moderate amount of inelasticity. We tried both the values 1550 and 1650 for W_{CDD}, and found 1550 to work somewhat better. This is not surprising because $\sin\delta \approx (W-1550)/(1236-1550)$ fits moderately well in the region *between* 1236 and 1550. Above 1550, the phase shift is complex.

Thus our *final form* for K is Eq. (5) above, with the result for the residue given there and in Table I.

We have also calculated these quantities with various modifications. The results are listed in Table I. It can be seen that the threshold factor combined with unitarity and real analyticity gives most of the effect. The imaginary part of R is only 2 MeV off, but the real part is 22 MeV low. Nucleon exchange, the CDD zero, and the kinematic double pole are all moderately important in the remaining real part, and all unimportant in the imaginary part. ρ exchange has little effect on the residue.

Probably the mass, and maybe the width, depend on more distant singularities than the residue does. Thus it may be difficult to quantitatively calculate these quantities. Moreover, the 3-3 resonance is a particularly favorable case for dynamic calculations, since the important forces are known in detail. Thus it is not reasonable to expect that a calculation (as opposed to a fit) of any other hadronic quantity, using considerations of analyticity and unitarity, can be carried out in the near future with the accuracy that we have been able to achieve.

We see from our calculations that the rather surprising value of the 3-3 residue can be understood in detail from entirely conventional ideas. We need only make good use of analyticity, unitarity, and crossing. We must take account of the major kinematic effects, the crossed-channel nucleon singularity, and the CDD effect caused by the opening of inelastic channels.

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Comment on experimental tests of *CPT* invariance*

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We discuss some general implications and further consequences of a weak-interaction model. They are related to a new test of *CPT* invariance, high-energy neutrino experiments, and so on. The model suggests that the violation of *CPT* invariance can only be effectively detected at high energies.

CPT invariance has been one of our most deeply believed laws of nature. Usually people test it by checking the equalities of the masses, the lifetimes, and the magnetic moments between a particle and its antiparticle. The accuracies of these measurements are very impressive, e.g.,

$$|[m(K^0)-m(\bar{K}^0)]/m(K^0)| \approx 10^{-17}.$$

Nevertheless, we note that *CPT* invariance is only a sufficient but not a necessary condition for these equalities. Moreover, if the usual weak interactions are mediated by some bosons, then the second-order processes such as K and μ decays may apparently conserve *CPT* even though the underlying basic weak interactions maximally violate *CPT* invariance.[1] Therefore, it is important to test this invariance in every possible way, especially now that violations of P, C, and CP have been established.

A plausible way of finding new tests of CPT invariance in weak interactions is to study the weak-interaction models. We have constructed a "pseudorenormalizable" model, based on the usual weak currents, in which interaction is mediated by spin-0 bosons $S^{\pm}$ and a neutral "aoraton" having spin 1 and not carrying energy and momentum. The model must maximally violate CPT invariance in order to be consistent with all existing data.[1] However, this maximum violation of CPT invariance in the basic (and the first-order) weak interactions cannot presently be detected by measuring the mass, the lifetime, or the magnetic moment difference between the observed particle and its antiparticle in low-energy experiments. In the model, first-order weak processes are Lorentz-invariant but maximally violate CPT invariance. The second-order processes, which include all the usually observed decay processes, violate CPT invariance and Lorentz invariance slightly provided that the S-boson mass is very large. It sounds very intriguing, but experiments have shown that the weak intermediate boson must have a large mass if it exists. Furthermore, the low-energy data of the kaon K^+ and the muon do show that their lifetimes appear to be shorter at higher energies. The data are consistent with a heavy intermediate boson with mass m_S between ~100 and ~300 GeV.[1,2] The anomalous energy dependence of $\pi^{\pm}$ lifetime and the lifetime difference between π^+ and π^- are of the order of $E_\pi/(E_\pi{}^2 - m_\pi{}^2 + m_S{}^2)^{1/2}$. This quantity is too small to be detected in low-energy experiments. Moreover, it is difficult to do experiments with high-energy pion beams because of the long decay length. A convenient way to test CPT is to measure the anomalous energy dependence of the lifetimes of baryon decays in flight at high energies.

The following are some general implications and further consequences of the model.

1. The manifestation of CPT violation. According to the model, the anomalous energy dependence of various particle lifetimes can be expressed in terms of one single parameter m_S:

$$\frac{1}{\tau(E)} = \frac{m}{E\tau_0}\left[1 \pm A\frac{E}{m_S} \pm O\left(\frac{E^2}{m_S{}^2}\right)\right], \tag{1}$$

where m and E are respectively the mass and the energy of the particle and A is some known number of order 1. The model cannot predict, in general, the sign in front of AE/m_S, because one may write $S^\dagger h_\lambda$ and $Sh_\lambda^\star$ instead of $S^\dagger h_\lambda^\star$ and Sh_λ in the interaction Lagrangian.[1] Strictly speaking, the model can only predict that if $+AE/m_S$ is associated with a particle then $-AE/m_S$ must be associated with its antiparticle, and vice versa. Using low-energy data for the lifetime difference between π^+ and π^- one obtains a larger value for m_S, i.e., ~800 GeV, which is not inconsistent with $m_S \approx 200$ GeV obtained from other data because of the unknown systematic error between experiments.

2. Test of CPT invariance. For baryon decays $B \to B' + \pi$, the model predicts

$$\frac{1}{\tau(E_B)} = \frac{m_B}{E_B\tau_{0B}}\left[1 \pm \frac{p_B}{m_S}\left(1 - \frac{m_{B'}{}^2}{m_B{}^2}\right)\right],$$
$$p_B{}^2 = E_B{}^2 - m_B{}^2 \ll m_S{}^2 . \tag{2}$$

We note that the anomalous energy dependence in (2) is suppressed by a factor $(1 - m_{B'}{}^2/m_B{}^2)$ and the sign in front of p_B/m_S cannot, in general, be predicted in the model. In a previous paper,[2] we give the positive sign in (2) based on the analogy between the leptonic decay and the nonleptonic decay of the hadrons. Whether this analogy is true can only be determined by future experiments. We may remark that for hadronic decay processes involving two or more pions in the final state the theoretical situation is not quite clear in the model.

3. The elastic scattering of neutrinos. The differential cross section of the elastic scattering $\nu_e + e^- \to \nu_e + e^-$ is

$$d\sigma_{\nu e} = \frac{G^2}{\pi}\left(1 + \frac{q^2}{m_S{}^2}\right)^{-2}\left\{1 + \frac{m_e{}^2 + m_e E_\nu - q^2}{[m_e{}^2 q^2 + (m_e{}^2 + m_e E_\nu - q^2)^2 + m_e{}^2 m_S{}^2]^{1/2}}\right\} dq^2 , \tag{3}$$

where E_ν is the incident neutrino energy in the rest system of the initial e^-, m_e is the electron mass, and q^2 is the square of the 4-momentum transfer. For $m_e \ll E_\nu < m_S$, we have essentially the usual result,

$$\sigma_{\nu e} \approx 2G^2 m_e E_\nu/\pi , \quad E_\nu < m_S . \tag{4}$$

However, when E_ν is extremely large, we have

$$\sigma_{\nu e} \approx G^2 m_S{}^2 \approx 10^{-31}\ \text{cm}^2 , \quad E_\nu m_e \gg m_S{}^2 . \tag{5}$$

(The latter condition gives $E_\nu \gg 2\times10^7$ to 2×10^8 GeV for $m_S = 100$ to 300 GeV.) For the process $\bar\nu_e(p) + e^-(p') \to \bar\nu_e(k') + e^-(k)$, the differential cross section is

$$d\sigma_{\bar\nu e} = \frac{G^2(q^2 - 2m_e E)^2}{4\pi m_e{}^2 E^2}\left[1 + \frac{E + m_e}{(E^2 + m_S{}^2)^{1/2}}\right]^2 \times\left(1 + \frac{2Em_e + m_e{}^2}{m_S{}^2}\right)^{-2} dq^2 , \tag{6}$$

where $q=k-p'$ and $E=p_0$. Because of the largeness of m_S, the "elastic" neutrino cross sections for $\nu_e n \to pe^-$ and $\bar{\nu}_e p \to ne^+$ are essentially the same as the usual results[3] for $E_\nu < 50$ GeV.

4. Implications of the rules $|\Delta \vec{I}| = \frac{1}{2}$ *and* $|\Delta S| \neq 2$. These rules suggest[4] that we may need at least two more neutral intermediate bosons S^0 and $\bar{S}^0$ with zero spin in the model. In this case, we may consider

$$(S^+, S^0) \text{ and } (S^-, \bar{S}^0) \tag{7}$$

as two isodoublets. We assume that $\mathcal{L}(\Delta S = \pm 1)$ conserves isospin and $\mathcal{L}(\Delta S = 0)$ violates isospin by an amount $|\Delta \vec{I}| = \frac{1}{2}$, where $\mathcal{L}(\Delta S = \pm 1)$ and $\mathcal{L}(\Delta S = 0)$ denote the interactions between the intermediate bosons and, respectively, the $\Delta S = \pm 1$ currents and the $\Delta S = 0$ currents. On the basis of symmetry considerations, we expect that the decay rates $\Lambda \to p\pi^-$ and $\Lambda \to n\pi^0$ would have the same anomalous energy dependence. We may remark here that the coupling between the neutral intermediate bosons and the neutral leptonic currents may be forbidden by a symmetry principle.[5]

In conclusion, we know that it is almost impossible to construct a *CPT*-violating model that satisfies Lorentz invariance and the usual causality. This does not necessarily imply that the suggestions of a *CPT*-violating model not satisfying Lorentz invariance should not be taken seriously. It demonstrates the close relationship between *CPT* invariance and Lorentz invariance. Also, it is clear that new accelerators provide a unique possibility of testing *CPT* and Lorentz invariances by measuring the lifetime of the particle in flight at very high energies.

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Measuring anomalous dimensions at high energies*

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A search for operators with anomalous dimensions is crucial in resolving the conflict of the light-cone algebra hypothesis with the results of explicit calculations in interacting-field theories. A likely place to find evidence for anomalous dimensions is in the large-ω region of high-energy inelastic eN and μN scattering.

A great deal of attention has been focused in recent years on the apparent scaling behavior of structure functions in deep-inelastic eN scattering.[1] These structure functions specify hadronic matrix elements of products of current operators and hence bear directly on the short-distance structure of hadrons. A general framework for investigations of this sort is provided by Wilson's hypothesis of scale invariance for the short-distance expansion of products of currents.[2] This hypothesis was motivated by and proven[3] to hold in various field-theory models. In this context the bold assumption of scale invariance for the leading light-cone singularities in the operator product is found to give a direct interpretation of the observed scaling behavior.[4] Furthermore, it lends itself to even more specific predictions where light-cone algebras of various naive field-theory models are incorporated.[5] In particular, the predictions of quark models have met with qualitative success in comparison with existing experimental data. It is also gratifying that most