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Large-transverse-momentum phenomena and Fermi's statistical model for multiparticle production at high energies

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Recent experimental findings at NAL and at the CERN ISR in connection with large-transverse-momentum phenomena are analyzed. It is observed that these data can be understood in the framework of Fermi's statistical model. A new scaling variable is proposed. Further experiments are suggested.

I. INTRODUCTION

Recent measurements at the CERN ISR[1-3] and at NAL[4,5] of inclusive cross sections $E\,d^3\sigma/dp^3$ at large transverse momentum ($p_T > 1$ GeV/c) and wide center-of-mass angles (θ near 90°) have attracted much attention in the study of multiparticle production processes. In this paper we present the result of an analysis of these data as well as of other experimental findings[6,7] relevant to the large-p_T phenomena. We show that these data can be understood in the framework of Fermi's statistical model.[8,9]

The most striking features of the large-transverse-momentum processes are that (a) the wide-angle inclusive cross section $E\,d^3\sigma/dp^3$ for large p_T is much larger than that expected from the simple extrapolation of the exponential behavior encountered for $p_T < 1$GeV/c, (b) this cross section increases with increasing total energy $\sqrt{s}$, (c) the energy dependence of this quantity is small at low p_T but becomes stronger as p_T increases, (d) at fixed energies the cross section at wide angles (θ near 90°) is almost independent of θ, (e) the pions have lost the overwhelming supremacy which they show over all other secondaries at p_T<1 GeV, and (f) the associated multiplicity at wide angles increases with increasing p_T. It has been reported[6] that the Pisa-Stony Brook group sees an increase of the charged multiplicity at wide angle when the transverse momentum of a specified wide-angle γ ray is increased. At the same time the multiplicity at small angle (very fast secondaries) drops. The CERN-Columbia-Rockefeller group finds that the charged multiplicity at 90° increases greatly when measurement of a 3-GeV/c π^0 is required. This increase is found not only in the direction opposite to the π^0 (factor 2.5, say) but also in the direction of π^0 (factor 1.8, say). Furthermore, (g) a strong "back-to-back" correlation between large-transverse-momentum π^0's is found.

Because of the apparent close connection [cf. item (f)] between the production of large-p_T particles and the angular distribution of multiplicities, it seems useful also to study the angular distribution in semi-inclusive processes. Angular distribution data, obtained *only* from processes in which at least one large-p_T particle is observed, are not available. However, such distributions *independent of this restriction* can be obtained from the existing data[10,6] for the single-particle η distribution at fixed charged multiplicities, $(1/\sigma_n)d\sigma_n/d\eta$. Here $\eta = -\ln\tan(\theta/2)$, n is the charged multiplicity, σ_n is the topological cross section,

$$\frac{1}{\sigma_n}\frac{d\sigma_n}{d\cos\theta} = \cosh^2\eta\left(\frac{1}{\sigma_n}\frac{d\sigma_n}{d\eta}\right), \quad (1)$$

and θ is the c.m. scattering angle. These data[10,6] are given in Fig. 1. We see that while the forward

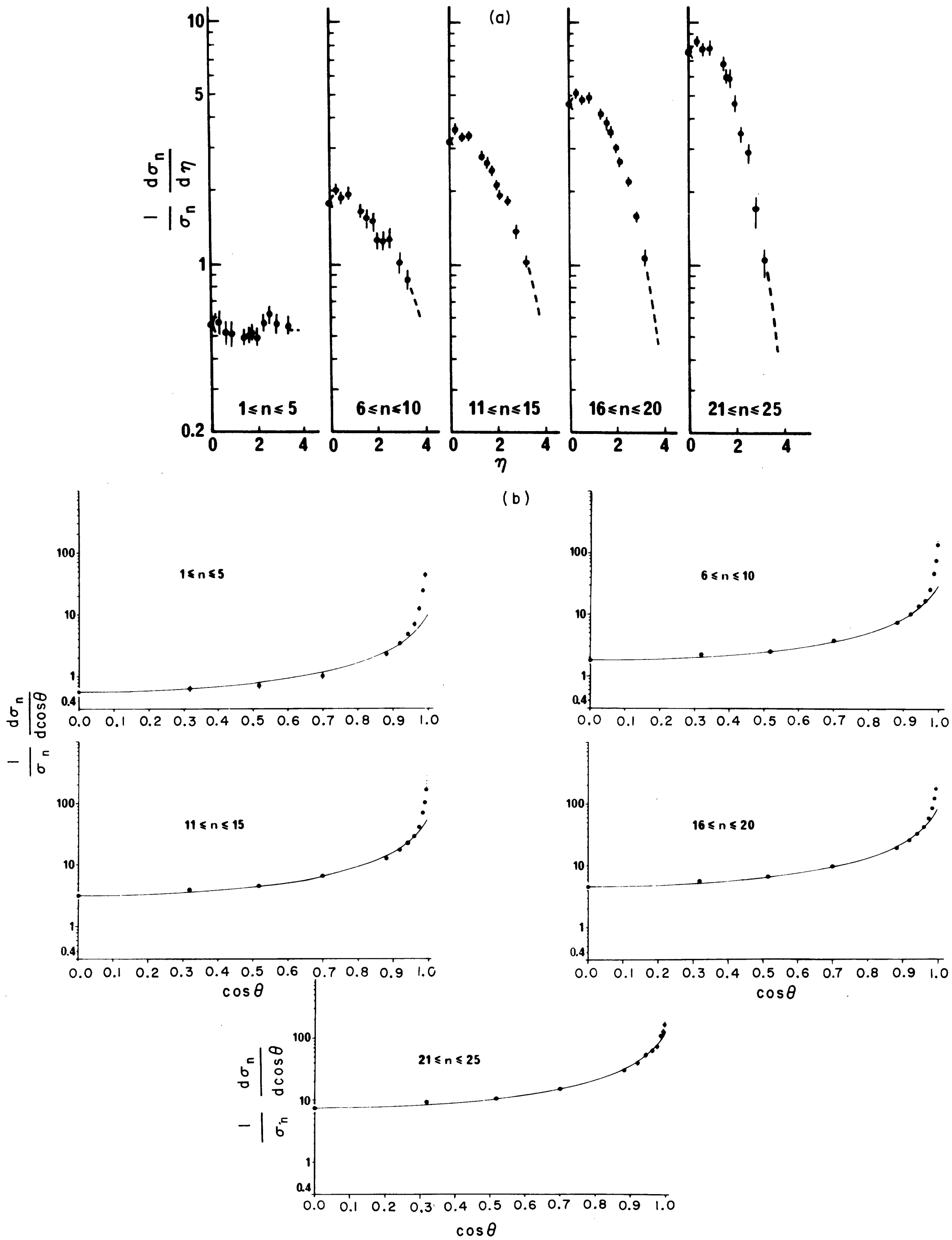

FIG. 1. (a) Single-particle η distributions for fixed charged multiplicities at ISR momentum = 15.4 GeV/c. Data are taken from Refs. 10 and 6. The ordinate normalization is arbitrary. The extrapolations to η = 3.8 are made by hand. (b) Inclusive angular distributions obtained from the data given in (a). The broken lines which extend to $\cos\theta = 0.9989$ correspond to the extrapolation in (a). The solid lines are obtained from Eqs. (22) and (23). They are separately normalized such that the point at $\cos\theta = 0$ coincides with the corresponding data point.

(and backward) directions ($\cos\theta=\pm 1$) are preferred by events of all multiplicities, the ratio of the population at small angles ($\cos\theta\approx\pm 1$) to that at large angles ($\cos\theta\approx 0$) becomes smaller for events with larger multiplicities. It seems that the large-p_T processes are predominantly associated with large-multiplicity events ($n>\langle n\rangle$ say, where $\langle n\rangle$ is the average multiplicity). As a working hypothesis we assume that this is the case. We shall show in the following that indeed both the transverse momentum distribution and the fixed-multiplicity angular distribution data can be understood in terms of the same (a modified Fermi) model.

According to the picture proposed by Fermi,[8,9] when two high-energy nucleons collide, the energy available in their center-of-mass system is released in a small volume having dimensions of the order of magnitude of the pion cloud surrounding the nucleons. The concentration of energy will rapidly dissolve and the particles into which the energy has been converted will fly in all directions. The basic assumptions made in Fermi's model are the following: (1) The interaction between pions (we recall that his papers were published in 1950/51) and nucleons is such that a statistical equilibrium is attained among all states that are compatible with the conservation laws. (2) The probability that the collision may result in the formation of one of the possible final states is proportional to the probability that the state in question will have all its particles contained at the same time inside the small volume into which the energy has been concentrated. (3) The volume V (the only adjustable parameter in this theory) into which the energy of the two colliding nucleons is dumped, and from which the produced particles fly out, is taken to be that of a Lorentz-contracted sphere, i.e.,

$$V(s)=V_0F(s)\,, \tag{2}$$

where

$$V_0=\tfrac{4}{3}\pi R^3\,, \tag{3}$$

$$F(s)=\frac{2M}{\sqrt{s}}\,, \tag{4}$$

M is the nuclear mass, and R is the radius of the sphere (1.4 fermi).

II. STATISTICAL MODEL FOR CENTRAL COLLISIONS

We first discuss the case in which the total c.m. energy $\sqrt{s}$ is the only conserved quantity that should be taken into account. (The total 3-momentum vanishes in the total center-of-mass system.) This is in fact the case when the collision of the two particles is considered to be central. The average number of particles in a quantum state of energy E within the volume V is ($-$ for bosons, $+$for fermions)

$$\frac{1}{e^{E/kT}\mp 1}, \tag{5}$$

where T is the "decay temperature" and k is the Boltzmann constant. For collisions at very high energies, we shall make the usual assumption that all produced particles are extremely relativistic. In such cases, the energy of the particle E in (5) can be replaced by the corresponding momentum $p=(p_T^2+p_L^2)^{1/2}$, where p_L stands for its longitudinal part. The number of particles having momenta with magnitudes between p and $p+dp$ is proportional to

$$\frac{V}{e^{p/kT}\mp 1}, \tag{6}$$

which gives

$$\frac{d^3\sigma}{dp^3}(p,s)\sim Ve^{-p/kT}\text{ for } e^{p/kT}\gg 1\,. \tag{7}$$

The decay temperature T can be found with the help of Stefan's law,

$$T^4\sim\frac{\sqrt{s}}{V(s)}\,. \tag{8}$$

That is, the energy dependence of the decay temperature is completely determined by that of the decay volume $V(s)$. The following cases are of particular interest:

$$T\sim s^{1/4}\,, \tag{9}$$

$$T\sim \text{const}, \tag{10}$$

and

$$T\sim s^{1/8}\,. \tag{11}$$

Obviously, (9) follows from (4) which is the formula used by Fermi[8,9] to calculate the total charged multiplicity observed in the cosmic-ray experiment. Equation (10) is the consequence of the Pomeranchuk model,[11,12] in which the decay volume is an increasing function of the total energy. The energy dependence given in (11) follows from the assumption that *V(s) is s-independent*. That is to say, the volume at the moment of decay—whatever the magnitude and the shape of this volume may be—does not depend upon the total energy of the system.

In terms of the variables p_T, θ, and s which have been used in presenting the experimental results, (7) can be written as

$$\frac{d^3\sigma}{dp^3}(p_T,\theta;s)\sim V(s)e^{-p_T/kT(s)\sin\theta}\,. \tag{12}$$

An analysis of the p_T-distribution data near $\theta=90°$

for γ rays and for pions as well as for all positively and negatively charged particles at different energies[1-5] shows that the formula (12) can indeed account for the features (a), (b), and (c) mentioned in Sec. I, *provided that the decay volume* $V(s)$ *is assumed to be independent of the total energy* s. A comparison of the above-mentioned data with the formula for $\theta = 90°$ obtained from (11) and (12),

$$\ln \frac{d^3\sigma}{dp^3}(p_T,\ \theta = 90°; s) \sim \text{const} \times p_T s^{-1/8}\,, \tag{13}$$

is given in Figs. 2–7. We see that

$$z_T = p_T s^{-1/8} \tag{14}$$

plays the role of a "scaling variable" for the large p_T processes and that in connection with the s dependence in p_T distribution, the present model seems to favor the data given in Refs. 2, 3, 4, and 5 against that given in Ref. 1. We think, this comparison shows that the $\theta = 90°$ straight-line parametrization (13) can be considered as a good first-order approximation of the p_T distribution data near 90° (see e.g. Refs. 3 and 7). Besides, we note that the slopes of these straight lines are approximately the same, and that the condition for the validity of the approximation given in (7) is fulfilled.

Furthermore, Eq. (12) implies

$$\ln\left[E\frac{d^3\sigma}{dp^3}(p_T,\theta;s)\right] - \ln\left[E\frac{d^3\sigma}{dp^3}(p_T,\ \theta = 90°; s)\right] = \frac{p_T}{kT(s)}\left(1 - \frac{1}{\sin\theta}\right). \tag{15}$$

Now, the function $1/\sin\theta$ is an extremely slowly varying function of θ near $\theta = 90°$ (e.g., $1/\sin 60° = 1.15$), and a crude estimate obtained from Figs. 2–7 gives

$$\frac{p_T}{kT} \approx 7.6 z_T\,. \tag{16}$$

Hence we can also qualitatively understand why the difference between the log plots for the p_T distributions near 90° cannot be large for small z_T (cf. Ref. 1 and (d) of Sec. I). We shall see in the next section that correction terms must be taken into account when noncentral collisions are included.

III. ANGULAR MOMENTUM CONSERVATION

Since it is not very probable that the collision of the two nucleons should be exactly centered, and that in cases in which the two particles collide somewhat on one side of each other the system is left with a considerable amount of angular momentum, the corresponding conservation law should be taken into account. A geometrical, semiclassical picture was proposed by Fermi[9] twenty-four years ago. Following Fermi, we in-

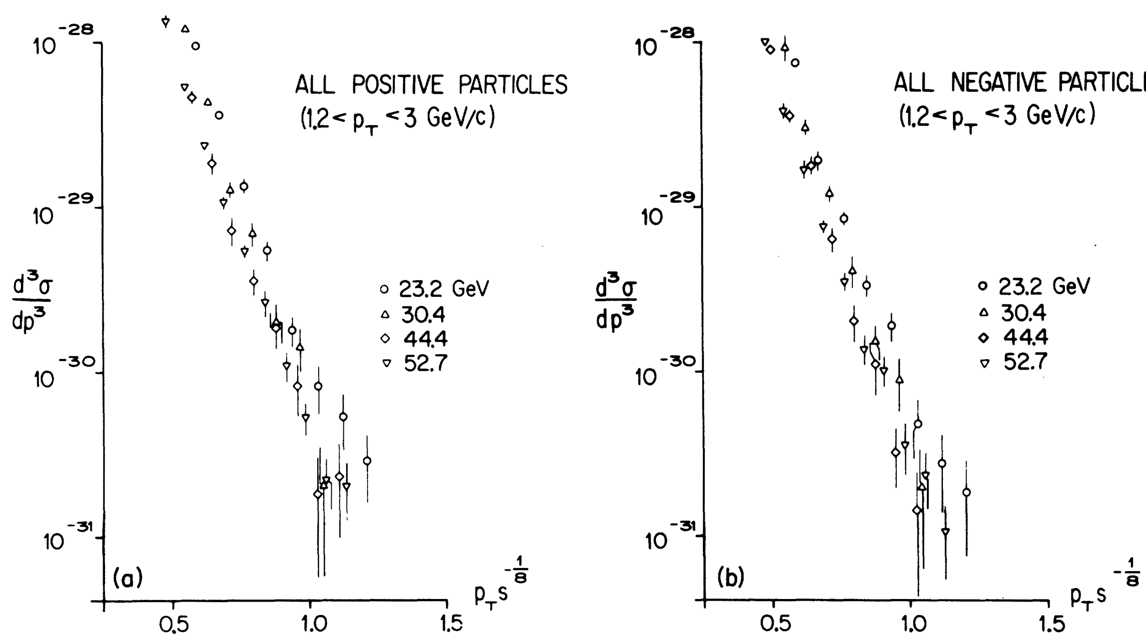

FIG. 2. Inclusive cross section for a single charged particle at $\theta = 90°$ in the center-of-mass system versus $z_T = p_T s^{-1/8}$, where p_T is the transverse momentum (in GeV/c) and $\sqrt{s}$ is the total c.m. energy (in GeV). $d^3\sigma/dp^3$ is given in $\text{cm}^2c^3/\text{sr}\,\text{GeV}^3$. The data are from Ref. 2.

troduce a coordinate system (x, y, z) with its origin on the center of the volume V (at Sec. I). The y axis is drawn parallel to the trajectories of the two nucleons. The z axis is perpendicular to the plane of these two particles and is therefore the direction of the angular momentum vector. Assume that the y dimension of the decay volume V (either s-dependent or s-independent) is flattened such that the z component of the angular momentum L_z of a particle with momentum p emerging at the point of coordinates x and z $(y \approx 0)$ may be approximately given by

$$L_z = xp\cos\theta\,, \tag{17}$$

where θ is the angle between p and the y axis. (This is precisely the c.m. system angle used in Secs. I and II.)

The average number of particles in a quantum state of energy E and the z component of the angular momentum L_z (the components of the total angular momentum in the x and y direction vanish) within the volume V is

$$\frac{1}{e^{\beta E - \lambda L_z} \mp 1}. \tag{18}$$

The constants β and λ are to be determined by the condition that the total energy and angular momentum must have their correct values $\sqrt{s}$ and $r\sqrt{s}$, respectively. Here, r is the distance from the center of the volume V. It is clear that the impact parameter will be different from collision to collision and that r will range in value from zero to R [cf. Eq. (3)]. The median value of r is therefore $R/\sqrt{2}$. Explicit calculations for the case

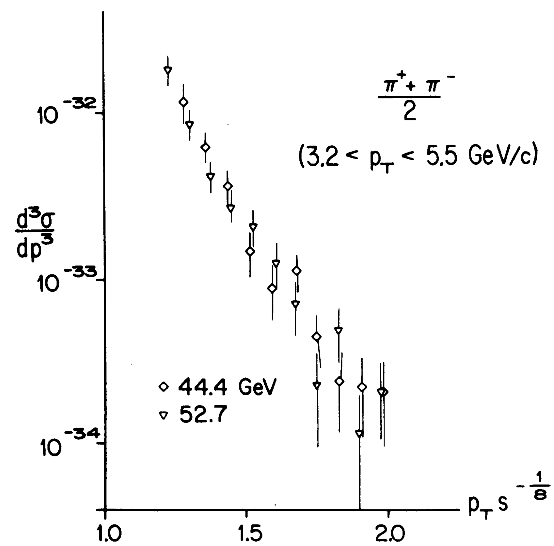

FIG. 3. Inclusive cross section for a single charged pion at $\theta = 90°$ versus z_T. Units are the same as those in Fig. 2.

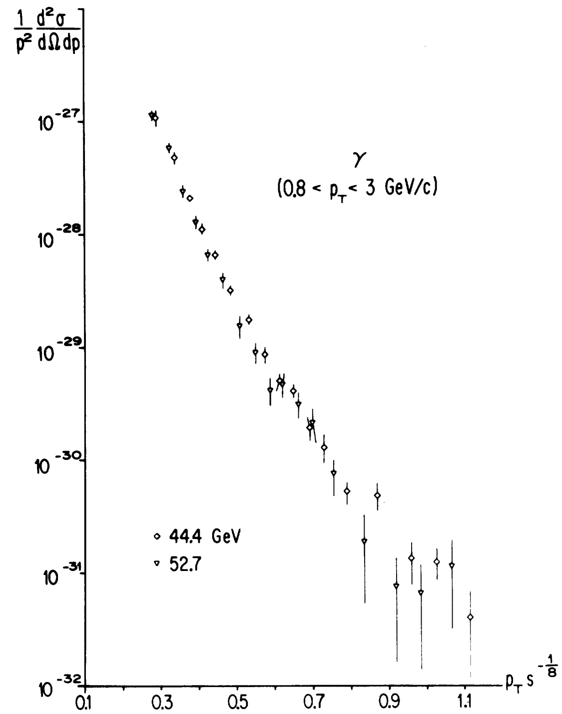

FIG. 4. γ spectra at $\theta = 90°$ from Ref. 2: $d^2\sigma/p^2 d\Omega dp$ versus z_T. $d^2\sigma/p^2 d\Omega dp$ is given in $cm^2 c^3/sr\,GeV^3$, p_T in GeV/c, and $\sqrt{s}$ in GeV.

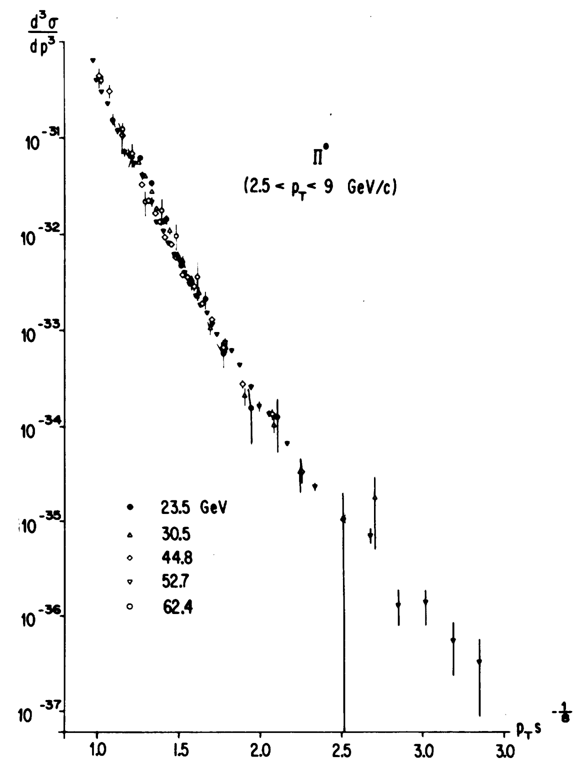

FIG. 5. Neutral-pion spectra near $\theta = 90°$ $(90° \pm 30°)$ from Ref. 3. Units are the same as those in Fig. 2.

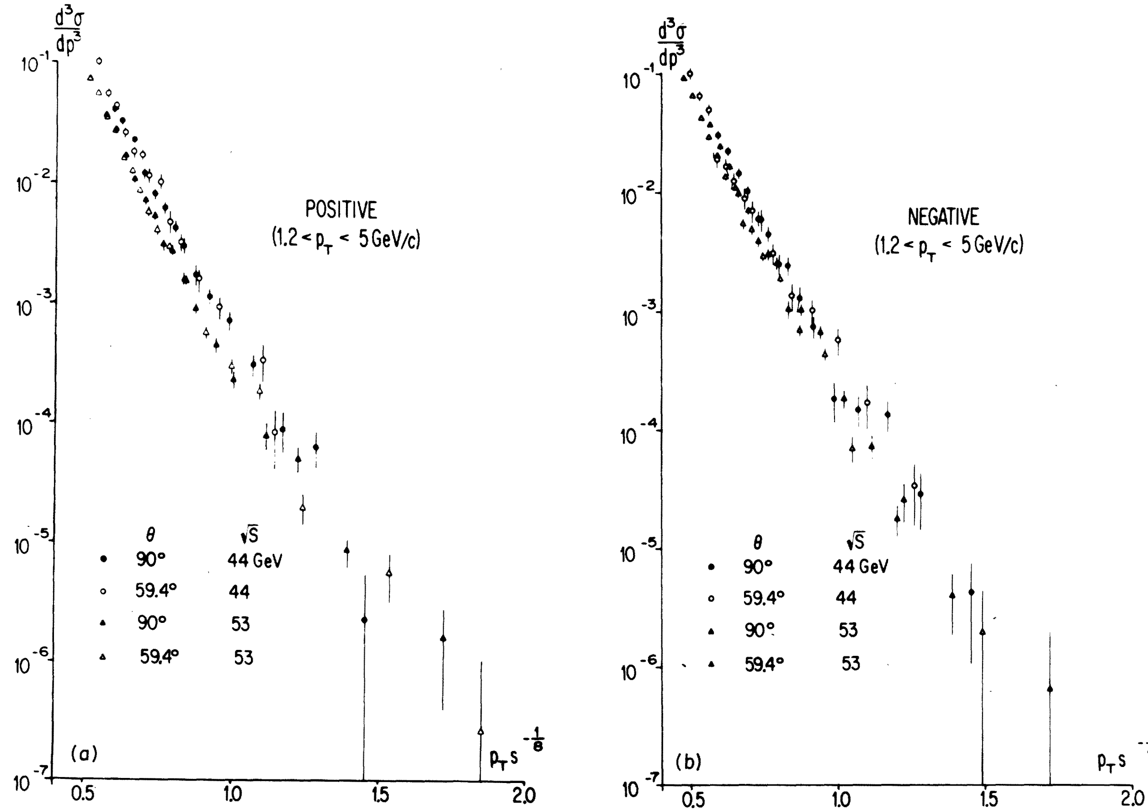

FIG. 6. Spectra for charged particles at $\theta = 90°$ and $\theta = 59.4°$ from Ref. 1. $d^3\sigma/dp^3$ is given in mb c^3/GeV^3, p_T in GeV/c, and $\sqrt{s}$ in GeV.

$F(s) = 2M/\sqrt{s}$ [see Eq. (4)] can be found in Fermi's paper.[9] The calculation for the case $F(s) = \text{const}$ is straightforward. The main result can be summarized as follows:

(a) For $\theta = 90°$, the inclusive single-particle distribution is also given by Eq. (13). That is, it is exactly the same as that for central collisions (cf. Sec. II).

(b) Instead of (15) we have

$$\ln\left[E\frac{d^3\sigma}{dp^3}(p_T, \theta; s)\right] - \ln\left[E\frac{d^3\sigma}{dp^3}(p_T, \theta = 90°; s)\right] = \frac{p_T}{kT(s)}\left(1 - \frac{1}{\sin\theta}\right) + \ln f\left(\frac{p_T}{kT}\cot\theta\right), \quad (19)$$

where

$$f(\alpha) = \frac{3}{2\alpha^3}\left[e^{\alpha}(\alpha - 1) + e^{-\alpha}(\alpha + 1)\right]. \quad (20)$$

Equations (19) and (20) are obtained under the condition

$$\frac{p_T}{e^{kT\sin\theta}}\left(1 - \frac{\rho x \cos\theta}{R}\right) \gg 1, \quad (21)$$

where ρ is 0.959 for $r = R/\sqrt{2}$ (cf. Table I in Ref. 9). From (21) and the rough estimate given in (16), we see that (19) and (20) can be used for wide angles ($\theta \gtrsim 60°$ say) and not too small z_T values ($z_T \gtrsim 0.5$ say). In order to compare (19) with the existing data,[1] we used (16) to calculate the right-hand side of Eq. (19), and obtained the values -0.2, 0.3, 1.1, 2.1, 3.3, and 4.5 for $z_T = 0.5$, 1, 1.5, 2, 2.5, and 3, respectively. This means, while the ratio of the invariant cross section at 59.4° and 90° is roughly between 0.8 and 3.1 for $0.5 \leq z_T \leq 1.5$, it becomes 25 for $z_T = 2.5$. Hence, wide-angle ($\theta \gtrsim 60°$, say) measurements at large z_T (>1.5 say) will be extremely interesting. In extending this kind of comparison to smaller angles, not only (19) must be modified but also an empirical separation of products of large-p_T processes from that of fragmentation processes may be necessary. This point will be discussed in some detail elsewhere.

(c) The fixed multiplicity angular distribution is given by

$$\frac{1}{\sigma_n}\frac{d\sigma_n}{d\cos\theta} = \text{const} \times g(\rho\cos\theta), \quad (22)$$

where

$$g(\alpha) = \frac{2}{\alpha^2(1 - \alpha^2)} - \frac{1}{\alpha^3}\ln\frac{1 + \alpha}{1 - \alpha}. \quad (23)$$

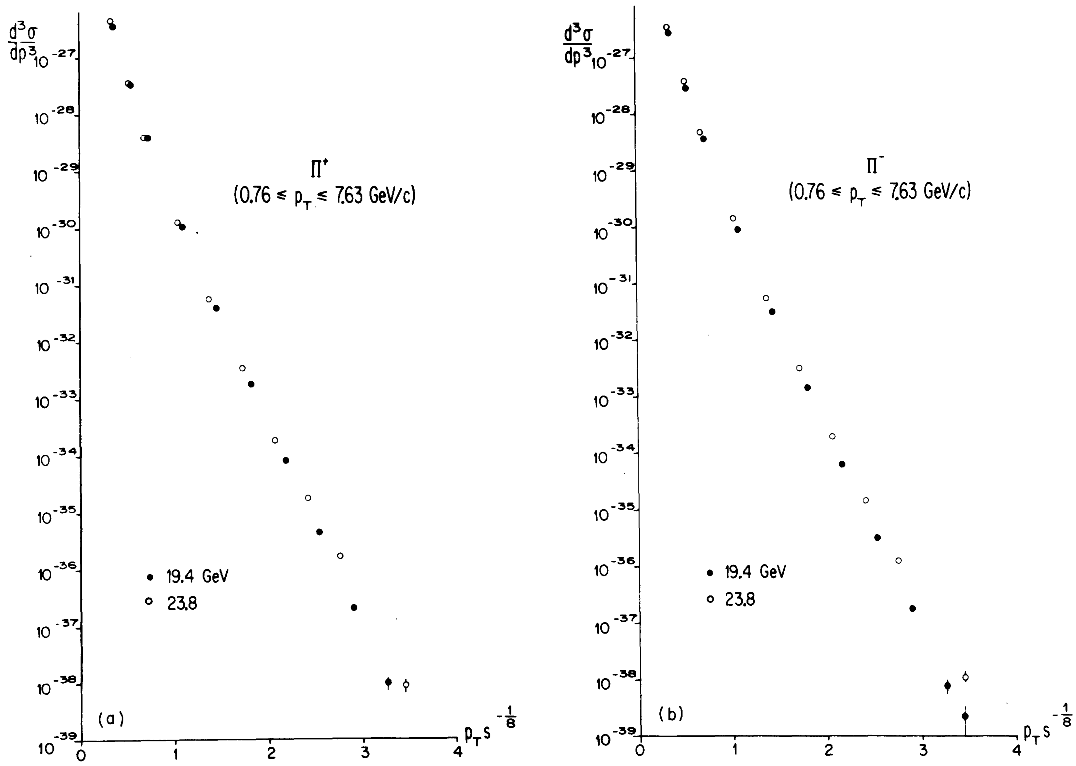

FIG. 7. π^+ and π^- spectra at $\theta \sim 90°$ from Ref. 4. Units are the same as those in Fig. 2.

This function is plotted in Fig. 1(b), where the curves are normalized to the experimental point at $\theta = 90°$. Here we see that the agreement with data for large-multiplicity events is remarkable.

IV. CONCLUDING REMARKS

In the last two sections we found that the wide-angle transverse momentum distribution data for $p_T > 2$ GeV/c with its most striking features [(a), (b), (c), and (d) of Sec. I] can be readily understood in the framework of a modified Fermi model. This proposal differs from that of Fermi's in the following two aspects: (A) The statistical method is *not* applied to *all* high-energy multiparticle production processes but *only* to the large-transverse-momentum processes (cf. Sec. I). (B) The decay volume [cf. Eqs. (2), (8), and (11)] is assumed to be *independent* of the total energy. These two assumptions have led us to propose the scaling variable z_T [Eq. (14)] for the large-p_T processes. Another immediate consequence of (A) and (B) is that the energy dependence of averaged multiplicity for large-transverse-momentum (LTM) processes $\langle n \rangle_{\text{LTM}}$ is given by

$$\langle n \rangle_{\text{LTM}} \sim VT^3 \sim s^{3/8}. \qquad (24)$$

In connection with the angular distribution of the associated multiplicities in such processes, an additional assumption has been made: (C) Large-transverse-momentum processes are predominantly associated with large-multiplicity events (cf. Sec. I). This means, the applicability of the present model to large-p_T processes implies that the same model must also be able to account for high-multiplicity events in general. A comparison with the existing multiplicity angular distribution data has been made in Sec. III. It shows that this is indeed the case.

The observed particle ratios in large-p_T processes [cf. (e) in Sec. I] have already been discussed in the literature.[7,13] It is found that they are consistent with the statistical approach.

In order to obtain further information on large-p_T processes in general, and to find the physical limitations of the present model in particular, we suggest the measurement of the following quantities *at different energies* under the condition that *at least one secondary particle with large transverse momentum ($p_T > 2$ GeV/c, say) is observed in the central region:* (i) average charged multiplicity, (ii) p_T distribution at different angles, (iii) angular distribution, (iv) distribution ratio[14] or angular distribution at fixed multiplicities, (v) multiplicity fluctuation, and (vi) two-particle correlation.

Experimentally, it seems that these measurements can be readily made by using, e.g., the detection of a large-p_T γ or pion as trigger. Theoretically, this information will be very helpful in differentiating models. For example, the present model can be ruled out if the quantities given in (i), (ii), (iii), and (iv) *obtained under the above-mentioned condition* turn out to be inconsistent with those predicted in Eqs. (24), (19), and (22). Fluctuation and correlation measurements indicated in (v) and (vi) will provide information on the possible existence of clustering effects in large-p_T processes. These problems will be discussed elsewhere.

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