in the present case gives no asymmetry, by the theorem of Sec. II, and the asymmetry derives from multiparticle states in the implied sum over states in the hadron tensors. The magnitude of the asymmetry is estimated by showing that both the high- and low-k^2 regions of the k integrand in Eq. (32) give no asymmetry. An upper limit on the contribution of the intermediate region may be estimated by assuming that the antisymmetric contribution of this region is approximately bounded by the symmetric contribution obtained in the calculation on bare hadrons. Consideration of the various simple graphs which contribute to the hadron tensors suggests that this may give a somewhat larger value than is the case. Nonetheless, this upper limit is the one presented here.

The effects on strong interactions are discussed by most authors.[13–26] The main tool in investigations of the ultraviolet behavior is the Bjorken limit.[26] In order that the limit give the correct divergent terms, it is necessary to assume that the ordinary time-ordered product is, in fact, the correct covariant-ordered product, that the amplitudes in $T_{\alpha\beta}$ satisfy a dispersion relation in k_0, with a finite number of subtractions, and are polynomially bounded in k^2 on the cuts,[27] and that there exists a local chiral algebra of the hadron currents. In the models considered, the Bjorken limit gives

$$\lim_{k_0\to\infty,\,|\vec{k}|\text{ fixed}} T^{(-)}_{\alpha\beta} = \frac{1}{k^0}\int d^3x\, e^{-ikx}\langle\Lambda|[J_\alpha(x,0), J^W_\beta(0)]|\Sigma^-\rangle$$
$$= \frac{1}{k^2}\left[k_\alpha t_\beta + k_\beta t_\alpha - \frac{k_\alpha k_\beta}{k^2} k\cdot t + \left(\frac{k_\alpha k_\beta}{k^2} - g_{\alpha\beta}\right) k\cdot\hat{t} + \epsilon\langle Q\rangle \epsilon_{\alpha\beta\rho\sigma} k^\rho \hat{t}^\sigma\right]. \tag{38}$$

The ultraviolet behavior of the corrections is controlled by the last term which, by isospin symmetry [Eq. (A25)] is the same for both $T^{(+)}$ and $T^{(-)}$. Thus the integrands of the two k integrals are effectively equal above the value of k^2 where the Bjorken limit begins to apply.

There is also a divergent contribution from the counterterm δt^σ:

$$\delta t^\sigma = -i\int dx\, e^{-iqx}\langle\Lambda|T^* J^\sigma_W(x)\delta g(0)|\Sigma^-\rangle . \tag{39}$$

Sirlin[24] and Preparata and Weisberger[25] have studied this, in an attempt to test the model dependence of the divergent piece of the radiative corrections. Several authors have studied δt^σ in specific models.[21–23] In general, there is found a close connection between the divergences in corrections to β decay and in the electromagnetic mass shifts of the hadrons. The precise relation is highly model-dependent, and the conditions under which the divergence in β decay is universal are very restrictive.[25] However, if the theory of the strong interactions is sufficiently well-behaved, the divergent contributions in $\delta g(x)$ have the same symmetry as the theory of hadrons. On assuming G invariance of the strong interactions, one may show then

$$U\delta g^{\text{div}} U^{-1} = \delta g^{\text{div}} , \tag{40}$$

and the counterterms give the same divergent contributions to $T^{(+)}$ and $T^{(-)}$.

The low-k^2 region of the integral in Eq. (31) is, of course, dominated by the lowest-mass states, the initial and final hadron states. This contribution is charge-symmetric. Thus, the charge asymmetry is seen to derive from the intermediate region, where multiparticle states contribute, and is infrared- and ultraviolet-convergent. Following the arguments of Ref. 16 the hadron tensors in this region are given by

$$g_{\alpha\beta}T^{\alpha\beta} = -2k_\sigma t^\sigma (k^2 + 2k\cdot p)^{-1} , \tag{41}$$

$$\epsilon_{\alpha\beta\rho\sigma}T^{\alpha\beta}k^\rho = -2\epsilon\langle Q\rangle t_\lambda (g^\lambda_\sigma - k^\lambda k_\sigma/k^2) \times (k^2 + 2k\cdot p)^{-1} . \tag{42}$$

Similar expressions exist for the M tensors. The resulting integrals are evaluated in the rest frame of the decaying hyperon and all terms proportional to lepton momenta are neglected. In this approximation, the contribution of the intermediate region to the vertex correction is proportional to the zeroth-order amplitude, and the upper limit obtained on the charge asymmetry is

$$t_\sigma - t'_\sigma \leq t_\sigma \frac{3\alpha}{4\pi}(\ln A + \tfrac{1}{2}), \tag{43}$$

where

$$A = \frac{m + M[(1 + 2m/M)^{1/2} + 1]}{m + \mu[(1 + 2m/\mu)^{1/2} + 1]} . \tag{44}$$

The mass M crudely represents the energy at which scaling sets in, μ is the mass of the lightest hadron (the pion), and m is the mass of the decaying hadron. Choosing M to be a few GeV, one obtains

$$\tfrac{3}{4}(\ln A + \tfrac{1}{2}) \approx 2\,. \tag{45}$$

The amplitude for bremsstrahlung of a single photon accompanying the decay of the Σ^- is

$$T_B = e\bar{u}(e)\left[\epsilon_\nu^* T^{\nu\lambda} + t^\lambda\left(\frac{2e\cdot\epsilon^* + \epsilon^*\cdot\gamma\not{q}}{2e\cdot q}\right)\right] \times\gamma_\lambda(1-\gamma_5)v(\bar{\nu})\,, \tag{46}$$

where q is the photon momentum. Separating from $T^{\nu\lambda}$ the Σ-pole term

$$T^{\nu\lambda} = -t^\lambda(p_1^\nu/p_1\cdot q) + T_m^{\nu\lambda}\,, \tag{47}$$

the bremsstrahlung amplitude may be written as the conventional pole approximation with a correction term:

$$T_B = -e\bar{u}(e)\left[t^\lambda\left(\frac{p_1\cdot\epsilon^*}{p_1\cdot q} - \frac{2e\cdot\epsilon^* + \epsilon^*\gamma\not{q}}{2e\cdot q}\right) - \epsilon_\nu^* T_m^{\nu\lambda}\right] \times\gamma_\lambda(1-\gamma_5)v(\bar{\nu})\,. \tag{48}$$

The conventional piece is clearly charge-symmetric. The remaining term interferes with the pole terms in the rate,[28] but because of the extra phase-space factors in the bremsstrahlung rate this interference term is of the order (energy release/m). Thus it may be neglected in lowest order, and in this order the bremsstrahlung rate is charge-symmetric.

IV. EFFECTS OF Σ-Λ MIXING

In the presence of electromagnetism, the physical neutral hyperons are no longer eigenstates of the total isospin.[29-32] The physical states diagonalize the Hamiltonian, whereas the states occurring in the various theories of the strong interactions diagonalize isospin. The two pairs of states are related by a transformation which is both unitary and real, since the electromagnetic Hamiltonian is Hermitian. The mixing angle, β, is defined as follows:

$$\begin{aligned}\Lambda_{\text{phys}} &= \Lambda^0\cos\beta + \Sigma^0\sin\beta\,,\\ \Sigma_{\text{phys}} &= \Sigma^0\cos\beta - \Lambda^0\sin\beta\,.\end{aligned} \tag{49}$$

The current matrix element responsible for Σ^- β decay may be written

$$\begin{aligned}t_\sigma^{(-)} &= \langle\Lambda_{\text{phys}}|J_\sigma^W|\Sigma^-\rangle\\ &= t_\sigma\cos\beta + \langle\Sigma^0|J_\sigma^W|\Sigma^-\rangle\sin\beta\,.\end{aligned} \tag{50}$$

Unfortunately the second matrix element is not accessible experimentally owing to the short Σ^0 lifetime. In the Cabibbo theory one obtains

$$t_\sigma^{(-)} = \bar{u}(\Lambda)[f^{(-)}\gamma_\sigma + g^{(-)}\gamma_5\gamma_\sigma]u(\Sigma)\,, \tag{51}$$

where

$$\begin{aligned}f^{(\pm)} &= \mp\sqrt{2}\,\sin\beta\,,\\ g^{(\pm)} &= \eta(\tfrac{2}{3})^{1/2}\cos\beta \mp (1-\eta)\sqrt{2}\,\sin\beta\,,\end{aligned} \tag{52}$$

and $f^{(+)}$, $g^{(+)}$ describe Σ^+ β decay, and $\eta\approx 0.6$ is related to the pion-nucleon coupling F/D ratio:

$$\frac{1-\eta}{\eta} = \frac{F}{D}\,. \tag{53}$$

The mixing angle is formally given by

$$\tan 2\beta = -\frac{2\langle\Lambda|H_{\text{em}}|\Sigma\rangle}{m_\Sigma - m_\Lambda}\,. \tag{54}$$

The Coleman-Glashow expression[33] for the electromagnetic mass splitting of the baryons may be used to estimate β if the matrix element is identified as the (3, 8) element of the mass matrix. This gives the estimate

$$\beta = -0.021 \pm 0.006\,, \tag{55}$$

where the errors reflect the accuracy of the assumption of octet breaking for these masses. Dalitz and von Hippel[31] have examined some of the consequences of such a large mixing angle. They showed that the $\Lambda\Lambda\pi$ coupling due to $\Sigma\Lambda$ and $\pi\eta$ mixing gives approximately the correct difference in binding energy for the hyperon in the hypernuclei ${}_\Lambda H^4$ and ${}_\Lambda He^4$. Since most of this coupling is due to hyperon mixing [cf. Eq. (10) of Ref. 31] we conclude that the mixing angle is indeed close to the Coleman-Glashow value.

V. CALCULATION OF THE DECAY RATE

The expression for the uncorrected decay rate, to lowest order in the lepton momenta, may be obtained from Eqs. (9), (10), and (11), retaining only the form factors $g_1^{(\pm)}$, all others being set to zero. In the rest frame of the decaying hyperon, the electron spectrum is

$$\frac{dN_0^{(\pm)}}{dE} = \frac{G}{2\pi^3}|g_1^{(\pm)}|^2 pE(E_m^{(\pm)} - E)^2\,, \tag{56}$$

where E and p are the electron's energy and momentum, and $E_m^{(\pm)}$ is its maximum energy. In the Cabibbo theory,

$$g_1^{(\pm)} = g \equiv 2^{1/2}\eta g_A\cos\theta_C\,, \tag{57}$$

and the spectra are charge-symmetric. The uncorrected rates are

$$R^{(\pm)} = G^2P^{(\pm)} \equiv G^2|g_1^{(\pm)}|^2\frac{(\Delta m^{(\pm)})^5}{60\pi^3}\,. \tag{58}$$

The electromagnetic corrections are calculated by first evaluating the radiative corrections to the decays of the charged Σ hyperons to the Λ hyperon. The effect of mixing is to multiply that amplitude by $\cos\beta$, and add to it the uncorrected amplitude for decay to the Σ^0 hyperon multiplied by $\sin\beta$. This gives the corrections to order α. The resulting electron spectrum is

$$\frac{dN^{(\pm)}}{dE}=\frac{dN_0^{(\pm)}}{dE}\left[1+\frac{\alpha}{\pi}(A_0+d)\right]+\frac{\alpha}{\pi}G^2(\rho_1\pm\rho_2), \quad (59)$$

where Eqs. (52) and (56) are to be used. A_0 and d are the divergent contributions of the vertex correction and the counterterm. The finite parts ρ_1 and ρ_2 are functions of E and $E_m^{(\pm)}$. Defining a, b, c, and m by

$$P^{(\pm)}(a\pm b)=\int_{m_e}^{E_m^{(\pm)}}\rho_1 dE\ , \quad (60)$$

$$P^{(\pm)}c=\int_{m_e}^{E_m^{(\pm)}}\rho_2 dE\ , \quad (61)$$

and

$$m=-2\sqrt{3}\,(1-\eta)\eta^{-1}\tan\beta\ , \quad (62)$$

the rates in the Cabibbo theory are given by

$$R^{(\pm)}=\int\left(\frac{dN^{\pm}}{dE}\right)dE$$
$$=G^2g^2\frac{(\Delta m^{(\pm)})^5}{60\pi^3}\left[1+\frac{\alpha}{\pi}(A_0+d+a\pm b\pm c)\pm m\right]. \quad (63)$$

Defining a renormalized Fermi constant by

$$G_R/G=1+\frac{\alpha}{2\pi}(A_0+d+a) \quad (64)$$

the rates are

$$R^{(\pm)}=R_0^{(\pm)}\left[1\pm m\pm\frac{\alpha}{\pi}(b+c)\right]\ , \quad (65)$$

where $R_0^{(\pm)}$ is the rate calculated according to the Cabibbo theory in the absence of electromagnetic corrections. This choice of G_R and the associated identification of $R_0^{(\pm)}$ are motivated by simplicity and are not unique. The experimental definition of G_R is

$$R^{(+)}(\Delta m^{(+)})^{-5}+R^{(-)}(\Delta m^{(-)})^{-5}=G_R{}^2(30\pi^3)^{-2}\ , \quad (66)$$

and we define a charge symmetry parameter, δ, by

$$\delta=\tfrac{1}{2}(R^{(+)}/R_0^{(+)}-R^{(-)}/R_0^{(-)})$$
$$=m+\frac{\alpha}{\pi}(b+c)\ . \quad (67)$$

The discussion of Sec. III indicates that an approximate upper bound on c is

$$|c|\lesssim 2\ . \quad (68)$$

The contribution b comes from integrating the radiative correction over the slightly different phase spaces available. A calculation of the radiative corrections to bare hadrons gives

$$b=+0.15\ . \quad (69)$$

Thus the conventional radiative corrections give a small asymmetry:

$$|\delta-m|\lesssim 0.005\ . \quad (70)$$

However, the large mixing angle discussed in Sec. IV gives a larger asymmetry:

$$m=-0.036\pm 0.010\ . \quad (71)$$

Thus the electromagnetic charge asymmetry is due mostly to mixing and enhances Σ^+ decay over Σ^- decay:

$$\delta=+0.036\pm 0.015\ . \quad (72)$$

VI. DISCUSSION

The sensitivity of δ to the mixing angle opens up the possibility that these decays may provide a way to measure the angle. The experimental ratio of the rates is[1]

$$R^{(+)}/R^{(-)}=0.62\pm 0.22\ , \quad (73)$$

and leads to a mixing angle consistent with zero, i.e.,

$$\tan\beta=0.0\pm 0.1\ . \quad (74)$$

An increase in statistics by a factor of 30 would narrow the error down to the present theoretical estimates of β. However, measurement of the rates defines β only within the Cabibbo theory, or a framework which predicts the matrix element

$$\langle\Sigma^0|J_\mu^W|\Sigma^-\rangle\ ,$$

since it is not accessible experimentally due to the short lifetime of the Σ^0 hyperon.

Because of mixing, the ratios of the form factors are

$$f_1^{(+)}/g_1^{(+)}=+0.05$$

and (75)

$$f_1^{(-)}/g_1^{(-)}=-0.04\ .$$

These may be compared with the result of Franzini *et al.*,[34] who obtained

$$f/g=-0.37\pm 0.2\ . \quad (76)$$

A ratio this large implies either a nonelectromagnetic violation of CVC, or that the Cabibbo theory predicts a value of g too large by an order of magnitude.

The sensitivity of the relative rates to mixing makes the detection of second-class currents harder to achieve, since the effects of mixing cannot be calculated uniquely. In order to detect a nonelectromagnetic asymmetry, one must show that one mixing angle will not describe the asymmetry of every form factor [cf. Eq. (12)]. This requires a measurement of at least two form factors for each decay, which, according to the CVC hypothesis [where f_1 is $O(\alpha)$], requires measurement of correlations among the decay products to an accuracy better than $O(\alpha)$. If a second-class effect is present here, hopefully it will be too large to be attributed to a mixing effect, which would remove the necessity for such a precise measurement of the form factors.

APPENDIX

The amplitude for Σ^- β decay to order α may be obtained in a straightforward way from Eq. (19):

$$T^{(-)}=\frac{G}{\sqrt{2}}\bar{u}(e)\left\{Z_2^{1/2}t_\lambda-\frac{i\alpha}{8\pi^3}\int dk D^{\mu\nu}(k)[T_{\mu\nu\lambda}(k,q)-2T_{\mu\lambda}(k,q)\gamma^\mu(\not{e}-\not{k}-m)^{-1}]\right\}\gamma^\lambda(1-\gamma_5)v(\bar{\nu}). \tag{A1}$$

The corresponding expression for Σ^+ decay is

$$T^{(+)}=\frac{G}{\sqrt{2}}\bar{u}(\nu)\gamma^\lambda(1-\gamma_5)\left\{Z_2^{1/2}t'_\lambda-\frac{i\alpha}{8\pi^3}\int dk D^{\mu\nu}(k)[T'_{\mu\nu\lambda}(k,q)-2T'_{\mu\lambda}(k,q)(-\not{e}+\not{k}-m)^{-1}\gamma^\mu]\right\}v(\bar{e}). \tag{A2}$$

The hadron tensors are defined by

$$t^\lambda=\langle\Lambda|J_W^\lambda(0)|\Sigma^-\rangle, \tag{A3}$$

$$T^{\mu\lambda}(k,q)=i\int dx\langle\Lambda|T^*J^\mu(x)J_W^\lambda(0)|\Sigma^-\rangle e^{ikx}, \tag{A4}$$

$$T^{\mu\nu\lambda}(k,q)=\int dx\,dz\langle\Lambda|T^*J^\mu(x)J^\nu(0)J_W^\lambda(z)|\Sigma^-\rangle \times e^{-ikx-iqz}, \tag{A5}$$

and the primed tensors are obtained from these by the replacement

$$J_W^\lambda(y)|\Sigma^-\rangle\to J_W^\lambda(y)^\dagger|\Sigma^+\rangle. \tag{A6}$$

The M tensors are obtained from the T tensors by the replacement

$$J_W^\lambda(y)\to -i\partial_\lambda J_W^\lambda(y). \tag{A7}$$

Ward identities may now be developed upon the assumption of the following commutators:

$$\delta(x_0)[J^0(x),J_W^\lambda(0)]=-\delta(x_0)[J_W^0(x),J^\lambda(0)] =i\delta^4(x)J_W^\lambda(0). \tag{A8}$$

Upon contracting $T^{\mu\nu\lambda}(k,p)$ with p, where p is an arbitrary vector, one obtains

$$p^\lambda T_{\mu\nu\lambda}(k,p)=M_{\mu\nu}(k,p)+T_{\mu\nu}(k,q) +T_{\nu\mu}(k+q-p,q). \tag{A9}$$

Differentiation with respect to p and setting $p=q$ gives

$$T_{\mu\nu\sigma}(k,q)=\frac{\partial}{\partial k^\sigma}T_{\nu\mu}(k,q)+\frac{\partial}{\partial q^\sigma}M_{\mu\nu}(k,q) -q^\lambda\frac{\partial}{\partial q^\sigma}T_{\mu\nu\lambda}(k,q). \tag{A10}$$

Further identities are

$$k^\nu T_{\nu\mu}(k,q)=-t_\mu, \tag{A11}$$

$$k^\mu T_{\nu\mu}(k,q)=-t_\nu+M_\nu(k,q)-q^\lambda T_{\nu\lambda}(k,q), \tag{A12}$$

$$T'_{\mu\nu\sigma}(k,q)=-\frac{\partial}{\partial k^\sigma}T'_{\nu\mu}(k,q)+\frac{\partial}{\partial q^\sigma}M'_{\mu\nu}(k,q) -q^\lambda\frac{\partial}{\partial q^\sigma}T'_{\mu\nu\lambda}(k,q), \tag{A13}$$

$$k^\nu T'_{\nu\mu}(k,q)=t'_\mu, \tag{A14}$$

$$k^\mu T'_{\nu\mu}(k,q)=t'_\nu+M_\nu(k,q)-q^\lambda T_{\nu\lambda}(k,q), \tag{A15}$$

and two spinor identities:

$$\bar{u}(e)\gamma_\mu(\not{e}-\not{k}+m)\gamma_\lambda(1-\gamma_5)v(\bar{\nu}) =\bar{u}(e)\gamma^\sigma(1-\gamma_5)v(\bar{\nu})H_{\mu\lambda\sigma}, \tag{A16}$$

$$\bar{u}(\nu)\gamma_\lambda(1-\gamma_5)(-\not{e}+\not{k}+m)\gamma_\mu v(\bar{e}) =\bar{u}(\nu)\gamma^\sigma(1-\gamma_5)v(\bar{e})H'_{\mu\lambda\sigma}, \tag{A17}$$

where

$$H_{\mu\lambda\sigma}=2e_\mu g_{\lambda\sigma}+k_\sigma g_{\mu\lambda} -(k_\mu g_{\lambda\sigma}+k_\lambda g_{\mu\sigma})-i\epsilon_{\mu\lambda\rho\sigma}k^\rho \tag{A18}$$

and

$$H'_{\mu\lambda\sigma}=-H^*_{\mu\lambda\sigma}. \tag{A19}$$

Use of these identities leads directly to the amplitude $T^{(-)}$ given in Eqs. (31) ff. The amplitude $T^{(+)}$ has the same form as that in Eq. (31) but where t_σ and δt_σ are replaced by primed quantities, and the integrands are as follows[35]:

$$E_1^{\sigma\prime}=-k^2\frac{\partial}{\partial k^\sigma}(g_{\alpha\beta}T^{\alpha\beta\prime}k^{-2})+2\epsilon^{\alpha\beta\rho\sigma}T'_{\alpha\beta}k_\rho k^{-2} -q^\lambda\frac{\partial}{\partial q^\sigma}(T'_{\alpha\beta\lambda}g^{\alpha\beta}), \tag{A20}$$

$$E_2^{\sigma\prime} = (k^2 - 2e\cdot k)^{-1} \times T'_{\alpha\beta}[-g^{\beta\sigma}(q+2e)^\alpha - 2e\cdot k k^{-2}(k^2 g^{\alpha\beta} + i\epsilon^{\alpha\beta\rho\sigma}k_\rho)], \tag{A21}$$

$$E_3^{\sigma\prime} = \frac{\partial}{\partial q^\sigma}(g^{\alpha\beta}M'_{\alpha\beta}) - 2(k^2 - 2e\cdot k)^{-1}M'_\sigma . \tag{A22}$$

The evaluation of the commutator obtained on taking the Bjorken limit Eq. (37) is based on various field-theoretic models. The tensor t_σ is obtained from t_σ by including only the contribution of terms in J_W^σ which are bilinear in the fundamental fermion fields of the assumed theory of hadrons. The isospin properties of the various terms in Eq. (37) in these models may be obtained by writing the electromagnetic current as the sum of two terms of isospin 1 and 0, respectively:

$$J_\mu = I_\mu + Y_\mu . \tag{A23}$$

Then, customarily,

$$[I^\alpha(x), J_W^\beta(0)] = +[I^\beta(x), J_W^\alpha(0)], \tag{A24}$$

$$[Y^\alpha(x), J_W^\beta(0)] = -[Y^\beta(x), J_W^\alpha(0)]. \tag{A25}$$

---

*Work supported in part by the National Science Foundation and by the United States Atomic Energy Commission.

†Work based on a dissertation presented in partial fulfillment of the requirements for a Ph.D. degree at Northwestern University.

[1]Particle Data Group, Rev. Mod. Phys. 45, S1 (1973).

[2]L. Michel, Nuovo Cimento 10, 319 (1953); T. D. Lee and C. N. Yang, *ibid.* 3, 749 (1956).

[3]S. Weinberg, Phys. Rev. 112, 1375 (1958).

[4]M. A. B. Bég and J. R. Bernstein, Phys. Rev. D 5, 714 (1972).

[5]N. Cabibbo, Phys. Rev. Lett. 10, 531 (1963).

[6]Polarizations are not observed.

[7]The charge asymmetry may be said to depend on the hypercharge densities of the hyperons.

[8]D. R. Yennie, S. Frautschi, and H. Suura, Ann. Phys. (N.Y.) 13, 379 (1961).

[9]R. E. Behrends, R. J. Finkelstein, and A. Sirlin, Phys. Rev. 101, 866 (1956).

[10]S. M. Berman, Phys. Rev. 112, 267 (1958).

[11]T. Kinoshita and A. Sirlin, Phys. Rev. 113, 1652 (1959).

[12]S. M. Berman and A. Sirlin, Ann. Phys. (N.Y.) 20, 20 (1962).

[13]G. Källén, Nucl. Phys. B1, 225 (1967).

[14]A. Sirlin, Phys. Rev. 164, 1767 (1967).

[15]E. S. Abers, D. A. Dicus, and R. E. Norton, Phys. Rev. Lett. 18, 676 (1967).

[16]E. S. Abers, D. A. Dicus, R. E. Norton, and H. R. Quinn, Phys. Rev. 167, 1461 (1968).

[17]N. Cabibbo, L. Maiani, and S. Preparata, Phys. Lett. 25B, 31, 132 (1967).

[18]K. Johnson, F. E. Low, and H. Suura, Phys. Rev. Lett. 18, 1224 (1967).

[19]A. Sirlin, Phys. Rev. Lett. 19, 877 (1967).

[20]N. Brene, M. Roos, and A. Sirlin, Nucl. Phys. B6, 225 (1968).

[21]C. G. Callan, Jr., Phys. Rev. 169, 1175 (1968).

[22]J. Schwinger, Phys. Rev. Lett. 19, 1501 (1967); 19, 1154 (1967).

[23]B. W. Lee and H. T. Nieh, Phys. Rev. 166, 1507 (1968).

[24]A. Sirlin, Phys. Rev. 176, 1871 (1968).

[25]G. Preparata and W. I. Weisberger, Phys. Rev. 175, 1965 (1968).

[26]J. D. Bjorken, Phys. Rev. 148, 1467 (1967).

[27]Reference 25, footnote 12.

[28]See also T. H. Burnett and N. M. Kroll, Phys. Rev. Lett. 20, 86 (1968).

[29]R. H. Dalitz, in *High Energy Physics, Les Houches, 1965,* edited by C. DeWitt and M. Jacob (Gordon and Breach, New York, 1966), p. 203.

[30]A. J. MacFarlane and E. G. C. Sudarshan, Nuovo Cimento 31, 1176 (1964).

[31]R. H. Dalitz and F. von Hippel, Phys. Lett. 10, 153 (1964).

[32]R. H. Dalitz; A. R. Bodmer, and D. M. Rote, in *Proceedings of the International Conference on Hypernuclear Physics, Argonne National Laboratory, 1969,* edited by A. R. Bodmer and L. G. Hyman (ANL, Argonne, Ill., 1969).

[33]S. Coleman and S. L. Glashow, Phys. Rev. Lett. 6, 423 (1961).

[34]P. Franzini *et al*., Phys. Rev. D 6, 2417 (1972).

[35]That $T^{(\pm)}$ give identical transition probabilities has been verified in a calculation on bare hyperons.