Electromagnetic properties of the neutrino from neutral-current experiments*

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(Received 28 January 1974)

Upper bounds on the charge radius and magnetic moment of the muon neutrino are inferred from recent experiments on neutrino-nucleon inclusive reactions and neutrino-electron elastic scattering.

The electromagnetic properties of the neutrino are of much significance, particularly in astrophysics. Traditionally, neutrinos are regarded as massless particles carrying no charge. However, because a neutrino can interact weakly with other charged particles, it can acquire a charge radius. Furthermore, if the neutrino mass is not strictly zero, it can also possess a magnetic moment. Unfortunately, calculations of these quantities based on conventional weak-interaction theories are not useful[1,2] since the results diverge. In the past, information on these properties has primarily been extracted[3] indirectly from astrophysical data, since the laboratory data has been rather meager. Recently however, CERN[4] and NAL have studied "μ-less" ν_μ-nucleon inclusive reactions and there are some data for elastic ν_μ-electron scattering.[5] In view of the obvious interest attached to the possible existence of neutral weak currents, these experiments would doubtless be repeated and refined. The purpose of this note is to show that these experiments can be used to investigate the electromagnetic properties of the neutrino.

If the neutrino has a charge radius and a magnetic moment, the inclusive reaction, $\nu_\mu(\bar{\nu}_\mu)+N \to \nu_\mu(\bar{\nu}_\mu)$ + anything, will receive a contribution from the Feynman diagram involving a photon exchange. We shall call this contribution electromagnetic although, as noted above, the charge radius and magnetic moment of the neutrino themselves could arise from weak-interaction effects. There are other pure weak-interaction contributions to these reactions. If there exists a neutral vector boson Z coupled to the neutrino and hadronic currents,[6] Z-exchange diagrams will contribute. Furthermore, the exchange of a pair of vector bosons like W^+W^- may also contribute. However, since we are interested in obtaining experimental upper bounds for the charge radius and magnetic moment, we may concentrate only on the electromagnetic contribution, and neglect all pure weak contributions. We may not be justified in dropping the interference term since we do not know the sign of this term; so we can only obtain a rough estimate of the bounds, if a genuine neutral weak current exists. To do better, if future data or some astrophysical consequences warrant it, one has to adopt a weak-interaction model. At the present time there are a large number of models in contention, and we would like to avoid specific model-dependent calculations.

The matrix element of the electromagnetic current in a one-neutrino state may be written as

$$\langle \nu(k')|V_\mu|\nu(k)\rangle = i\bar{\nu}(k')[\gamma_\mu F_1(q^2) + \sigma_{\mu\nu}q_\nu F_2(q^2)]\nu(k), \quad (1)$$

where $q=k-k'$, and F_1, F_2 are the standard charge and magnetic moment form factors. Since the neutrino carries no charge, $F_1(0)=0$. Note that if one uses the conventional two-component neutrino theory, the magnetic coupling vanishes,[1,3] and the matrix element (1) would be given only in terms of the charge form factor. A two-component neutrino must be massless, but the present experimental data for the muon neutrino (with which we are concerned here) does not rule out a small mass,[1] so for the sake of generality we shall retain the magnetic-moment term here. For the neutrino form factors, it is reasonable to set

$$F_1(q^2) \simeq \tfrac{1}{6}a^2q^2, \quad F_2(q^2) \simeq \frac{f}{2m_e}, \quad (2)$$

where a^2 is the mean square charge radius and f is the magnetic moment in units of electron-Bohr magnetons.

The one-photon-exchange contribution to the cross section $\nu_\mu + N \to \nu_\mu$ + anything can be computed using Eqs. (1) and (2). In the lab frame, we obtain

$$\frac{d^2\sigma_\gamma}{dx\,dy}(\nu_\mu + N \to \nu_\mu + \text{anything}) = \pi\alpha^2\left\{\tfrac{2}{9}a^4ME\left[xy^2F_1^{\gamma N}(x) + \left(1-y-\frac{Mxy}{2E}\right)F_2^{\gamma N}(x)\right] + \left(\frac{f}{2m_e}\right)^2\left[-2yF_1^{\gamma N}(x) + \frac{(2-y)^2}{xy}F_2^{\gamma N}(x)\right]\right\}, \quad (3)$$

where E is the incident energy of the neutrino in the lab frame, M is the mass of the nucleon, and x, y are the scaling variables

$$x=\frac{q^2}{2M\nu},\qquad y=\frac{\nu}{E},\tag{4}$$

ν being the energy transfer between leptons in the nucleon rest frame, $\nu=E-E'$. $F_1^{\gamma N}$ and $F_2^{\gamma N}$ are the standard structure functions for the target nucleon which in the scaling limit depend only on the dimensionless variable x. Note that the charge-radius contribution to Eq. (3) becomes more important as E becomes larger. Also, since electromagnetic interactions are invariant under charge conjugation, the cross section for the antineutrino reaction $\bar{\nu}_\mu+N\to\bar{\nu}_\mu+$ anything is also given by Eq. (3).

Typically, experiments measure the ratios

$$R(\nu_\mu)=\frac{\sigma(\nu_\mu+N\to\nu_\mu+\text{anything})}{\sigma(\nu_\mu+N\to\mu^-+\text{anything})},\tag{5}$$

$$R(\bar{\nu}_\mu)=\frac{\sigma(\bar{\nu}_\mu+N\to\bar{\nu}_\mu+\text{anything})}{\sigma(\bar{\nu}_\mu+N\to\mu^++\text{anything})}.\tag{6}$$

The reactions $\nu_\mu(\bar{\nu}_\mu)+N\to\mu^-(\mu^+)+$ anything are weak processes and are mediated through the usual charged-current interactions. In the scaling region, the differential cross section has the well-known form[7]

$$\frac{d^2\sigma}{dx\,dy}(\nu_\mu(\bar{\nu}_\mu)+N\to\mu^-(\mu^+)+\text{anything})=\frac{G^2ME}{\pi}\left[xy^2F_1^{\nu N}(x)+\left(1-y-\frac{M}{2E}xy\right)F_2^{\nu N}(x)\mp y(1-\tfrac{1}{2}y)xF_3^{\nu N}(x)\right],\tag{7}$$

where the last term is the V,A interference term, which changes sign under $\nu_\mu\leftrightarrow\bar{\nu}_\mu$ interchange. $F_{1,2,3}^{\nu N}$ are the usual nucleon structure functions for weak currents, depending only on the variable x, and G is the Fermi coupling constant. To obtain the ratios (5) and (6), one has to integrate the formulas (3) and (7). Note that both x and y vary between 0 and 1. The magnetic-moment contribution in Eq. (3), however, leads to an infrared divergence on integration, which arises due to the photon propagator. Note that the charge-radius contribution to Eq. (3) does not show this behavior because the propagator singularity is canceled by the q^2 factor in the form factor $F_1(q^2)$ [see Eq. (2)]. In fact the charge-radius contribution is effectively given by a contact interaction of a pure vector type and one can check that with obvious replacements, it can be reproduced from the weak-interaction formula (7). For the last term in Eq. (3), the $q^2=0$ value can be avoided by integrating over q^2 from some minimum value $(q^2)_{\min}$ accessible to the experiments.

For further simplification we shall use the Callan-Gross relation[8] for the structure functions:

$$2xF_1(x)=F_2(x).\tag{8}$$

From Eqs. (3), (7), and (8), we then write the one-photon contribution to (5) and (6) as

$$(2+B)R_\gamma(\nu_\mu)=(2-B)R_\gamma(\bar{\nu}_\mu)=\frac{3\pi^2\alpha^2}{(GM^2)^2}\left\{\frac{4}{27}(aM)^4\frac{\int F_2^{\gamma N}(x)dx-\frac{3}{8}(M/E)\int xF_2^{\gamma N}(x)dx}{\int F_2^{\nu N}(x)dx-(M/4E)\int xF_2^{\nu N}(x)dx}+\frac{f^2}{4}\left(\frac{M}{m_e}\right)^2\frac{M}{E}\frac{\int\int dx\,dy[(-y/x)+(2-y)^2/xy]F_2^{\gamma N}(x)}{\int F_2^{\nu N}(x)dx-(M/4E)\int xF_2^{\nu N}(x)dx}\right\},\tag{9}$$

where

$$B=\frac{-\int xF_3^{\nu N}(x)dx}{\int F_2^{\nu N}(x)dx}.$$

We now proceed with the numerical analysis of Eq. (9). For this purpose, we adopt the following values from the literature[9]:

$$B=0.86\pm0.04,\tag{10}$$

$$\int F_2^{\gamma N}(x)dx=0.14\pm0.02,\tag{11}$$

$$\int F_2^{\nu N}(x)dx=0.49\pm0.07.\tag{12}$$

The value of B is obtained from the formula

$$R\equiv\frac{\sigma(\nu_\mu+N\to\mu^-+\text{anything})}{\sigma(\bar{\nu}_\mu+N\to\mu^++\text{anything})}=\frac{2-B}{2+B},$$

which follows from Eq. (7). Using the average experimental value of R one obtains[7] the result (10). The integral (11) is obtained from the SLAC-MIT electron scattering data. The result (12) has been obtained by Perkins[7] using the conserved-vector-current (CVC) hypothesis together with chiral-symmetry arguments. The double integral in Eq. (9) is evaluated by integrating from a minimum value of q^2, and we choose $(q^2)_{\min} = (0.3\text{ GeV})^2$, which is presumably the lower-limit criterion for q^2 in rejecting the background in the CERN experiment. One estimates[10] then, using the SLAC-MIT data for $F_2^{\gamma N}$,

$$\int\int dx\,dy\left[-\frac{y}{x}+\frac{(2-y)^2}{xy}\right]F_2^{\gamma N}(x)\simeq 8.05\,. \qquad (13)$$

Furthermore, for $E \gg M$, we can neglect[11] the terms proportional to the integrals $\int x F_2^{\nu(\gamma)N}(x)dx$ from the right-hand side of Eq. (9). We now use the CERN results[4] for the neutral-current events $R(\nu_\mu) = 0.21 \pm 0.03$ and $R(\bar{\nu}_\mu) = 0.45 \pm 0.09$, and set these values as the upper bounds on $R_\gamma(\nu_\mu)$ and $R_\gamma(\bar{\nu}_\mu)$, respectively. It is easy to see that for B given by (10), the two bounds for the right-hand side of Eq. (9) are roughly the same. The bounds on the charge radius and magnetic moment of the muon neutrino are then numerically given by

$$\begin{aligned} |a| &< 6.2\times 10^{-16}\text{ cm}, \\ |f| &< 1.1\times 10^{-7}, \end{aligned} \qquad (14)$$

where we have taken the incident neutrino energy $E = 5$ GeV. From astrophysical data, Bernstein *et al.*[3] had estimated a somewhat smaller bound for the magnetic moment $|f| < 10^{-10}$, but a larger bound for the charge radius $|a| < 4.5\times 10^{-13}$ cm, provided $m_{\nu_\mu} < 1$ kev.

It would be of considerable interest if future experiments can separate out the photon-exchange effect in neutral-current events. Note in this connection that the magnetic-moment contribution to the differential cross section (3) is proportional to $1/q^2$. The charge-radius effect, on the other hand, would be much harder to distinguish from the purely weak-interaction contributions.

At this point, it is interesting to speculate what happens if for some reason the one-photon-exchange contribution dominates all others. In this case, one would predict

$$\frac{\sigma(\nu_\mu + N \to \nu_\mu + \text{anything})}{\sigma(\bar{\nu}_\mu + N \to \bar{\nu}_\mu + \text{anything})} = 1\,. \qquad (15)$$

Using the CERN data for the ratios (5) and (6), together with the observed value[12]

$$\frac{\sigma(\bar{\nu}_\mu + N \to \mu^+ + \text{anything})}{\sigma(\nu_\mu + N \to \mu^- + \text{anything})} = 0.26 \pm 0.03\,,$$

we see that the right-hand side of Eq. (15) should be compared with the value $1.79^{+1.11}_{-0.64}$. Note that for a $V-A$ neutral weak current, one would expect the ratio (15) to be ~3. We may remark that the relation (15) has been predicted by some authors[13] on the basis of other models.

In view of the possible numerical uncertainties in the above analysis introduced through the various structure functions of the nucleon, perhaps more reliable bounds on the charge radius and magnetic moment of ν_μ can be obtained from the recent data[5] on the ν_μ-e elastic scattering. The one-photon contribution to this scattering process in the lab frame is given by

$$\frac{d\sigma_\gamma}{dq^2}(\nu_\mu(\bar{\nu}_\mu) + e \to \nu_\mu(\bar{\nu}_\mu) + e)$$

$$= \pi\alpha^2\left\{\frac{a^4}{9}\left[1-\frac{q^2}{2m_e E}\left(1+\frac{m_e}{2E}\right)+\frac{q^4}{8m_e^2E^2}\right] + \left(\frac{f}{2m_e}\right)^2\frac{4}{q^2}\left(1-\frac{q^2}{2m_eE}\right)\right\}, \qquad (16)$$

where E is the neutrino energy in the rest frame of the electron and in terms of the scattering angle

$$q^2 = \frac{4E^2\sin^2(\frac{1}{2}\theta)}{1+(2E/m_e)\sin^2(\frac{1}{2}\theta)}\,.$$

In the CERN experiment,[5] one possible $\bar{\nu}_\mu$ event has been observed. Furthermore, since the estimated background for $\bar{\nu}_\mu$ data is much smaller, we shall use the quoted upper bound for the $\bar{\nu}_\mu$-e cross section for our purposes. In the experiment the neutrino energy is peaked between 1 and 2 GeV, so we will take $E \simeq 1.5$ GeV. Also, a lower limit of 300 MeV was set on the energy of the scattered electron in order to eliminate low-energy background. This fixes the lower bound on q^2, and integrating Eq. (16) one can obtain the cross section numerically. Experimentally, the upper bound for $\sigma(\bar{\nu}_\mu + e \to \bar{\nu}_\mu + e)$ is quoted as $0.88E\times 10^{-41}$ cm^2 (with 90% confidence), which then leads to the result

$$\begin{aligned} |a| &< 5.9\times 10^{-16}\text{ cm}, \\ |f| &< 8.1\times 10^{-9}. \end{aligned} \qquad (17)$$

The $\bar{\nu}_\mu$-e scattering data thus improve the bound on f by an order of magnitude over the result (14).

In conclusion, it may be of interest to speculate that if electron neutrinos also possess magnetic moment, the neutrino flux on the earth coming from the sun would be expected to be reduced due to the effect of solar and terrestial magnetic interactions. The rather low solar-neutrino flux observed recently[14] might merit taking this idea seriously.

We would like to thank Professor W. F. Fry, Professor H. L. Helfer, Professor A. K. Mann, Professor A. Melissinos, and Professor S. L. Olsen for stimulating conversation.

*Work supported by the U. S. Atomic Energy Commission.

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PHYSICAL REVIEW D VOLUME 9, NUMBER 11 1 JUNE 1974

Measurement of multivariate scaling and factorization in exclusive multiparticle production*

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(Received 10 December 1973)

Several new model-independent techniques for the analysis of multidimensional data are presented and applied to exclusive multiparticle production. For the reaction $pp \to pp\pi^+\pi^+\pi^-\pi^-$ from 12 to 28 GeV/c, an algorithm that directly compares two multidimensional point distributions is used to show that the shape of $d^{12}\sigma/d(\vec{p}_\perp)^{12}$ is energy-independent, while $d^6\sigma/dx^6$ ($x = p_\parallel/p_{\max}$) varies dramatically with beam energy. A multidimensional test for independence is used to show that the multivariate differential cross section approximately factors into its cylindrical momentum components, $d^{18}\sigma/d(\vec{p})^{18} \simeq (d^6\sigma/dp_\parallel^6)\,[d^6\sigma/d(p_\perp^2)^6]\,d^6\sigma/d\varphi^6$. The shape of $d^6\sigma/d\varphi^6$ is also shown to be compatible with that predicted solely by kinematics.

INTRODUCTION

Multiparticle production presents a difficult problem in data analysis owing to the large number of independent observables necessary to completely describe the data. Excluding spin information, an n-particle final state requires $3n-4$ independent measurables for a complete description. The normalized multivariate differential cross section

$$\rho(x_1, x_2, \ldots, x_{3n-4}) = \frac{1}{\sigma}\,\frac{d^{3n-4}\sigma}{dx_1\,dx_2\cdots dx_{3n-4}}$$

can be thought of as a probability density function in the chosen measurables, $x_1 x_2 \cdots x_{3n-4}$, defined over their physically allowed values.

Each event can be represented as a point in the multidimensional space whose coordinates are the measurables of the event. This point contains all the information in the event and, thus, the collection or swarm of experimental points in this space contains all the information from the experiment. The purpose of data analysis is to infer the properties of the unknown probability density function (PDF) from the experimental point sample in the multidimensional space. For exclusive experiments where all $3n-4$ independent observables are measured, this PDF is directly related to the transition-matrix element squared for the reaction.

This report describes the results of applying several newly developed nonparametric (model-