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Antinucleon production and its relation to the rise in the proton-proton total cross section

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Arguments suggesting that the rise in the pp total cross section is due to antinucleon production are reviewed and questioned. While this hypothesis cannot be ruled out, the important coupling between the $N\bar{N}$ channel and meson channels complicates the dynamical picture. We consider multiperipheral cluster models generally, but include some of the effects of unitarity by discussing final-state interactions. We conclude that, either multiperipheral cluster models for $N\bar{N}$ production are inapplicable or, within the context of such models, $N\bar{N}$ production alters the production of other particles. These facts render previous model calculations of dubious applicability.

INTRODUCTION

Recent measurements indicate a rapid rise of the antiproton production cross section over the CERN ISR energy range.[1] It has been speculated[2,3] that this is to be associated with the observed rise in $\sigma_{tot}(pp)$.[4] In the framework of multiperipheral models, the onset of significant $\bar{N}N$ production is naturally delayed somewhat. However, as we shall show in Sec. I, it would seem insufficient to account for experimental observations. It has been also assumed that, when it occurs, $\bar{N}$ production makes an independent additive contribution to the purely mesonic multiperipheral diagrams. If the latter account for the roughly constant pp cross section at pre-ISR energies, it has been argued[2,3] that a rise over the ISR energy range might be understandable.

In the following we investigate such models and, in particular, discuss the possible effects of final-state interactions mixing $\bar{N}N$ and mesonic annihilation channels. We find that such effects are significant and modify the analysis. It is quite conceivable that $\bar{p}$ production will rise without a corresponding increase in $\sigma_{tot}(pp)$, just as other channels such as $K\bar{K}$, 4π, 6π, etc., have done. Although the conjecture that the rise in $\sigma_{tot}(pp)$ is due to a high effective $\bar{N}N$ threshold remains a viable and interesting possibility, we conclude that it strongly depends on the detailed dynamics of the $\bar{N}N$ channel near and below its threshold $M^2 = 4M_N^2$.

The plan of the present paper is the following: In Sec. I, we take up some important kinematical points. In Sec. II we briefly discuss multiperipheral models for $\bar{p}$ production following, in particular, Suzuki's model.[2] Some interesting alternative possibilities are also raised. In Sec. III we indicate some of the modifications required by "final-state" annihilations and the correlations which are then imposed on the mesonic systems. In Sec. IV, we show how, through unitarity, $N\bar{N}$ production affects meson production. We illustrate this in the particular "extreme" case when $N\bar{N}$ production is driven solely by its coupling (via annihilation) with meson channels. We conclude in Sec. V with a few additional comments and a summary.

I. SOME KINEMATICAL CONSIDERATIONS

A popular approach to inelastic processes with considerable phenomenological success is the separation of diffractive and nondiffractive mechanisms.[5] The nondiffractive component is generally regarded from a multiperipheral viewpoint, but, rather than assuming independent-particle emission, it is assumed that independent clusters of particles form the links of the chain.[2,3,5] (For example, see Fig. 1.) This enables the models to fit the correlations between particles which are observed experimentally.

Let us consider a multiperipheral cluster model, such as depicted in Fig. 1, and focus our attention on a cluster from which an $N\bar{N}$ pair emerges. It has been suggested[3] that the strong damping of the momentum in links entering the cluster will naturally inhibit the production of heavier masses, such as $p\bar{p}$. This is true to some extent, but the analysis presented was oversimplified and leads to much too large a threshold value from this effect.

To see this, suppose the $N\bar{N}$ pair has invariant mass M, whereas the adjacent clusters produce mesons with total average invariant mass μ. Kinematically, then, we face the situation indicated in Fig. 2. Let s_i be the invariant subenergy of particles produced from clusters exchanging momentum transfer t_i. We assume that, owing to form factors or propagators, the momentum transfers along the chain are restricted in magnitude to be smaller than some maximum (negative) value

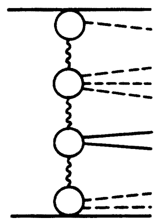

FIG. 1. A typical multiperipheral cluster graph.

t_0 ($|t_i| \leq |t_0|$). Kinematically, the minimum momentum transfer allowed across the ith link is (approximately) given by

$$t_i \leq (t_i)_{\min} = \frac{-(\mu^2 - t_{i-1})(M^2 - t_{i+1})}{s_i} . \quad (1)$$

On the other hand, we must have s_i to be sufficiently large so that

$$|(t_i)_{\min}| \leq |t_0| .$$

Interpreted somewhat differently, this means there is a *minimum* subenergy $(s_i)_{\min}$ before the production is kinematically allowed, where

$$(s_i)_{\min} \equiv \frac{(\mu^2 - t_{i-1})(M^2 - t_{i+1})}{|t_0|} . \quad (2)$$

Since the momentum transfers must be spacelike, the smallest permissible $(s_i)_{\min}$ will occur for $t_{i-1} \approx t_{i+1} \approx 0$, so[6]

$$(s_i)_{\min} \approx \frac{\mu^2 M^2}{|t_0|} . \quad (3)$$

What does this mean in terms of the minimum longitudinal rapidity spacing Δ between clusters? The relation between Δ and $(s_i)_{\min}$ is

$$(s_i)_{\min} = M^2 + \mu^2 + 2M\mu \cosh\Delta . \quad (4)$$

It then follows that

$$4\sinh^2\left(\frac{\Delta}{2}\right) = \frac{\mu M}{|t_0|} - \frac{(M+\mu)^2}{M\mu} . \quad (5)$$

The simplest multiperipheral cluster models assume meson clusters corresponds to ρ, ω, or σ mesons, so typically $\mu \lesssim 0.8$ GeV. If the $N\bar{N}$ system is produced nearly at rest, then $M \approx 2$ GeV. Consequently,

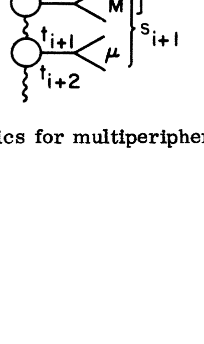

FIG. 2. Kinematics for multiperipheral cluster production.

$$4\sinh^2\left(\frac{\Delta}{2}\right) \approx \frac{1.6}{|t_0|} - 4.9 . \quad (6)$$

Note, for example, that the second term is quite significant and that for $|t_0| \geq \frac{1}{3}$ GeV^2, there is *no* minimum rapidity gap necessary, i.e., threshold would simply be $s_i \approx (M+\mu)^2 = 8$ GeV^2. Perhaps a somewhat smaller $|t_0|$ is physically justified, but even the smallest assumed in Ref. 3 is $|t_0| \approx 0.2$. This corresponds to $\Delta \approx 1.6$.[7] Assuming that one needs a minimum total rapidity of about 2Δ to produce $N\bar{N}$ pairs, we would find in pp collisions, a threshold energy $s_{\text{th}} = 4M^2\cosh^2\Delta \approx 20$ GeV^2. Even if $|t_0| = 0.1$, we find $\Delta = 2.6$ so that $s_{\text{th}} \approx 160$ GeV^2, which still seems a bit small. Whether a multiperipheral cluster model is capable of producing the high threshold observed[8] ($s_{\text{th}} \approx 400$) remains an open question and is very sensitive to details of the model.

A second kinematical point we wish to make is that, in proton-antiproton annihilation, the momentum of the produced pions, in particular, the relative longitudinal rapidity spacing between adjacent pions, is quite comparable to that of pions produced in the pionization region in pp collisions. (This kinematical fact will be useful for considerations later.)

We know from $N\bar{N}$ interactions near threshold that the system tends to annihilate, usually to five or six pions.[9] Estimates indicate that the longitudinal rapidity spacing between neighboring pions is between 0.4 and 1. For example, in $(N\bar{N})_{\text{rest}} \rightarrow 6\pi$, if we assume each pion carries off an equal energy, we find that the longitudinal rapidity spacing (along an arbitrary axis) is between 0.52 and 0.65. Even if pions cluster together in pairs, their relative rapidity separation is about 0.6.

We want to compare these spacings to the mean spacing of pionization products in production reactions. The latest fit[10] including the NAL 400-GeV/c bubble chamber data gives the mean charged particle production as $\langle n_{\text{ch}} \rangle = 1.7 \ln s - 2.9$ (s in units of GeV^2). Assuming that 80 percent of the produced particles are pions and that the number of π^0's is half the number of charged pions, we conclude that $\langle n_\pi \rangle = 2.2 \ln s - 3.5$. Thus, asymptotically, the mean rapidity spacing of pionization products would be

$$\Delta y = \lim_{s\rightarrow\infty} \frac{\ln s}{\langle n_\pi \rangle - 1} = 0.45 .$$

At finite energies, one's estimate may be somewhat larger because the rapidity space available to pions is $\ln(s/\mu^2)$ and because of the constant term in $\langle n_\pi \rangle$. A contrary correction, however, is due to the fact that the probability of finding pions near the center of the rapidity region is greater than near the ends. A way to estimate this average spacing is to look at the length of the rapidity plateau in the ISR inclusive experiments and at the number of pions produced. One then finds that the average rapidity spacing between pions is about 0.43, assuming they are uncorrelated.[11]

In the final analysis, we believe it is safe to say that the relative rapidity separation between pions produced in production reactions at high energy overlaps the rapidity separations found in $N\bar{N}$ annihilation near threshold.

II. MULTIPERIPHERAL CLUSTER MODELS AND ALTERNATIVES

Following some recent analyses,[5] let us assume that $\sigma_{\text{tot}}(pp)$ is dominated, throughout the high-energy range of interest, by multiperipheral particle emission rather than by "diffractive" processes. The suggestion made in Refs. 2 and 3 is that the rise in $\sigma_{\text{tot}}(pp)$ should be attributed to this first "short-range" component and differs, therefore, from most other explanations which involve processes with long-range correlations in rapidity.

The production process is described by independent emission of clusters of various kinds from a multiperipheral chain (Fig. 1). The cross section for N-cluster emission is

$$\sigma_N(s) = \frac{\beta s^{2\alpha_R-2}}{N!}\left(\sum_i f_i \ln s\right)^N, \tag{7}$$

where α_R is the effective intercept or spin of the exchanged trajectory or particle. The effective coupling f_i for the ith cluster is related to the average multiplicity of clusters by

$$n_k = f_k \ln s + c_k . \tag{8}$$

Summing Eq. (1) over N we find[12] $\sigma_{\text{tot}}(s) \approx \beta s^{\alpha_P - 1} \approx \beta s^{2\alpha_R - 2 + \Sigma f_i}$, so

$$\alpha_P - 1 = 2\alpha_R - 2 + \sum f_i . \tag{9}$$

Suzuki's model[2] is specified by choosing $\alpha_R = \frac{1}{2}$ and three types of clusters "ρ" and "ω" mesonic (m) clusters, $\bar{K}K$ and $\bar{N}N$ clusters. It may be worth noting that the experimentally observed strong correlations between charged and neutral pions suggest that a large fraction of the pions are indeed produced in ρ- and ω-like clusters.[13]

A fit[2] to the observed multiplicities can then be obtained by choosing

$$\begin{aligned} n_\omega &\approx n_\rho^+ \approx n_\rho^- \approx n_\rho^0 = f_m \ln s - 0.16 , \\ n_{K\bar{K}} &= f_{K\bar{K}} \ln s - 0.66 , \\ n_{N\bar{N}} &= f_{N\bar{N}} \ln s - 1.06 , \end{aligned} \tag{10}$$

with

$$f_m \approx f_{K\bar{K}} \approx f_{N\bar{N}} \approx 0.18 .$$

(It is interesting to note the apparently universal value of the effective couplings f_i.) As indicated in Sec. I, copious $\bar{N}$ production is expected to occur only for sufficiently high energies s, compatible with the damping of momentum transfers.

Consistent with experiment, Suzuki assumes that there is some high dynamical threshold, above which $N\bar{N}$ production is no longer hindered. Such $N\bar{N}$ pair production is assumed to be dynamically independent of other clusters and, throughout the energy range of interest, it is further assumed that only one such pair is produced. Consequently, its production cross section increases as $\ln(s/s_{\text{th}})$, proportional to the expansion of its rapidity plateau. At these energies in such a model, $\bar{N}N$ clusters will, therefore, effectively contribute to the α_P intercept sum rule of Eq. (9). Using the f_i from Eq. (10), one finds the effective intercept $\alpha_P \gtrsim 1$, corresponding to a rise in $\sigma_{\text{tot}}(pp)$.

It seems to us that various features of the model are unsatisfactory. The saturation of the sum rule ($\alpha_P \approx 1$) at the "$\bar{N}N$ level" depends strongly on the choice $\alpha_R = \frac{1}{2}$ and the assumption of no direct pion emission. A detailed model should also explain the large $f_{N\bar{N}}$ coupling and explain how it is compatible with the high threshold.

Rather than dwell on these (relevant, but detailed) topics, we will focus on the basic feature of such models, which singles out the opening up of the $\bar{N}N$ production channel (from, say, n-meson production) as a candidate for explaining the rise in $\sigma_{\text{tot}}(pp)$. We question the assumption that $\bar{N}N$ production involves a new cluster in the chain which is dynamically independent of any of the other mesonic clusters. In the following sections, we illustrate our point within the framework of the multiperipheral cluster models by discussing physical reasons for relaxing this assumption and the sort of complications this leads to. However, before so doing, we would like to mention another possible point of view, which is somewhat antithetical to the assumptions underlying these cluster models.

In multiperipheral cluster models for meson production, reasonable phenomenological success is achieved with clusters decaying to 3 or 4 mesons per cluster on the average. On the other hand, we know experimentally[9] that, in $N\bar{N}$ annihilation near threshold, 5 or 6 mesons are usually produced, whose relatively rapidity spacing is roughly the same as in inelastic meson production, regardless of the precise details of the cluster model. Since the force in the annihilation channels is so strong near threshold, one would expect that the $N\bar{N}$ pairs might come from a "supercluster" consisting of two or more meson clusters. To put it another way, one might expect final-state interactions of the $N\overline{N}$ system to induce correlations between what at first appear to be independent clusters for meson production. In the presence of correlations, the simple sum rule (3) has to be generalized to[14–16]

$$\alpha_P - 1 = 2\alpha_R - 2 + c_1 - \frac{c_2}{2!} + \frac{c_2}{3!} - \cdots, \tag{11}$$

where c_n are the coefficients of ln$s$ in the n-particle correlation integrals. (Since we still assume only short-range rapidity correlations, all these integrals are asymptotically proportional to $Y = \ln s$.) In particular, c_1, the single-particle term, is to be identified with f_m the effective coupling for the "m" cluster emission defined above. A positive two-body correlation(c_2) will then tend to decrease α_P from the naive estimate based on independent "m" emission alone.

The physical significance of this is simple: Positive c_2 implies that two m's tends to be emitted at neighboring y_i, y_{i+1}. The two m's will thus behave part of the time as a single mm supercluster. The effective number of independently emitted objects (per unit of rapidity) is therefore reduced, and the original formula[9] for $\alpha_P - 1$ in term of independently emitted objects is decreased.[17]

Indeed, what we have here is a general mechanism by which a multiperipheral cluster model (i.e., a model with short-range correlations) may avoid violating the Froissart bound even as we get beyond the effective threshold for producing $\overline{N}N$, $\overline{\Lambda}\Lambda$, ... channels with available mm, mmm, clusters and may generate enough mm correlations to offset the increase of α_P naively expected. As should be clear from the above discussion, the basic mechanism involved is unitarity in the $\overline{N}N$-mesons coupled channels.

It remains to be investigated whether the best way to implement this philosophy is simply to enlarge the basic cluster in multiperipheral models or to develop an entirely different approach to short-range correlations. In the remainder of this paper, however, we shall discuss only those complications arising within the confines of multiperipheral cluster models.

III. A MODEL WITH FINAL-STATE $\overline{N}N$ INTERACTIONS

A striking feature of the $\overline{N}N$ system at "low" energies ($M_{\overline{N}N}{}^2 < 8$ GeV^2) is the large inelastic and, in particular, the large annihilation cross section. This results in a largely absorptive elastic $\overline{N}N$ amplitude at low energies. For $P_{\text{lab}}(\overline{p}) = 3.28$ GeV/c, the amplitude[9] corresponds to an almost black disk of radius $R = 1.3$ fm in b space with a transmission amplitude (or S matrix)

$$S(b) = 0.36, \quad b \le R. \tag{12}$$

An alternative, more reasonable, description is given by

$$S(b) = 1 - ce^{-b^2/R^2}, \quad R^2 = 2 \text{ fm}^2 \tag{13}$$

where $S(0) = 1 - c$ is very small (≈ 0.0–0.2). In particular $S(0)$ is smaller for $M_{\overline{N}N}{}^2$ near threshold. The large annihilation radius R contradicts naive expectations that $R \approx 1/m_N$. It is due to the strongly attractive potential that all exchanges, in particular the long-range π exchange, generate in the $\overline{N}N$ system.[18] This tends to yield bound states and resonances in the threshold region and, in any case, draws the incident N and $\overline{N}$ together so that annihilation can occur.[19]

The strong, long-range $\overline{N}N$ interaction suggests an approximate treatment of final state $\overline{N}N$ interactions by treating the whole production as a two-step process, in analogy to a two-potential problem: (a) Multiperipheral production of m, $\overline{K}K$, and $\overline{N}N$ clusters. In particular, since the latter involves a nucleon exchange, the $\overline{N}N$ pair is expected to emerge from this primary stage at a low subenergy M^2 and at a small ($\approx 1/m_N$) relative impact parameter $|b_i - b_{\text{in}}|$ (or low $\overline{N}N$ partial waves). (b) Final-state interactions, especially annihilation, in the $\overline{N}N$ channel. We follow the simple multiperipheral model in neglecting final-state interactions between any other clusters, arguing that the relatively large subenergies and/or relative impact parameters make these interactions relatively weak and of short duration as compared to the $\overline{N}N$ final-state interactions. Thus the conditions for the final-state (Watson's) theorem are, qualitatively at least, satisfied in the present case. This suggests a final-state phase[20,21]

$$e^{i\delta_J(M^2)} \equiv [S_J(M^2)]^{1/2}.$$

To the extent that the assumed decoupling of the $\overline{N}N$ system from the other final-state particles is indeed correct, the evolution of this subsystem is governed by a separate unitary matrix. Thus, any final-state interaction will not effect the total cross section for the "original" production of any combination of clusters, which will then still be given by the multiperipheral expression of Eq. (7) above. The so-called "$\overline{N}N$" cluster will, however, emerge only as a bona fide asymptotic final state $\overline{N}N$ only with a small probability

$$P_{``\overline{N}N" \to \overline{N}N} \approx |[S_J(M^2)]^{1/2}|^2 = |S_J(M^2)|. \tag{14}$$

The probability that the "bare" $\overline{N}N$ cluster yields inelastic ($\bar{N}N+$ mesons) rather than annihilation (mesons) final states in the M^2 channel is

$$\approx (1-S_J)\,\frac{\sigma_{\text{inel-nonannihilation}}(M^2)}{\sigma_{\text{inel-tot}}(M^2)}\;.$$

The last ratio is small (< 0.25) over the region of interest[20] and quickly approaches zero as $M^2 \to 4M_N^2$. Thus we will neglect in the following the nonannihilation channels and consider only annihilation channels which, in this energy region, contain on the average ≈ 6.0–6.5 mesons.[9] The probability that the bare "$\bar{N}N$" cluster will evolve via final-state interactions into annihilation mesons is thus given by

$$P_{\bar{N}N\to\text{ "annihilation mesons"}} = 1-P_{\text{"}\bar{N}N\text{"}\to\bar{N}N} \approx 1-S_J(M^2)\,. \tag{15}$$

Comparing Eqs. (14) and (15) we find that, on the average, we expect for every final state $\bar{N}N$, $k\approx[1-S(\bar{b})]/S(\bar{b})$ annihilation-meson clusters. [$\bar{b}$ is the relevant average (presumably small) relative impact parameter of the "intermediate" $\bar{N}$ and N emerging from the chain.] From Eqs. (12) or (13), we therefore expect a large k and at any rate $k\geq 2$.

At this point two "limiting" theoretical possibilities arise with respect to the nature of the annihilation clusters:

(1) The "annihilation mesons clusters" are completely distinct from any combination *mm*, *mmm*, etc., of the original single-meson clusters. This distinction can be reflected by different multiplicities for given invariant M^2 in $\bar{N}N$ annihilation clusters and in multi-*m* systems emitted from the multiperipheral chain, different angular momenta in rest frame, etc.

(2) The annihilation meson cluster are dynamically very similar to the *mm*, *mmm*, combinations emitted directly from the multiperipheral chain.

Even if we adopt the first possibility (which we believe to be more in the spirit of the model of Sec. II), the analysis is qualitatively changed. The multiplicity of "$\bar{N}N$" primary clusters should be multiplied by three (at least) as compared with the estimate of Eq. (4) in order that the yield of observed $\bar{N}N$ will remain unchanged. Since it is the coefficient of the log in the multiplicity of independently produced clusters, $f_{\text{"}\bar{N}N\text{"}}\approx 0.54$, which enters into the sum rule [Eq. (3)], the saturation of the "$\bar{N}N$ level" will be much more dramatic in this case ($\alpha_P\approx 1.4$ instead of 1.04). And since each of the ($0.36\ln s + c$) annihilation clusters produces on the average six pions as compared to 2.25 pions only in the decay of m (ρ or ω) clusters, we expect the coefficient of $\ln s$ in the pion multiplicity to be doubled, at least for $s > 400$ GeV^2. While there may be some indication of an increasing slope in plots of $n(s)$ vs $\ln s$, the effects estimated here are somewhat too big in absolute terms.

Notice, however, that if we want to keep roughly the same pionization plateau height as implied by Eq. (10), we have to change f_m to a new value f'_m so that

$$(2.25)4f'_m + 6f_{\bar{N}N} \approx (2.25)4f_m\,, \tag{16}$$

i.e., $f'_m\approx 0.06$ (instead of 0.18). This in turn will prevent the saturation of the α_P sum rule.

The possible correctness of the second assumption (2) above is also a detailed dynamical question and will be discussed in Sec. IV. We would like to emphasize, however, that if there is any overlap between the annihilation mesons and *mm*, *mmm* clusters, etc., the basic analysis of the proceeding chapter must be modified.

IV. COUPLED-CHANNEL MODEL FOR $N\bar{N}$ PRODUCTION

The very strong coupling of the $N\bar{N}$ and multimeson systems suggests that we investigate how, tnrough unitarity, the meson and $N\bar{N}$ pair production cross sections are related. Since it appears that $N\bar{N}$ near threshold couples strongly to as many as five or six mesons, the basic cluster might involve several meson clusters. For example, in Suzuki's model,[2] described in Sec. II, the basic meson cluster was a ρ or ω meson. Since $N\bar{N}\to\rho\omega, \omega\omega, \rho\rho$, etc., would not be negligible, the basic cluster for our purposes might include two ρ or ω mesons. (To some extent, we have in mind the sort of supercluster described toward the end of Sec. II.) To illustrate the mechanism of unitarity and to be able to solve the model explicitly, we suppose that each channel consists of two particles $m\bar{m}$ and $N\bar{N}$ (m will be a general term for a meson or meson resonance such as ρ, ω, K^*, etc.). To further simplify the discussion, we shall neglect spin and other kinematic complications and assume particles m and N have zero spin and equal mass. (We indicate in an appendix how this analysis must be modified for the unphysical region in the case of unequal masses.)

So our multiperipheral chain consists of uncorrelated clusters which "decay" into either an $m\bar{m}$ or $N\bar{N}$ pair (Fig. 3). Let $F^J_m(M^2)$ be the amplitude for the cluster to produce an $m\bar{m}$ pair with invariant mass M^2 and angular momentum J. Similarly define $F^J_N(M^2)$. (We could replace J by relative impact parameter b without affecting the discussion.) In this notation, the asymptotic be-

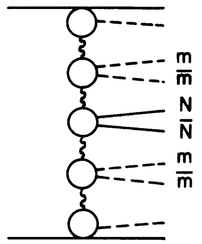

FIG. 3. Simplified cluster model for production of meson pairs ($m\bar{m}$) and nucleon-antinucleon pairs ($N\bar{N}$).

havior of the multiplicity for producing $m\bar{m}$ pairs will be proportional to

$$n_{m\bar{m}} \sim \ln s \int dM^2 \sum_J |F_m^J(M^2)|^2 \tag{17}$$

and similarly for $n_{N\bar{N}}$. For a fixed value of the invariant mass M^2, the right hand discontinuities of F_m and F_N obey a coupled set of equations depicted in Fig. 4. (We neglect possible complications due to multiparticle states and suppose that these are the only intermediate states.) For a fixed angular momentum J (or impact parameter b), the coupled equations become algebraic

$$\mathrm{Im}\begin{pmatrix} F_m^J \\ F_N^J \end{pmatrix} = \tfrac{1}{2} T_J^* \begin{pmatrix} F_n^J \\ F_N^J \end{pmatrix} \tag{18}$$

or

$$\begin{pmatrix} F_m^J \\ F_N^J \end{pmatrix} = S_J \begin{pmatrix} F_m^{J*} \\ F_N^{J*} \end{pmatrix} , \tag{19}$$

where S_J is the strong-interaction S matrix for the two-channel system. It can be parameterized[23] in terms of two real phases and the inelasticity strength η_J:

$$S = \begin{pmatrix} (1-\eta^2)^{1/2} e^{2i\delta_m} & i\eta e^{i(\delta_m+\delta_N)} \\ i\eta e^{i(\delta_m+\delta_N)} & (1-\eta^2)^{1/2} e^{2i\delta_N} \end{pmatrix} . \tag{20}$$

(Here we have suppressed the angular momentum label J.) Equation (2) has the well-known form of a system of singular integral equations.[24]

Although many properties of the solution of such a system are known, there is no general technique

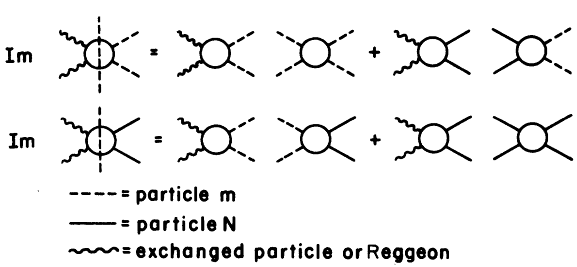

FIG. 4. Unitarity relations between meson pairs ($m\bar{m}$) and baryon pair ($N\bar{N}$) production.

for finding the solution. Consequently, we are forced to make further assumptions and approximations. Although F_m and F_N have left-hand discontinuities as well, we shall neglect them. This seems reasonable, since our current interest is in how the production of one channel affects the production of others to which it is coupled, i.e., what are the effects of final-state interactions? We imagine that, because of baryon number conservation for example, there is no such mixing along the left-hand cut. Even if we are willing to neglect the left-hand cuts of F_m and F_N [so that their only discontinuity is given by Eq. (18)], we cannot explicitly solve the system. We shall obtain an approximate solution below, but first let us discuss some interesting features of the solution. Let X be a fundamental matrix[25] for Eq. (19),

$$X = \begin{pmatrix} F_{1m} & F_{2m} \\ F_{1N} & F_{2N} \end{pmatrix} . \tag{21}$$

Then the most general solution is

$$F = XP = P_1(M^2)\begin{pmatrix} F_{1m} \\ F_{1N} \end{pmatrix} + P_2(M^2)\begin{pmatrix} F_{2m} \\ F_{2N} \end{pmatrix} , \tag{22}$$

where $P_1(M^2)$ are real entire functions[26] of the invariant mass M^2.

Corresponding to the idea that the $N\bar{N}$ system is strongly absorptive, we imagine that the production of $N\bar{N}$ pairs is via meson-pair intermediate states, so that, as $\eta \to 0$, $F_N \to 0$. If F_N is of the order of the inelasticity η, we see that, via the unitarity equation, Eq. (18), the inelasticity affects F_m to second order in η^2. If we imagine turning on the inelasticity η, then as $N\bar{N}$ pairs are produced, a *change* is produced in the production of $m\bar{m}$ pairs of order η^2, which contributes to the same order in the multiplicity as $|F_N|^2$. Whether the coefficient of $\ln s$ in the total multiplicity increases, decreases, or is unchanged to lowest order in η^2 is a detailed dynamical question, as we shall see further below. Similarly, whether the total cross section rises, falls, or is unchanged because of the production of $N\bar{N}$ pairs depends on how, through unitarity, this influences other channels. Prior to the rise in σ_{tot} seen at the ISR, it seemed that the rise with energy of any one channel was offset by the decrease of other channels, so that σ_{tot} was nearly energy independent. The point of view suggested in Refs. 2 and 3 is that this energy independence was an accident. While this may be correct, it is not a very pretty picture to say the least.

We will now illustrate the above remarks by constructing an approximate solution to our system. Our fundamental matrix X satisfies

$$X = SX^* .$$

Because S is a symmetric, unitary matrix, it can be diagonalized with a real, orthogonal matrix O.

$$OSO^T = D \equiv e^{i(\delta_m + \delta_N)} e^{i\Delta\lambda\sigma_3}, \tag{23}$$

where σ_3 is the usual Pauli matrix, and the difference of the eigenphases, $\Delta\lambda$, satisfies

$$\cos(\Delta\lambda) = (1-\eta^2)^{1/2}\cos(\delta_m - \delta_N) . \tag{24}$$

We may choose

$$O = e^{(i\Theta/2)\sigma_2}, \tag{25}$$

where the real angle Θ is defined through the relations

$$\begin{aligned} &\sin(\Delta\lambda)\sin\Theta = \eta ,\\ &\sin(\Delta\lambda)\cos\Theta = (1-\eta^2)^{1/2}\sin(\delta_m - \delta_N) . \end{aligned} \tag{26}$$

Now, the matrix $Y = OXO^T$ satisfies

$$Y = DY^* . \tag{27}$$

Since D is diagonal, Eq. (27) would appear to be easily solvable. However, even if the left-hand cut of X is ignored, the analyticity of Y may be complicated because of the analyticity properties of O, that is, Y will have singularities other than the discontinuity represented by Eq. (27).

Suppose Z is a fundamental matrix for Eq. (27), assuming that this right-hand cut is the *only* discontinuity of Z. Then $W = Z^{-1}\,Y$ has no right-hand cut and any other singularities of Y are reflected in W. Thus we may write

$$X = O^T Y O = (O^T Z O)(O^T W O), \tag{28}$$

and the general solution is [recall Eq. (5)]

$$F = XP = (O^T Z O) G, \tag{29}$$

where

$$G \equiv (O^T W O) P .$$

The matrix $O^T Z O$ expresses how the S matrix on the right-hand cut mixes the two channels. The properties associated with possible left-hand cuts and the ambiguities associated with entire functions are contained in the vector G. (Note that, in any case, G is *real* for M^2 real and positive.)

A fundamental matrix Z is given by

$$Z = e^{\Sigma(M^2)/2} e^{\Delta(M^2)\sigma_3/2}, \tag{30}$$

where

$$\begin{aligned} \Sigma(M^2) &\equiv \frac{M^2}{\pi}\int_{M_0{}^2}^{\infty} \frac{dx\,[\delta_m(x) + \delta_N(x)]}{x(x-M^2-i\epsilon)} ,\\ \Delta(M^2) &\equiv \frac{M^2}{\pi}\int_{M_0{}^2}^{\infty} \frac{dx\,\Delta\lambda(x)}{x(x-M^2-i\epsilon)} . \end{aligned} \tag{31}$$

From this, one can calculate

$$\begin{aligned} O^T Z O = e^{\Sigma/2}\,[\,&\cosh(\tfrac{1}{2}\Delta) + \cos\Theta\,\sinh(\tfrac{1}{2}\Delta)\sigma_3 \\ &+ \sin\Theta\,\sinh(\tfrac{1}{2}\Delta)\sigma_1] . \end{aligned} \tag{32}$$

Now, the coefficient of $\ln s$ in the total multiplicity for the production of $m\bar{m}$ or $N\bar{N}$ pairs is proportional to

$$F^\dagger F = G^\dagger (O^T Z^\dagger Z O) G . \tag{33}$$

Letting Σ_p and Δ_p be the real parts of Σ and Δ, we find

$$\begin{aligned} O^T Z^\dagger Z O = e^{\Sigma/2}[&\cosh\Delta_p + \cos\Theta\,\sinh\Delta_p \sigma_3 \\ &+ \sin\Theta\,\sinh\Delta_p \sigma_1] . \end{aligned} \tag{34}$$

The precise properties of the result (33) clearly depends on the precise nature of the dynamics of the mixing [Eq. (34)], the contributions of possible left-hand cuts, and the asymptotic behavior assumed for P, as summarized in G. Recall that, in the absence of correlations between clusters, we have

$$\alpha_P - 1 = 2\alpha_R - 2 + \int dM^2 \sum_J F_J^\dagger(M^2) F_J(M^2) . \tag{35}$$

Previous authors have assumed that the production of $N\bar{N}$ pairs would necessarily increase α_P. This is true if they were simply added independently, but we see in our model how the effects of unitarity may complicate matters. We explore this further below.

Let us consider, in the context of this discussion, the following simple model. Suppose that, as the inelasticity $\eta \to 0$, the production of $N\bar{N}$ pairs also goes to zero. We have in mind that, since annihilation into mesons dominate the $N\bar{N}$ interaction near threshold, $N\bar{N}$ production should also be predominantly via meson channels. Clearly as $\eta \to 0$, the angle $\Theta \to 0$ and $\Delta\lambda \to (\delta_m - \delta_N)$. Consequently, in this limit

$$\begin{aligned} O^T Z^\dagger Z O &= e^{\Sigma_p}(\cosh\Delta_p + \sinh\Delta_p\,\sigma_3) \\ &= \begin{pmatrix} e^{\Sigma_p + \Delta_p} & 0 \\ 0 & e^{\Sigma_p - \Delta_p} \end{pmatrix} . \end{aligned} \tag{36}$$

Hence, we want to choose $G_2 = 0$, so that $F_N = 0$, in this limit.

Now, if we ignore left-hand cuts so that $G = P$, we could argue that since P_2 is, on the one hand, an entire function of M^2 and, at the same time, a function of $\eta(M^2)$ which must vanish at $\eta = 0$, it must be that $G_2 = 0$, *for all* η. This is no longer the case when left-hand cuts are included, but assuming $G_2 = 0$ anyway provides a simple interesting model. In this case, we find

$$F_m = G_1 e^{\Sigma/2}[\cosh(\tfrac{1}{2}\Delta) + \cos\Theta \sinh(\tfrac{1}{2}\Delta)],$$

$$F_N = G_1 e^{\Sigma/2} \sin\Theta \sinh(\tfrac{1}{2}\Delta),$$

and (37)

$$F^\dagger F = |G_1|^2 e^{\Sigma_p}(\cosh\Delta_p + \cos\Theta \sinh\Delta_p).$$

Even in this case, it is not at all clear whether increasing η from zero increases or decreases $F^\dagger F$.

To get some feeling for these fairly complicated formulas, imagine that, as η increases from zero, the phases δ_m and δ_N remain unchanged. Since Θ is of order η, we see from Eq. (37) that $F^\dagger F$ will decrease (increase) as η increases if Δ_p is positive (negative). The sign of Δ_p is rather complicated to ascertain generally. For example, if $\Delta\lambda$ is independent of M^2, then

$$\Delta_p = -\frac{\Delta\lambda}{\pi} \ln\left|\frac{M^2 - M_0^2}{M_0^2}\right| .$$

Thus if $\Delta\lambda$ is positive, as M^2 increases from threshold, Δ_p decreases from $+\infty$, crosses zero at $M^2 = 2M_0^2$, and is negative for higher M^2. So, even with these assumptions, the correction to the multiplicity is ambiguous. If $\Delta\lambda \approx \delta_m - \delta_N$ is positive and the low-M^2 region dominates the integral, then $\int dM^2 F^\dagger F$ will *decrease* with increasing η. The situation here is reminiscent of the sign of diffractive rescattering corrections.[27] Essentially F_N is of order η, but the correction to F_m is like $(1-\eta^2)$ so that

$$|F_m|^2 + |F_N|^2 \approx (1-\eta^2)^2 + \eta^2$$

$$\approx 1 - \eta^2 .$$

Because of the kinematical complications of spin, internal symmetries, and unequal masses, as well as the dynamical complication of having many coupled, multiparticle channels, the real world is considerably more difficult to analyze than the models discussed above. On the other hand, we see no reason to suppose that the real world is substantially *simpler* than our models. Since the total cross section does *not* increase every time another threshold is crossed, the coupling between channels *must* be essential in general.

In the case of unequal masses, an important complication arises because of the presence of unphysical regions. In the context of our model, if the $N\overline{N} \to m\overline{m}$ amplitude is known is the unphysical region, the solution of this new problem may be reduced to the foregoing. For those interested, the method is discussed in an appendix.

V. CONCLUSIONS

In the preceding sections, we have argued against treating the production of antibaryons as somehow unique and different from other inelastic channels. To emphasize this point, we reproduce in Fig. 5 a plot of the average multiplicity for different species of particles.[8] There is certainly nothing in this figure which urges us to account for the 10 percent rise at the ISR in the pp total cross section in terms of $\overline{p}$ production alone. Indeed, Suzuki[2] shows that, if one assumes that only one $p\overline{p}$ pair is produced throughout the energy range of interest, then the rise in $\sigma_{\overline{p}}$, the antiproton cross section, is sufficient to account for the rise in σ_{tot}. Assuming that $\sigma_{\overline{n}}$, the antineutron cross section, is of the same order as the antiproton cross section, we conclude that, if truly independent of other channels, antinucleon production would have led to a much larger increase in σ_{tot}. This fact alone reinforces our arguments that antinucleon production *cannot* be considered irrespective of its coupling to other production channels.

One shortcoming of multiperipheral models (including cluster models) is that the Pomeron quite by accident appears to have intercept nearly equal to one. The reason for the approximate constancy of the pp total cross section is quite mysterious, for the exclusive components of the total cross section manifest very significant variations with energy. A much more elegant point of view[14] is that the Pomeron is somehow fixed near unity and the rise of certain exclusive channels *must* be counterbalanced by the fall of others.

In Sec. I, we cast further doubt on multiperipheral models of $N\overline{N}$ pair production by indicating that, contrary to lore, the very high threshold is not easily accounted for by a peripheral requirement. In subsequent sections, we tried to indicate mechanisms by which the production of one channel, such as $N\overline{N}$ pairs, feeds back through unitarity on other channels. We also argued that correlations are undoubtedly important for understanding the buildup of the total cross section. Indeed, if the total cross section is asymptotically increasing with energy, then models which assume that there are independently produced clusters are suspect.

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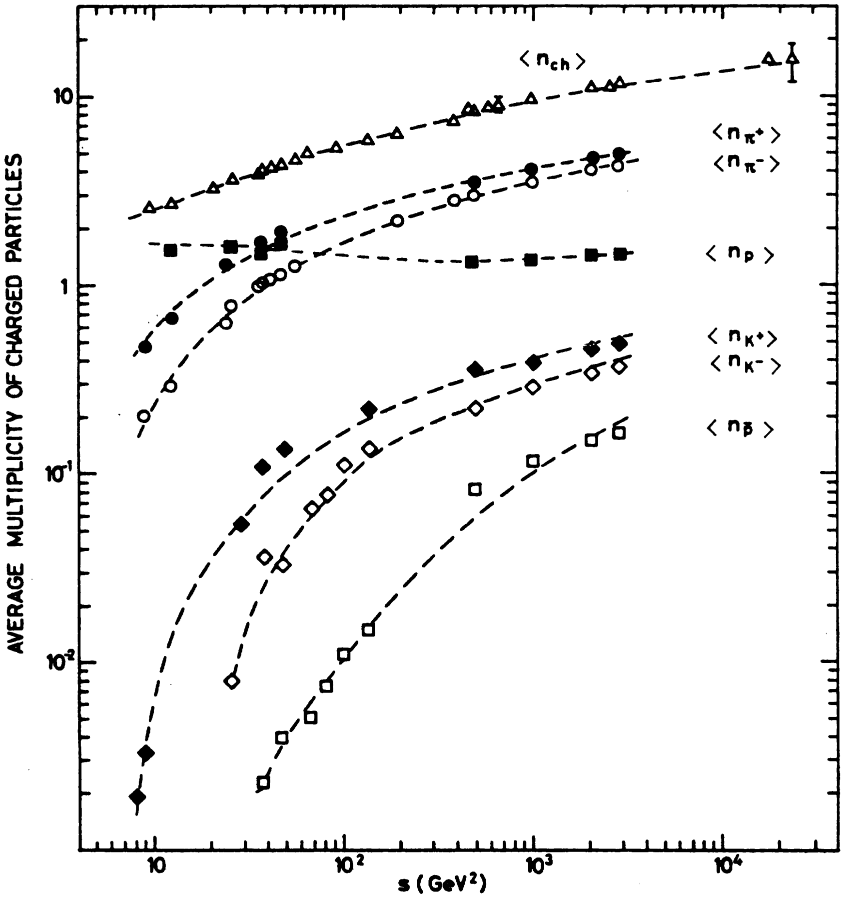

FIG. 5. Average charged particle multiplicities versus center-of-mass energy squared (from Ref. 8).

APPENDIX

In this appendix, we indicate how the two-channel problem with an unphysical region can be reduced to the problem treated in Sec. IV. Suppose that the mass of the meson m is m_0; of the spinless "baryon" N, m_1. For invariant mass $M^2>4m_1^2$, the unitarity equations are precisely as given in Eq. (18) and Fig. 4. However, in the unphysical region, $2m_0<M<2m_1$, the second terms on the right-hand side in Fig. 4 are absent. Assuming we know the $m\overline{m}\to N\overline{N}$ amplitude in the unphysical regions, we have the corresponding equations, for $4m_0^2<M^2<4m_1^2$,

$$\begin{pmatrix} F_m^J \\ F_N^J \end{pmatrix} = \begin{pmatrix} e^{2i\delta_m} & 0 \\ i\zeta e^{i\delta_m} & 1 \end{pmatrix} \begin{pmatrix} F_m^J \\ F_N^J \end{pmatrix}^* . \tag{A1}$$

The solution to the combined set of discontinuity equations may be constructed as follows:

Consider the alternative problem of finding the functions F_{0m}, F_{0N} with a right-hand discontinuity for $M^2>4m_0^2$ specified by Eq. (18). In the unphysical region, $4m_0^2<M^2<4m_1^2$, we define our effective S matrix by arbitrarily setting $\eta=0$ and $\delta_N=0$. This hypothetical problem may be treated as in Sec. IV. Then the general solution to the original problem posed by Eqs. (18) and (A1) is obtained by setting

$$F_n = F_{0m}$$

and

$$F_N = F_{0N} + \frac{1}{2\pi}\int_{4m_0^2}^{4m_1^2} \frac{\zeta(x)|F_{0m}(x)|\,dx}{x-M^2} .$$

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[1]E. Lillethum, in *Proceedings of the XVI International Conference on High Energy Physics, Chicago-Batavia, Ill., 1972,* edited by J. D. Jackson and A. Roberts (NAL, Batavia, Ill., 1973), Vol. 1, p. 211.

[2]M. Suzuki, Univ. of California, Berkeley preprint, 1973 (unpublished).

[3]T. K. Gaisser and C.-I Tan, Phys. Rev. D 8, 3881 (1973).

[4]U. Amaldi *et al.*, Phys. Lett. 43B, 231 (1973); 44B, 112 (1973); S. R. Amendolia *et al.*, *ibid.* 44B, 119 (1973).

[5]An excellent, recent review of this approach, with references to earlier literature, has been presented by C. Quigg, in Lectures given at Canadian Institute for Particle Physics Summer School, McGill University, 1973, available as SUNY, Stony Brook Report No. ITP-SB-73-42 (unpublished).

[6]The alert reader will observe that the situation depicted in the figure is symmetric about the $N\bar{N}$ cluster so that, in fact, t_{i+1} also has a minimum value. It then follows that $t_{i\min} = (\mu^2 M^2/s_i)(1+\mu^2/s_{i+1})/(1-\mu^4/s_i s_{i+1})$. We suppose that, even for the s_i, s_{i+1} having their minimum values

$$\frac{\mu^2}{s_{i+1}}, \frac{\mu^2}{s_i} \ll 1.$$

[7]This should be compared to the estimate of $\Delta=3$, given in Ref. 3, for this same value of t_0. Since this is a log scale, a factor of 2 corresponds to the *square* of the corresponding energies.

[8]For a compilation and comparison of results on particle multiplicities, see M. Antinucci *et al.*, Nuovo Cimento Lett. 6, 121 (1973). One of their figures is reproduced below in Fig. 5. See also Ref. 5.

[9]See R. Armenteros and B. French, in *High Energy Physics,* edited by E. H. S. Burhop (Academic, New York, 1969).

[10]Using data for $P_{\text{lab}} > 50$ GeV/c, J. J. Whitmore (private communication) finds $\langle n_{\text{ch}}\rangle = (1.79 \pm 0.05)\ln s - (2.9 \pm 0.3)$. These numbers agree with those presented by J. Vander Velde in *High Energy Collisions—1973,* proceedings of the fifth international conference on high energy collisions, Stony Brook, 1973, edited by C. Quigg (A.I.P., New York, 1973), pp. 24ff.

[11]We would like to thank G. Giacomelli for assistance in making this estimate, as well as for several useful conversations.

[12]G. F. Chew and A. Pignotti, Phys. Rev. 176, 2112 (1969).

[13]D. Horn and A. Schwimmer, Nucl. Phys. B52, 627 (1973).

[14]H. Harari, Phys. Rev. Lett. 29, 1708 (1972).

[15]H. D. I. Abarbanel and M. B. Einhorn, Phys. Rev. Lett. 30, 404 (1973).

[16]G. Veneziano, Phys. Lett. 43B, 413 (1973).

[17]It is very likely that the whole sum rule [Eq. (11)] can be understood as an analog of Eq. (9), where the various types of "particles" i constitute a single cluster m, 2 correlated m's, etc.

[18]J. Ball and G. F. Chew, Phys. Rev. 109, 1385 (1958).

[19]In particular, a stronger attractive potential in the $I=1$'s odd G parity (or like quantum number state) could contribute to even-odd effects in annihilation in flight—a point which was discussed from a somewhat different point of view by H. Rubinstein (private communication).

[20]This corresponds to the absorbed "Born term" procedure of Sopkovich, Dar, and Gottfried and Jackson in the case that we are interested in the absorptive effects in a particular subsystem.

[21]A similar final-state interaction approach for relating $\bar{p}p$ annihilation and nonannihilation process was used by J. Jacob and S. Nussinov, Nuovo Cimento 14A, 335 (1973).

[22]In particular, the increase of this ratio with $M_{\bar{N}N}{}^2$ may contribute to delaying the effective $\bar{N}N$ production threshold to high ($s \approx 400$ GeV^2) energies.

[23]As a matter of principle, the magnitude of inelasticity η could be determined from measurements of the cross section $N\bar{N} \to m\bar{m}$. Then the phases δ_m, δ_N could be determined (modulo π and their signs) from measurements of the elastic cross sections.

[24]N. P. Vekua, *Systems of Singular Integral Equations* (Noordhoff, Groningen, 1967). These are sometimes referred to by physicists as the coupled-channel Omnès equations.

[25]A fundamental matrix is a matrix whose two column vectors are solutions of the equation such that every other solution of the equation may be written as a linear combination of these two solutions. Moreover, the coefficients in the linear combination are entire functions. See Ref. 24.

[26]Their degree at ∞ is limited by the asymptotic behavior required of F, e.g., if F is to be polynomially bounded at ∞, then generally $P_i(M^2)$ will be polynomials. Generally, these functions would not be entire but would reflect the left-hand singularities of F.

[27]See R. Blankenbecler, Phys. Rev. Lett. 31, 964 (1973), and references therein.