[20]B. Lørstad *et al.*, Nucl. Phys. B14, 63 (1969).
[21]J. Campbell *et al.*, Phys. Rev. Lett. 22, 1204 (1969).
[22]M. Gell-Mann, D. Sharp, and W. G. Wagner, Phys. Rev. Lett. 8, (1962); S. Hori, S. Oneda, S. Chiba, and H. Hiraki, Phys. Lett. 1, 81 (1962).
[23]We do not use asymptotic SU(3) for this problem, since our treatment is already approximate. The error could be as large as 20–30%.
[24]Seisaku Matsuda and S. Oneda, Phys. Rev. 174, 2106 (1968).
[25]F. J. Gilman and M. Kugler, Phys. Rev. Lett. 30, 518 (1973); F. J. Gilman, M. Kugler, and S. Meshkov, Phys. Lett. 45B, 481 (1973).
[26]See the remark of Ref. 25 (GK) made in this respect.
[27]In the quark model this corresponds to the statement that the D consists solely of $\Lambda\bar{\Lambda}$ quarks.
[28]For more details see T. Laankan, Ph.D. thesis, Univ. of Maryland, 1973 (unpublished).
[29]R. W. Jacobel *et al.*, Phys. Rev. Lett. 29, 671 (1972).

PHYSICAL REVIEW D VOLUME 9, NUMBER 11 1 JUNE 1974

$K_L \to 2\mu$, $K_L \to 2\gamma$, and quark masses in gauge models

Ernest Ma*
Department of Physics, University of California, Santa Barbara, California 93106
(Received 17 December 1973)

Experimental knowledge of the decay rates for $K_L \to 2\mu$ and $K_L \to 2\gamma$ is combined with theoretical understanding of strangeness-changing neutral-current effects in unified gauge models of the weak and electromagnetic interactions for a determination of quark masses. Given that the Weinberg parameter $\sin^2\theta_W$ is between 0.3 and 0.4, the mass of the proton quark becomes roughly 900 MeV, while that of the "charmed" quark is about 5 GeV.

As a part of weak-interaction phenomenology, it is well known that strangeness-changing neutral current effects are extremely small. For example, the decay rate for $K_L \to \mu^+\mu^-$ is less than 10^{-8} times that for $K^+ \to \mu^+\nu$. To understand this dramatic suppression, in the language of unified gauge models[1] of the weak and electromagnetic interactions, one has to assume that the neutral vector boson does not couple directly to a quark-antiquark pair carrying strangeness; in other words, no elementary transition is to occur between an $\mathfrak{N}$ quark and a λ quark. As a result, the lowest-order contribution to the process $K_L \to 2\mu$ becomes rather the exchange of two charged vector bosons (Fig. 1).[2] To get an idea of the magnitude of this contribution, we calculate the box graph of Fig. 1, using the appropriate Feynman rules[3] corresponding to a simple extension[4] of the Weinberg-Salam model.[1] We find, in the limit of small momenta for the external quarks, the following effective interaction:

$$\frac{G_F^2 M_W^2}{2\pi^2}\sin\theta_C\cos\theta_C\,\bar{\lambda}\gamma_\mu\tfrac{1}{2}(1-\gamma_5)\mathfrak{N}\bar{\mu}\gamma^\mu\tfrac{1}{2}(1-\gamma_5)\mu\,, \tag{1}$$

where $G_F \simeq 10^{-5}/(\text{proton mass})^2$ is the weak coupling constant, M_W is the mass of the charged vector boson, and θ_C is the Cabibbo angle. Comparing this with the amplitude of $K^+ \to \mu^+\nu$, and using for the branching ratio $\Gamma(K_L \to 2\mu)/\Gamma(K_L \to \text{all})$ the value[5] 11×10^{-9}, we get[6]

$$2\left(\frac{G_F M_W^2}{4\pi^2}\right)^2 \simeq 1.9\times10^{-9}\,,$$

indicating that $M_W \simeq 10.4$ GeV, which is contrary to the theoretical input condition of the Weinberg-Salam model[1] that $M_W > 37$ GeV.

As was pointed out by Lee, Primack, and Treiman,[2] this apparent contradiction can be removed with the introduction of an extra quark $\mathcal{P}'$, the so-called "charmed" quark of Glashow, Iliopoulos, and Maiani,[7] such that cancellations occur between the resulting additional graph and the original one. This is most naturally done in unified gauge models by letting the proton quark $\mathcal{P}$ couple to $\mathfrak{N}\cos\theta_C + \lambda\sin\theta_C$ and the charmed quark $\mathcal{P}'$ couple to the orthogonal combination $-\mathfrak{N}\sin\theta_C + \lambda\cos\theta_C$.[4] We now consider two possibilities: (a) $m_{\mathcal{P}'} \simeq m_{\mathcal{P}} \equiv m \ll M_W$ and $\Delta m^2 \equiv |m_{\mathcal{P}'}^2 - m_{\mathcal{P}}^2| \ll m^2$, and (b) $m_{\mathcal{P}} \ll m_{\mathcal{P}'} \ll M_W$. For (a), the effective interaction (1), with the inclusion of single neutral vector-boson exchange, is modified by the factor

$$\frac{\Delta m^2}{M_W^2}\left(\ln\frac{M_W^2}{m^2} - 2\right)\,, \tag{2}$$

whereas for (b), it is

$$\frac{m_{\mathcal{P}'}^2}{M_W^2}\left(\ln\frac{M_W^2}{m_{\mathcal{P}'}^2} - 1\right)\,. \tag{3}$$

We have, therefore, further suppression of the

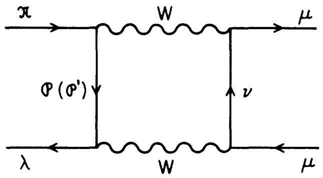

FIG. 1. $\mathfrak{N}\bar{\lambda} \to 2\mu$.

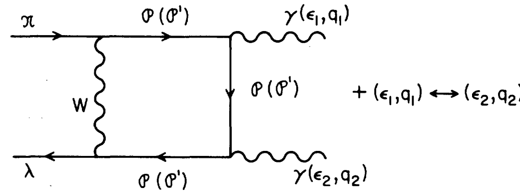

FIG. 2. $\mathfrak{N}\bar{\lambda} \to 2\gamma$.

decay rate for $K_L \to 2\mu$, and using the same argument as before, we obtain either

$$\text{(a)}\quad \Delta m\left(\ln\frac{M_W^2}{m^2} - 2\right)^{1/2} \simeq 10.4\ \text{GeV} \tag{4}$$

or

$$\text{(b)}\quad m_{\mathcal{P}'}\left(\ln\frac{M_W^2}{m_{\mathcal{P}'}^2} - 1\right)^{1/2} \simeq 10.4\ \text{GeV}, \tag{5}$$

which is, however, insufficient for a unique determination of all the masses. The next logical step then is to consider the process $K_L \to 2\gamma$ which is governed to lowest order by the box graph shown in Fig. 2.[8] We find, again in the limit of small momenta for the external quarks, the following effective interaction:

$$\frac{G_F}{\sqrt{2}}\frac{e^2}{4\pi^2}\sin\theta_C\cos\theta_C \times \ln\frac{m_{\mathcal{P}'}^2}{m_{\mathcal{P}}^2}\,\epsilon_{abcd}\,\epsilon_2^a(q_2-q_1)^b\,\epsilon_1^c\,\bar{\lambda}\gamma^d\,\tfrac{1}{2}(1-\gamma_5)\mathfrak{N}, \tag{6}$$

where the suppression factor due to $\mathcal{P}'$ is $\ln(m_{\mathcal{P}'}^2/m_{\mathcal{P}}^2)$ rather than the factors (2) or (3), which was the case for $K_L \to 2\mu$. This makes a very important difference, as the following discussion will show.

Neglecting the presumably small CP-violating effects,[9] we take the real part of the amplitude for $K_L \to 2\mu$ to be (1) times (2) or (3), while the imaginary part is assumed to be entirely given by the two-photon intermediate state.[10] We then have, allowing for phase-space corrections, and using[5] the experimental value 2.2×10^{-5} for the branching ratio $\Gamma(K_L \to 2\mu)/\Gamma(K_L \to 2\gamma)$, either

$$\text{(a)}\quad m\left(\ln\frac{M_W^2}{m^2} - 2\right)^{1/2} \simeq 5.6\ \text{GeV} \tag{7}$$

or

$$\text{(b)}\quad m_{\mathcal{P}'}\left(\ln\frac{M_W^2}{m_{\mathcal{P}'}^2} - 1\right)^{1/2}\left(\ln\frac{m_{\mathcal{P}'}^2}{m_{\mathcal{P}}^2}\right)^{-1/2} \simeq 5.6\ \text{GeV}. \tag{8}$$

Comparing (7) with (4), we see clearly that assumption (a) is not being justified, whereas dividing (5) by (8), we get

$$\frac{m_{\mathcal{P}'}}{m_{\mathcal{P}}} \simeq 5.5 \tag{9}$$

and, solving for M_W in (5), we have

$$M_W^2 \simeq m_{\mathcal{P}'}^2 e^{1+(10.4/m_{\mathcal{P}'})^2}. \tag{10}$$

If we furthermore take, from the recent report on high-energy neutrino experiments,[11] the value of $\sin^2\theta_W \equiv (37.2\ \text{GeV}/M_W)^2$ to be between 0.3 and 0.4, we obtain a very narrow range of allowed values for $m_{\mathcal{P}'}$, centered at[12]

$$m_{\mathcal{P}'} \simeq 5\ \text{GeV} \tag{11}$$

and from (9),

$$m_{\mathcal{P}} \simeq 900\ \text{MeV}. \tag{12}$$

In conclusion, we note that whereas the specific numbers quoted in the above are not model-independent, the qualitative assertion that $m_{\mathcal{P}} \ll m_{\mathcal{P}'} \ll M_W$ does seem to be on firm ground in general. Further consideration of similar processes such as $K^+ \to \pi^+e^+e^-$ is expected to yield valuable information and they are being investigated.

Finally, upon the essential completion of this work, it became known to the author that the same results were also being obtained by Gaillard and Lee.[13]

---

*Work supported by the National Science Foundation.

[1]S. Weinberg, Phys. Rev. Lett. 19, 1264 (1967); A. Salam, in *Elementary Particle Theory: Relativistic Groups and Analyticity (Nobel Symposium No. 8)*, edited by N. Svartholm (Almqvist and Wiksell, Stockholm, 1968); B. W. Lee, in *Proceedings of the XVI International Conference on High Energy Physics, Chicago-Batavia, Ill., 1972*, edited by J. D. Jackson and A. Roberts (NAL, Batavia, Ill., 1973), Vol. 4, p. 249.

[2]B. W. Lee, J. R. Primack, and S. B. Treiman, Phys. Rev. D 7, 510 (1973). There are also graphs (not shown) involving the exchange of a single neutral vector boson in the Weinberg-Salam model.

[3]These are given by K. Fujikawa, B. W. Lee, and A. I. Sanda, Phys. Rev. D 6, 2923 (1972).

[4]S. Weinberg, Phys. Rev. D 5, 1412 (1972).

[5]W. C. Carithers *et al.*, Phys. Rev. Lett. 30, 1336 (1973).

[6]The contribution of the 2γ intermediate state to the decay rate is taken to be 6×10^{-9}. See Ref. 10.

[7]S. Glashow, J. Iliopoulos, and L. Maiani, Phys. Rev. D 2, 1285 (1970).

[8]Other graphs exist to this order, but under the assumptions (a) or (b), the one shown in Fig. 2 is dominant. Whereas the graph for $\mathfrak{N}\bar{\lambda} \to 2p \to 2\gamma$ is proportional to $\ln(m_{\mathcal{P}'}^2/m_{\mathcal{P}}^2)$, the graphs for $\mathfrak{N}\bar{\lambda} \to 2W \to 2\gamma$ are proportional to $m_{\mathcal{P}'}^2/M_W^2$, which is assumed to be much smaller.

[9]K_L has a small $CP=+1$ component which is assumed not to have an anomalously large CP-violating decay rate into 2μ.

[10]L. M. Sehgal, Phys. Rev. 183, 1511 (1969).

[11]F. J. Hasert *et al.*, Phys. Lett. 46B, 138 (1973).

[12]Because of the functional form of Eq. (10), $m_{\mathcal{P}'}$ changes less than 0.2 GeV within the range in question.

[13]M. K. Gaillard and B. W. Lee, Phys. Rev. D (to be published).

PHYSICAL REVIEW D VOLUME 9, NUMBER 11 1 JUNE 1974

Charge symmetry and electromagnetic corrections to $\Sigma^\pm \to \Lambda e^\pm \nu$ decays*

David Eimerl†

Department of Physics, University of California, San Diego, La Jolla, California 92037

(Received 5 October 1973; revised manuscript received 24 January 1974)

Electromagnetic corrections to $\Sigma^\pm \to \Lambda e^\pm \nu$ decays are investigated with a view to the detection of a possible nonelectromagnetic charge asymmetry. A single renormalization of the Fermi constant renders both rates finite. The electromagnetic asymmetry is due mostly to electromagnetic $\Sigma\Lambda$ isospin mixing, and may be as much as 10%. The remaining electromagnetic asymmetry depends on the hypercharge densities of the hyperons, and approximate bounds on it are estimated.

I. CHARGE SYMMETRY AND THE Σ DECAYS

The semileptonic decays of the Σ hyperons to the Λ hyperon[1] provide a possible experimental test for charge asymmetry in the weak interaction. There are several reasons for considering these decays for such a test. The absence of any nuclear structure corrections and the isospin selection rule on the weak current ($\Delta I = 1$) are perhaps the most persuasive, while the large energy release (~80 MeV) opens up the possibility of measuring the weak magnetism and pseudotensor components of the current matrix element. The subsequent decay of the Λ hyperon to a proton and a pion serves to measure the helicity of the Λ, allowing an almost complete reconstruction of the Σ decay. (The electron helicity is fixed by the $V-A$ form of the lepton current.) Also, these matrix elements of the current are interesting in that they are the only accessible examples of the matrix element of the current between states of zero hypercharge, owing to the instability of the neutral Σ hyperon.

The semileptonic Hamiltonian density is

$$\mathcal{H}_W = \frac{G}{\sqrt{2}}(J_W^\mu l_\mu^\dagger + \text{H.c.}), \tag{1}$$

where the weak hadronic current responsible for the Σ decays to Λ carries isospin 1 and may be written as the sum of a first-class and a second-class[2-4] piece:

$$J_W^\mu \equiv J^{\mu(+)} = J_1^{\mu(+)} + J_2^{\mu(+)}. \tag{2}$$

These satisfy

$$J_{1,2}^{\mu(-)} = e^{i\pi I_2} J_{1,2}^{\mu(+)} e^{-i\pi I_2} = \pm J_{1,2}^{\mu(+)\dagger}, \tag{3}$$

where the negative sign refers to J_2, the second-class piece. The weak current is defined to be charge-symmetric if it contains no second-class term, in which case

$$\langle\Lambda|J_W^\mu|\Sigma^-\rangle = \langle\Lambda|J_1^{\mu(+)}|\Sigma^-\rangle = \langle\Lambda|J_W^{\mu\dagger}|\Sigma^+\rangle. \tag{4}$$

The decay amplitudes are

$$T^{(+)} = \bar{u}(\nu)\gamma_\mu(1-\gamma_5)v(\bar{e})\langle\Lambda|J_W^{\mu\dagger}|\Sigma^+\rangle \tag{5}$$

and

$$T^{(-)} = \bar{u}(e)\gamma_\mu(1-\gamma_5)v(\bar{\nu})\langle\Lambda|J_W^\mu|\Sigma^-\rangle. \tag{6}$$

On forming the transition probabilities, and carrying out all hadron spin summations, one obtains

$$\overline{\sum}|T^{(\pm)}|^2 = \tfrac{1}{2}H_{\mu\nu}^{(\pm)}L^{\mu\nu(\pm)}, \tag{7}$$

where

$$L_{\alpha\beta}^{(\pm)} = 4[g_{\alpha\sigma}\nu_\beta + g_{\beta\sigma}\nu_\alpha - g_{\alpha\beta}\nu_\sigma \pm i\epsilon_{\alpha\beta\sigma\lambda}\nu^\lambda] \times (e \mp m_e s)^\sigma, \tag{8}$$

and s is the lepton helicity vector. If time-reversal invariance holds, as we shall assume, then