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Coherent exponentials in $\bar{p}p$ elastic scattering

H. B. Crawley, N. W. Dean, and W. J. Kernan
Ames Laboratory-USAEC and Department of Physics, Iowa State University, Ames, Iowa 50010
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It is shown that three coherent exponentials can represent the entire center-of-mass angular distribution in $\bar{p}p$ elastic scattering for the most accurate available experiments. The parametrization is significantly better determined if the ratios of the slopes of the exponentials are constrained. The most effective constraints are those corresponding to a simple absorption picture in which the absorbing (Pomeron) amplitude is only half as steep in t as the unabsorbed amplitude.

Experimental investigations of the $\bar{p}p$ elastic differential cross section, for momenta ranging from 1.4 to 16 GeV/c, consistently reveal the presence of a minimum around $-t=0.4$ (GeV/c)^2, followed by a secondary maximum beyond which the slope of the differential cross section is considerably flatter. In a recent series of papers,[1-3] we have shown that two coherent interfering exponentials can describe this structure even out to very large values of $-t$. In one of these papers,[1] we commented that beyond $-t=1.8$ (GeV/c)^2, there was additional structure which appeared to require a third exponential term, but that the data did not seem to be sufficiently precise to permit accurate determination of this structure.

In this paper we wish to explore the presence and possible significance of this third exponential term. When we had completed the analysis in terms of two exponentials, we realized that the three most accurate and extensive experiments, those at 2.32,[4] 2.33,[2] and 2.85 GeV/c,[1] all clearly indicated this third exponential term. We therefore attempted to fit these differential cross sections with the form

$$\frac{d\sigma}{dt}=\left(\frac{d\sigma}{dt}\right)_0\left|\frac{e^{b_1t/2}+A_1e^{i\phi_1}e^{b_2t/2}+A_2e^{i\phi_2}e^{b_3t/2}}{1+A_1e^{i\phi_1}+A_2e^{i\phi_2}}\right|^2 . \tag{1}$$

All three experiments could be well represented by this form, but many parameters were poorly determined. Even for the best data set, the 2.32-GeV/c data of the Argonne-Oxford collaboration, the parameters of the third term, A_2, ϕ_2, and b_3, could be varied simultaneously over large ranges (in a correlated way) without significantly changing the quality of the fit.

A similar difficulty had arisen in our earlier two-exponential analysis[3] [equivalent to Eq. (1) with $A_2=0$] for a number of sets of experimental data. We found in that case that it was possible to constrain $b_2/b_1=\frac{1}{3}$ and still represent the data adequately. The choice of $\frac{1}{3}$ for this ratio is consistent with the values found for b_2/b_1 at eight energies where both slope parameters were well determined; the ratio was generally between 0.3 and 0.4, and in no case was it as large as 0.5. With $b_2/b_1=\frac{1}{2}$, it was not possible to fit the data satisfactorily.

One simple way to understand this constraint is to assume that the first exponential term represents unabsorbed scattering, while the second is an absorption (or rescattering) correction. [In Regge language, the former corresponds to an effective pole trajectory term combining the exchanges of all relevant poles; the latter is the cut contribution, arising from the iteration of these poles with primarily elastic rescattering, as shown in Fig. 1.] Suppose the elastic amplitude has a t dependence of the form e^{b_et}. Then using any simple form of absorption model to calculate the second slope b_2 as an absorptive correction to b_1 will yield

$$b_2=b_1b_e/(b_1+b_e). \tag{2}$$

The ratio $b_2/b_1=\frac{1}{3}$ corresponds to $b_e=\frac{1}{2}b_1$, while $b_2/b_1=\frac{1}{2}$ follows from $b_e=b_1$. Since only the former is in good agreement with the data, we may conclude that $b_e<b_1$ is preferred.

If this model describes the first two terms of Eq. (1), then the third term should logically arise from a second absorption term, corresponding to the diagram of Fig. 1(c). The same analysis leading to Eq. (2) would then predict that

$$b_3=b_1b_e/(b_e+2b_1) . \tag{3}$$

If $b_2/b_1=\frac{1}{3}$, for example, then it follows from Eq. (3) that $b_3/b_1=\frac{1}{5}$. Using these ratios, Eq. (1) can be rewritten as

$$\frac{d\sigma}{dt}=\left(\frac{d\sigma}{dt}\right)_0\left|\frac{e^{b_1t/2}+A_1e^{i\phi_1}e^{b_1t/6}+A_2e^{i\phi_2}e^{b_1t/10}}{1+A_1e^{i\phi_1}+A_2e^{i\phi_2}}\right|^2 . \tag{4}$$

We have attempted to fit all the data of the differential cross section at each of the three ener-

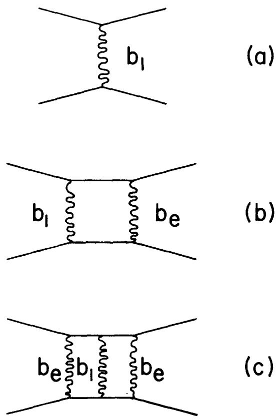

FIG. 1. Diagrams illustrating the interpretation of Eq. (1) as a combination of (a) unabsorbed scattering, with slope b_1; (b) single absorption, with slope b_e for the rescattering amplitude; and (c) double absorption.

gies using Eq. (4). The results were very satisfactory, and the parameters which result from the fit are listed in Table I. In Fig. 2 the data for each of the three experiments are shown with the results of the fit superimposed. A log-log scale was chosen so that the quality of the fit in the low-t region is easily judged. In Fig. 3 the same data and fits are replotted on a semilogarithmic scale for $-t \gtrsim 0.30$ (GeV/c)^2 to illustrate the fit in the higher-t region. The low-t data are left out of this plot since the points would be virtually indistinguishable from the curve. These parameters are in excellent agreement with those determined using Eq. (1), but the uncertainties in the parameters are much smaller in this more constrained fit. The values of χ^2, χ^2 per degree of freedom, and the confidence level for each data set are also shown in Table I. For the 2.32- and 2.33-GeV/c data, χ^2 is essentially unchanged between the eight-parameter fit to Eq. (1) and the six-parameter fit to Eq. (4). For the 2.85-GeV/c data the χ^2 increases by approximately two when the parameters are constrained.

In order to test the degree to which the predicted relationship between b_1, b_2, and b_3 is fulfilled, we performed a fit to the 2.32-GeV/c data in which the ratio b_2/b_1 was fixed at $\frac{1}{3}$, but b_3 was left free. This fit yielded a χ^2 of 70.5 with best-fit parameters not significantly different from those listed in Table I; the resulting value of the ratio b_3/b_1 was 0.20 ± 0.02, in exact agreement with the prediction of Eq. (3).

We also performed a fit at 2.32 GeV/c in which the ratios implied by Eq. (2) and Eq. (3) were assumed to hold, but the value of b_e was left free. The best fit in this case was obtained for $b_e/b_1 = 0.49 \pm 0.10$, in excellent agreement with the ratio $\frac{1}{2}$ (which corresponds to $b_2/b_1 = \frac{1}{3}$) in our earlier two-exponential fits.

We may draw the following conclusions from these constrained fits, even though the data are not sufficiently precise to allow accurate determination of all eight parameters in Eq. (1). First, the data are in excellent agreement with a three-exponential amplitude parametrization. Second, the ratios b_2/b_1 and b_3/b_1 both strongly prefer values which are fully consistent with the absorptive-model results of Eq. (2) and Eq. (3). Third, the ratios yielding the best description of the data correspond to an elastic rescattering amplitude which has a much flatter t dependence than that of the unabsorbed amplitude, the best fit corresponding quite well to $b_e = \frac{1}{2}b_1$.

A theoretical interpretation of these conclusions may be phrased in terms of Regge theory. Our leading term corresponds to all pole terms contributing to $\bar{p}p$ elastic scattering, including both the Pomeron and the standard ρ, ω, f, and A_2 Regge poles. Since the total cross section in this energy range is approximately double its apparent asymptotic value, the latter poles are contributing about half the amplitude. The rescattering or absorp-

TABLE I. The parameters providing the best fits of Eq. (4) to the data of Refs. 1, 2, and 4, and the values of χ^2, χ^2 per degree of freedom, and the confidence level (C. L.) obtained in those fits.

	2.32 GeV/c	2.33 GeV/c	2.85 GeV/c
$(d\sigma/dt)_0$ [mb/GeV/c^2)]	347.8 ± 4.3	368.5 ± 7.6	291.0 ± 4.7
b_1 (GeV/c)$^{-2}$	9.36 ± 0.12	9.50 ± 0.20	9.02 ± 0.15
A_1	0.509 ± 0.021	0.527 ± 0.025	0.505 ± 0.021
A_2	0.214 ± 0.011	0.223 ± 0.011	0.203 ± 0.010
ϕ_1	$(190.0 \pm 0.9)^\circ$	$(189.6 \pm 1.91)^\circ$	$(187.7 \pm 1.5)^\circ$
ϕ_2	$(33.11 \pm 2.5)^\circ$	$(30.2 \pm 4.1)^\circ$	$(27.8 \pm 3.6)^\circ$
χ^2	70.7	52.6	75.8
χ^2/ν	0.89	0.89	0.98
C.L.	74%	71%	52%

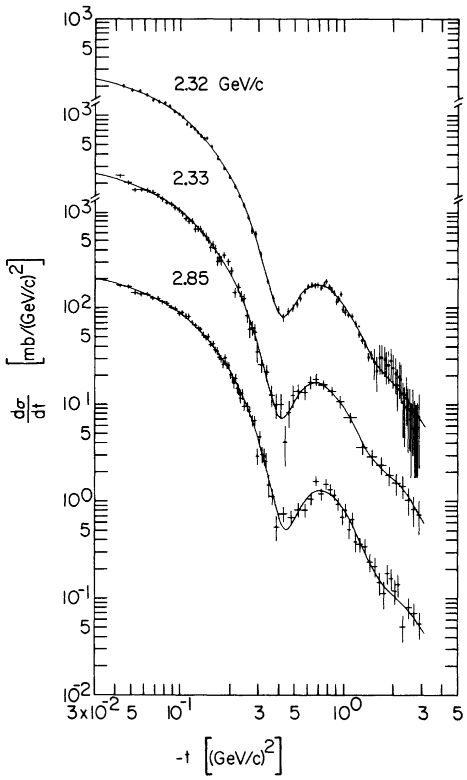

FIG. 2. Comparison of our fits using Eq. (8) with the experimental data of Refs. 1, 2, and 4. A log-log scale has been used in order to emphasize the quality of the fit at smaller values of $-t$.

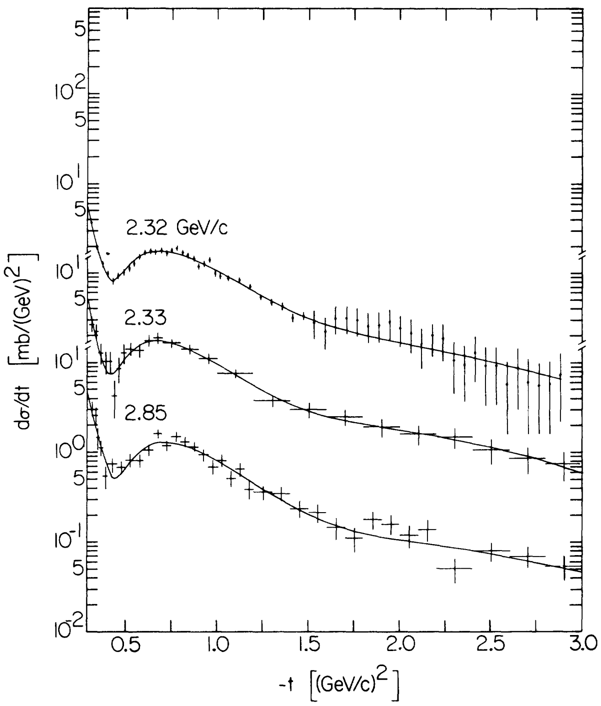

FIG. 3. The region $-t > 0.3$ (GeV/c)^2 of the fits shown in Fig. 2, plotted against the usual semilog scale in order to show clearly the effects of the third exponential term.

tion, however, arises predominantly from purely elastic scattering, corresponding to the Pomeron only. That is, Regge-Pomeron cuts are much more important than Regge-Regge cuts. The result $b_e \approx \frac{1}{2} b_1$ therefore implies that the Pomeron amplitude is flatter in t than the full $\bar{p}p$ pole amplitude (and, *a fortiori,* than the other pole terms).

This conclusion is in qualitative agreement with the observation that in the medium-energy region the $\bar{p}p$ elastic diffraction peak is steeper than that of pp elastic scattering, which is Pomeron-dominated. The ratio of these two experimental quantities is, however, somewhat greater than $\frac{1}{2}$, perhaps indicating some difference between the Pomeron part of the leading term and that involved in the absorption process.

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