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Gauge-invariant signal for gauge-symmetry breaking*

L. Dolan and R. Jackiw

Laboratory for Nuclear Science and Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139

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The effective potential is computed to order $\hbar$ in an Abelian gauge theory—scalar electrodynamics. The calculation is performed first in the ghost-requiring R_ξ gauges. The corresponding expression is also derived from the unitary Lagrangian. We discuss the gauge dependence of the effective potential and its minima in connection with spontaneous symmetry breakdown; and we interpret the unitary computation to be the physically relevant one.

I. INTRODUCTION

Minima in the field-theoretic effective potential V indicate symmetry properties of solutions for the theory.[1] Unfortunately, an exact calculation of V is rarely possible; often the best answer to be had is the first few terms in a loop expansion.[2-6] In general, one goes beyond the lowest tree approximation, so that effects due to accidental symmetries,[4,5] finite temperature,[7] or radiative corrections[3] can be examined. However, any approximate calculation may be unreliable; it may exhibit unphysical minima. (For example, we show in the Appendix that in ordinary quantum mechanics one frequently commits errors when the exact V is approximated by a finite series since the series does not converge in the region of the true minimum.)

In gauge theories the effective potential is gauge-dependent.[3,4,6] This presents difficulty in assessing the validity of any approximation to the complete V, since the gauge dependence may create false minima. Also, a direct physical interpretation cannot be given to a gauge-dependent quantity. Furthermore, it has been alleged that in some gauges (the R_ξ gauges[8]) V cannot be defined.[4]

In this paper we compute the effective potential to order $\hbar$ for an Abelian gauge theory—scalar electrodynamics. We show how even in the R_ξ gauges a potential *can* be defined. The calculation in this gauge is of additional interest as it involves a treatment of ghost loops. The problems with gauge dependence are vividly portrayed in our calculation. In the R_ξ gauge, V is already gauge-dependent in the tree approximation and possesses stationary points which do not correspond to physical solutions of the theory.

We suggest that the difficulty of the gauge dependence may be resolved by considering the unitary Lagrangian $\mathcal{L}_U$ (frequently called the Lagrangian in the unitary gauge). This unitary Lagrangian can be obtained as the limit of the corresponding object in the R_ξ gauge. However, we shall argue that $\mathcal{L}_U$ may be viewed not merely as a Lagrangian in a special gauge, but also as the Lagrangian for the theory when all gauge degrees of freedom have been removed.[9] The unitary Lagrangian reflects the physical spectrum for its fields, and the effective potential V_U associated with it merits the physical interpretation given by Symanzik.[10]

The danger with computations based on the unitary Lagrangian is that they may not be renormalizable. V_U, being the generator of single-particle irreducible Green's functions at zero momentum, does not appear to be directly related to physical S-matrix elements. Consequently, *a priori* one does not know whether V_U is renormalizable. Happily, we find that a finite answer emerges to the one-loop approximation.

In Sec. II we derive the renormalized effective potential to order $\hbar$ in the R_ξ gauge and discuss its properties. Section III is devoted to a direct evaluation of V_U from $\mathcal{L}_U$.[11] We also study the unitary limit of the effective potential in the R_ξ gauge. The expression which emerges in the limit coincides with V_U, provided the limit is taken at fixed cutoff; otherwise an ambiguous additional term is present. Concluding remarks comprise Sec. IV, while in an appendix we discuss the problems that are encountered in ordinary quantum mechanics when series approximations to V are attempted.

II. The R_ξ-GAUGE CALCULATION

A. The effective potential

We compute the renormalized one-loop approximation to the effective potential for scalar electrodynamics in the R_ξ gauge. The unrenormalized Lagrangian is

$$\mathcal{L} = \tfrac{1}{2}\partial_\mu\phi_a\partial^\mu\phi_a - \tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} - \tfrac{1}{2}m^2\phi^2 - \frac{\lambda}{4!}\phi^4 - e\epsilon_{ab}\partial_\mu\phi_a\phi_b A^\mu + \tfrac{1}{2}e^2\phi^2A^2, \tag{2.1}$$

where $\phi^2 = \phi_1{}^2 + \phi_2{}^2$, $\phi^4 = (\phi^2)^2$, and ϕ_a $(a=1, 2)$ is a real field. Specification of the R_ξ gauge conventionally involves adding to (2.1) a gauge-determining term which depends on ϕ_a^0, the vacuum expectation value of ϕ_a, in the case that the symmetry is spontaneously broken.[8] However, the effective-potential approach requires calculations to be performed before a commitment is made to symmetry breakdown; and symmetry breaking emerges as a displaced minimum of V. Consequently we must not, at this stage, assume symmetry breaking in the definition of the gauge. Thus we are led to a gauge function

$$-\frac{1}{2\alpha}(\partial\cdot A + v\cdot\phi)^2. \tag{2.2}$$

Here v_a is an *arbitrary* external 2-vector, not related to any properties of ϕ_a. [Previous definitions of the R_ξ gauge[8] correspond to $\alpha = 1/\xi$. $v_a = (1/\xi)e\epsilon_{ab}\phi_b^0$.] The gauge (2.2) requires a ghost-compensating term: One adds to (2.1)

$$\partial_\mu\psi^*\partial^\mu\psi - e\psi^*\psi v\times\phi, \tag{2.3}$$

where ψ and ψ^* are spinless fermions, and $v\times\phi = v_a\epsilon_{ab}\phi_b$.

It is the above procedure, of divorcing v_a from ϕ_a^0, that permits one to define an effective potential, and to circumvent the difficulty, observed by Weinberg,[4] that V does not exist if $v_a \propto \epsilon_{ab}\phi_b^0$. We shall continue to call our modified gauge the R_ξ gauge.

The complete unrenormalized Lagrangian is given by the sum of (2.1), (2.2), and (2.3). Renormalization can be performed before a commitment to symmetry breaking is made. The renormalization of the symmetric Lagrangian has been given by Lee, with the following result.[12] One renormalizes the theory as if v_a were zero. Then one adds the complete gauge-determining and gauge-compensating expressions in terms of renormalized fields without any further renormalization. Finally, ϕ_a is shifted by a renormalization counterterm w_a. Thus the renormalized Lagrangian is

$$\mathcal{L} = Z_2[\tfrac{1}{2}\partial_\mu\phi_a\partial^\mu\phi_a - e\epsilon_{ab}\partial_\mu\phi_a(\phi_b + w_b)A^\mu + \tfrac{1}{2}e^2A^2(\phi_a + w_a)^2] - Z_3\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2\alpha}(\partial\cdot A + v\cdot\phi + v\cdot w)^2$$
$$-\tfrac{1}{2}(m^2 + \delta m^2)(\phi_a + w_a)^2 - \frac{1}{4!}(\lambda + \delta\lambda)[(\phi_a + w_a)^2]^2 + \partial_\mu\psi^*\partial^\mu\psi - e\psi^*\psi v\times(\phi + w). \tag{2.4}$$

(Here all fields and parameters are renormalized.) As we are computing only to order $\hbar$, it is sufficient to keep the counterterms only to that accuracy. Setting $Z_3 = 1 + z_3$, $Z_2 = 1 + z_2$, and recalling that z_2, z_3, δm^2, $\delta\lambda$, and w_a are each of order $\hbar$, we may replace (2.4) by the simpler expression

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_x, \tag{2.5a}$$

where

$$\mathcal{L}_0 = -\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} + \tfrac{1}{2}\partial_\mu\phi_a\partial^\mu\phi_a - e\epsilon_{ab}\partial_\mu\phi_a\phi_bA^\mu + \tfrac{1}{2}e^2A^2\phi^2 - \frac{1}{2\alpha}(\partial\cdot A + v\cdot\phi)^2 + \partial_\mu\psi^*\partial^\mu\psi - e\psi^*\psi v\times\phi - \tfrac{1}{2}m^2\phi^2 - \frac{\lambda}{4!}\phi^4, \tag{2.5b}$$

$$\mathcal{L}_x = -\tfrac{1}{4}z_3 F_{\mu\nu}F^{\mu\nu} + z_2(\tfrac{1}{2}\partial_\mu\phi_a\partial^\mu\phi_a - e\epsilon_{ab}\partial_\mu\phi_a\phi_b A^\mu + \tfrac{1}{2}e^2A^2\phi^2) - e\epsilon_{ab}\partial_\mu\phi_a w_b A^\mu + e^2A^2\phi\cdot w$$
$$-\frac{1}{\alpha}(\partial\cdot A + v\cdot\phi)w\cdot v - \tfrac{1}{2}\delta m^2\phi^2 - m^2\phi\cdot w - \frac{\delta\lambda}{4!}\phi^4 - \tfrac{1}{6}\lambda\phi\cdot w\phi^2 - e\psi^*\psi v\times w. \qquad (2.5c)$$

$V(\hat\phi)$ to zero order and the $O(\hbar)$ counterterm follow immediately from (2.5). These are

$$V_0(\hat\phi) = \tfrac{1}{2}(m^2+\delta m^2)\hat\phi^2 + \frac{1}{4!}(\lambda+\delta\lambda)\hat\phi^4 + \frac{1}{2\alpha}(v\cdot\hat\phi)^2 + \hat\phi\cdot w(m^2 + \tfrac{1}{6}\lambda\hat\phi^2) + \frac{1}{\alpha}(\hat\phi\cdot v)(w\cdot v). \qquad (2.6)$$

The nontrivial $O(\hbar)$ contribution is evaluated by the functional method. We shift $\phi_a(x)$ in (2.5b) by $\hat\phi_a$, and retain terms quadratic in the quantum fields.[6] [Only quadratic terms are needed for an $O(\hbar)$ result.] The above procedure defines an action given by

$$I(\hat\phi;\phi_a,A^\mu,\psi^*,\psi) = \int d^4x d^4y[\tfrac{1}{2}\phi_a(x)iD^{-1}{}_{ab}(\hat\phi;x-y)\phi_b(y) + \tfrac{1}{2}A^\mu(x)i\Delta^{-1}{}_{\mu\nu}(\hat\phi;x-y)A^\nu(y) + A^\mu(x)M_{\mu a}(\hat\phi;x-y)\phi_a(y) + \psi^*(x)iS^{-1}(\hat\phi;x-y)\psi(x)]. \qquad (2.7)$$

In momentum space the propagators are

$$iD^{-1}{}_{ab}(\hat\phi;k) = (k^2 - m^2 - \tfrac{1}{6}\lambda\hat\phi^2)\delta_{ab} - \tfrac{1}{3}\lambda\hat\phi_a\hat\phi_b - \frac{1}{\alpha}v_a v_b,$$
$$i\Delta^{-1}{}_{\mu\nu}(\hat\phi;k) = (-k^2 + e^2\hat\phi^2)g_{\mu\nu} + \left(1-\frac{1}{\alpha}\right)k_\mu k_\nu,$$
$$M_{\mu a}(\hat\phi;k) = ie\epsilon_{ab}\hat\phi_b k_\mu - \frac{i}{\alpha}k_\mu v_a, \qquad (2.8)$$
$$iS^{-1}(\hat\phi;k) = k^2 - ev\times\hat\phi.$$

According to the general procedure previously described,[6] the $O(\hbar)$ part of V is determined by the functional integral

$$\int d\phi_a d\psi^* d\psi dA^\mu e^{(i/\hbar)I} = \mathrm{Det}\,iS^{-1}\mathrm{Det}^{-1/2}(i\Delta^{-1})\mathrm{Det}^{-1/2}(iD^{-1}+iN). \qquad (2.9)$$

The A^μ integration was performed first and N is defined by

$$N_{ab}(\hat\phi;k) = M_a^\mu(\hat\phi;k)\Delta_{\mu\nu}(\hat\phi;k)M_b^\nu(\hat\phi;-k). \qquad (2.10)$$

The $O(\hbar)$ effective potential which follows from (2.9) is

$$V_1(\hat\phi) = i\hbar\int\frac{d^4k}{(2\pi)^4}\ln[iS^{-1}(\hat\phi;k)] - \frac{i\hbar}{2}\int\frac{d^4k}{(2\pi)^4}\ln\det[i\Delta^{-1}{}_{\mu\nu}(\hat\phi;k)] - \frac{i\hbar}{2}\int\frac{d^4k}{(2\pi)^4}\ln\det[iD^{-1}{}_{ab}(\hat\phi;k) + iN_{ab}(\hat\phi;k)]. \qquad (2.11)$$

We substitute (2.8) and (2.10) in (2.11) and rotate the integration contour to find

$$V_1(\hat\phi) = \frac{\hbar}{32\pi^2}\int_0^\infty dk^2k^2[-2\ln(k^2+ev\times\hat\phi) + 3\ln(k^2+e^2\hat\phi^2) + \ln(k^6 + a_2k^4 + a_1k^2 + a_0)], \qquad (2.12)$$

where the coefficients a_2, a_1, and a_0 are given by

$$a_2 = \tfrac{2}{3}\lambda\hat\phi^2 + 2m^2 + 2ev\times\hat\phi,$$
$$a_1 = \hat\phi^4(\tfrac{1}{12}\lambda^2 + \tfrac{1}{6}\alpha e^2\lambda) + \hat\phi^2(\alpha e^2m^2 + \tfrac{2}{3}\lambda m^2 + v^2e^2 + \lambda ev\times\hat\phi) + m^4 + 2m^2ev\times\hat\phi - e^2(v\cdot\hat\phi)^2,$$
$$a_0 = \hat\phi^6(\tfrac{1}{12}\lambda^2\alpha e^2) + \hat\phi^4(\tfrac{2}{3}\alpha e^2m^2\lambda + \tfrac{1}{6}e^2v^2\lambda) + \hat\phi^2[\alpha e^2m^4 + v^2e^2m^2 + \tfrac{1}{3}\lambda(\hat\phi\times v)^2e^2].$$

It is now possible to evaluate (2.12) by introducing a cutoff at $k^2=\Lambda^2$ and dropping terms which vanish for large Λ^2:

$$V_1(\hat\phi) = \frac{\hbar}{32\pi^2}\{(\tfrac{2}{3}\lambda\hat\phi^2 + 3e^2\hat\phi^2)\Lambda^2 + [\hat\phi^4(-\tfrac{3}{2}e^4 - \tfrac{5}{36}\lambda^2 + \tfrac{1}{6}\alpha e^2\lambda) + \hat\phi^2(\alpha e^2m^2 - \tfrac{2}{3}\lambda m^2) - \tfrac{1}{3}\lambda e(v\times\hat\phi)\hat\phi^2 - 2m^2ev\times\hat\phi]\ln\Lambda^2$$
$$- e^2(v\times\hat\phi)^2\ln ev\times\hat\phi + \tfrac{1}{2}e^2(v\times\hat\phi)^2 + \tfrac{3}{2}e^4\hat\phi^4\ln e^2\hat\phi^2 - \tfrac{3}{4}e^4\hat\phi^4 + \tfrac{1}{4}(r_1^2\ln r_1^2 + r_2^2\ln r_2^2 + r_3^2\ln r_3^2) - \tfrac{1}{4}(r_1^2+r_2^2+r_3^2)\}. \qquad (2.13)$$

Here the r_i are roots of the cubic polynomial in k^2 appearing in (2.12); we shall not compute them because of the tedium involved.

Renormalization is achieved by combining the counterterms in (2.6) with the cutoff-dependent parts of (2.13). With the identification

$$\frac{1}{2}\delta m^2=\frac{-\hbar}{32\pi^2}[(3e^2+\tfrac{2}{3}\lambda)\Lambda^2+(\alpha e^2 m^2-\tfrac{2}{3}m^2\lambda)(\ln\Lambda^2)],$$

$$\frac{\delta\lambda}{4!}=\frac{-\hbar}{32\pi^2}(-\tfrac{3}{2}e^4-\tfrac{5}{36}\lambda^2+\tfrac{1}{6}\alpha e^2\lambda)\ln\Lambda^2, \tag{2.14}$$

$$w_a=-e\epsilon_{ab}v_b\frac{\hbar}{16\pi^2}\ln\Lambda^2,$$

the one-loop approximation to the effective potential is completely renormalized and defined up to an arbitrary finite polynomial.

In the limit $m=0$, $v_a=0$, the calculation is in agreement with the result for a massless theory in Lorentz gauges.[6] Another interesting limit is $\lambda=0$ and $m=0$ (recall that λ is conventionally of order e^4). In that case (2.13) simplifies enormously. We find apart from polynomials in ϕ^2,

$$V_1(\hat{\phi})=\frac{3\hbar}{64\pi^2}e^4\hat{\phi}^4\ln\hat{\phi}^2. \tag{2.15}$$

It is remarkable that all gauge dependence has disappeared to this order. (For gauges which do not require ghost-compensating the same phenomenon has already been observed.[6])

B. Gauge dependence and symmetry breaking

The effective potential in the R_ξ gauge is gauge-dependent in zeroth order [see (2.6)]. There are now several solutions of $\partial V(\hat{\phi})/\partial\hat{\phi}_a=0$ in the tree approximation. From (2.6) it follows that

$$\frac{\partial V_0(\hat{\phi})}{\partial\hat{\phi}_a}=m^2\hat{\phi}_a+\frac{\lambda}{6}\hat{\phi}_a\hat{\phi}^2+\frac{1}{\alpha}v_a(v\cdot\hat{\phi}). \tag{2.16}$$

(We have deleted the counterterms, as they are irrelevant.) In addition to the symmetric minimum at $\hat{\phi}_a=0$, the above expression possesses the usual symmetry-breaking minimum for negative m^2:

$$\hat{\phi}_a=\epsilon_{ab}\hat{v}_b\left(-\frac{6m^2}{\lambda}\right)^{1/2},\qquad \hat{v}_a=\frac{v_a}{(v^2)^{1/2}}. \tag{2.17}$$

This corresponds, in the familiar fashion, to a vector-meson mass $\mu_V^2=-6e^2m^2/\lambda$.

However, for $(v^2/\alpha)+m^2<0$, we also find that $\partial V_0(\hat{\phi})/\partial\phi_a$ vanishes for

$$\hat{\phi}_a=\hat{v}_a[-(6/\lambda)(m^2+v^2/\alpha)]^{1/2}. \tag{2.18}$$

Clearly this solution is unphysical. It may be present even when $m^2>0$, where there is no known mechanism for symmetry breaking. Furthermore, (2.18) would imply a gauge-dependent mass for the vector meson. Note also $\hat{\phi}_a$ in (2.18) points in a direction perpendicular to the usual symmetry-breaking one, (2.17). Presumably the unphysical solution is absent in the complete effective potential, and an explicit expression for V is needed for further analysis.

Computations of effective potentials for field theories at finite temperature[7] make use of statistical methods which are readily applicable only to physical quantities. The gauge dependence of V frustrates such calculations.

There is a need to eliminate the confusion of multiple, gauge-dependent zeros in (2.16). Also it is desirable to define an effective potential whose physical content is transparent. Consequently, we are led to a consideration of the unitary Lagrangian which appears to satisfy all requirements.

III. THE UNITARY CALCULATION

A. Definition of the unitary theory

Conventionally the unitary Lagrangian is obtained as a limit of the R_ξ Lagrangian. It thus appears that this limit corresponds to a specific albeit singular choice of gauge—hence the nomenclature "unitary gauge." We propose that the unitary Lagrangian is not merely a choice of gauge; rather it is the unique description of the physical dynamics of the system from which the gauge degrees of freedom have been removed by a functional integration.[9] Therefore the effective potential for the physical system is properly computed from the unitary Lagrangian, $\mathcal{L}_U$.

To substantiate our point of view, we show that for *any choice of gauge* we can arrive at $\mathcal{L}_U$ by a change of variables and an elimination of the gauge degrees of freedom. This procedure may be effected for the symmetric theory as well as for a spontaneously broken theory. We begin by representing the vacuum expectation value of a physical (gauge-invariant) quantity by a Feynman-path integral.

$$\langle F\rangle=\int d\phi_a dA^\mu F e^{(i/\hbar)I}. \tag{3.1}$$

Here F is the gauge-invariant quantity of interest and I is the complete action, i.e., the classical gauge-invariant action I_0 plus the gauge-determining and -compensating part I_g. I_g is a functional of the fields ϕ_a and A^μ; its form defines the gauge. I_g is arbitrary except that I_g^θ, the gauge transform of I_g, is normalized by

$$\int d\theta \exp\left(\frac{i}{\hbar} I_g^\theta\right) = 1 . \quad (3.2)$$

Let us now change variables in (3.1):

$$\phi_1 = \rho \cos\theta ,$$
$$\phi_2 = \rho \sin\theta , \quad (3.3)$$
$$A^\mu = B^\mu - \frac{1}{e}\partial^\mu\theta .$$

This is *not* a gauge transformation, but a change of variables in the functional integral. [In a gauge where ϕ_a and A^i are canonical coordinates, (3.3) defines a change of canonical coordinates to ρ, θ, and B^i. Hence it is a canonical transformation.[13]] However, since the form of Eq. (3.3) is similar to that of a gauge transform, we know that the transformed F and I_0 are independent of θ, while I_g becomes I_g^θ. Since (3.3) is not a gauge transformation, the Jacobian is not unity, but rather

$$d\phi_a dA^\mu = \mathrm{Det}\rho\, d\rho\, d\theta\, dB^\mu . \quad (3.4a)$$

Thus (3.1) becomes

$$\langle F\rangle = \int \mathrm{Det}\rho\, d\rho\, d\theta\, dB^\mu F \exp\left[\frac{i}{\hbar}(I_0 + I_g^\theta)\right]$$
$$= \int d\psi^* d\psi\, d\rho\, dB^\mu F \exp\left[\frac{i}{\hbar}(I_0 + \psi^*\rho\psi)\right] . \quad (3.4b)$$

We have used the normalization condition (3.2) and have represented the functional determinant $\mathrm{Det}\rho$ by a ghost field action $\int d^4x\, \psi^*(x)\rho(x)\psi(x)$. (More precisely we consider matrix elements $\langle F\rangle$ in the charge-zero sector of the theory. Then F in (3.1) is explicitly gauge-invariant and the steps from (3.1) to (3.4) are manifestly valid. In sectors with nonzero charge, physical quantities, to be sure, are gauge-independent. However, they are computed from matrix elements of charge-bearing operators which are not gauge-invariant. We do not wish to enter here upon the question of whether or not it is possible to represent all physical quantities with gauge-invariant operators. A related point is that whereas (3.3) can be a canonical transformation, the removal of the θ degrees of freedom by the functional integration (3.2) is not equivalent to a canonical transformation. The charge density operator involves Π_θ, the momentum conjugate to θ, as is seen from the commutation relation

$$i[\Pi_\theta(x), \rho(y)e^{\pm i\theta(y)}]_{x^0=y^0} = i[\Pi_\theta(x), \phi_1(y) \pm i\phi_2(y)]_{x^0=y^0}$$
$$= \pm i\rho(x)e^{\pm i\theta(x)}\delta^3(\vec{x}-\vec{y})$$
$$= \pm i[\phi_1(x) \pm i\phi_2(x)]\delta^3(\vec{x}-\vec{y}).$$

The removal of the θ degrees of freedom thus implies the removal of the charge operator.)

The unitary unrenormalized Lagrangian is obtained, for any choice of gauge, from (3.4b); it is

$$\mathcal{L}_U = -\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} + \tfrac{1}{2}\partial_\mu\rho\partial^\mu\rho - \tfrac{1}{2}m^2\rho^2 - \frac{\lambda}{4!}\rho^4$$
$$+ \tfrac{1}{2}e^2B^2\rho^2 + \psi^*\rho\psi . \quad (3.5)$$

B. Computation from the unitary Lagrangian

The renormalized effective potential is found as usual by shifting $\rho \to \rho + \hat\rho$, to $O(\hbar)$ keeping only quadratic terms, and performing the appropriate functional integral[14]:

$$V_U(\hat\rho) = \frac{m^2 + \delta m^2}{2}\hat\rho^2 + \frac{\lambda + \delta\lambda}{4!}\hat\rho^4 + i\hbar\int\frac{d^4k}{(2\pi)^4}\ln\hat\rho$$
$$-\frac{i\hbar}{2}\int\frac{d^4k}{(2\pi)^4}\ln\det[(-k^2 + e^2\hat\rho^2)g_{\mu\nu} + k_\mu k_\nu]$$
$$-\frac{i\hbar}{2}\int\frac{d^4k}{(2\pi)^4}\ln\left(k^2 - m^2 - \frac{\lambda}{2}\hat\rho^2\right) + O(\hbar^2) .$$
$$(3.6a)$$

The first two integrals above, the ghost and photon contributions, combine and thereby eliminate a possible quartic divergence. The result of an elementary calculation is

$$V_U(\hat\rho) = \tfrac{1}{2}m^2\left(1 - \frac{\hbar}{64\pi^2}\lambda\right)\hat\rho^2 + \frac{\bar\lambda}{4!}\hat\rho^4$$
$$+\frac{\hbar}{64\pi^2}\left[3e^4\hat\rho^4\ln\frac{\hat\rho^2}{m^2}\right.$$
$$\left. + \left(m^2 + \frac{\lambda}{2}\hat\rho^2\right)^2\ln\left(1 + \frac{\lambda}{2}\frac{\hat\rho^2}{m^2}\right)\right] .$$
$$(3.6b)$$

The mass counterterm has been determined in (3.6b) by the requirement $\partial^2 V_U(\hat\rho)/\partial\hat\rho^2|_{\hat\rho=0} = m^2$; $\bar\lambda$ is a finite quantity which cannot be determined because of infrared divergences. The computation is especially simple since the procedure of changing variables, (3.3), eliminates a trilinear interaction between the vector-meson and the scalar particles. We note the important result $V_U(\hat\rho)$ to $O(\hbar)$ is renormalizable.

The coefficients of the logarithmic expressions in (3.6b) reflect the number of degrees of freedom available to the respective particles multiplied by the square of the induced mass: 3 times $e^4\hat\rho^4$ for the vector-meson and 1 times $(m^2 + \frac{1}{2}\lambda\hat\rho^2)^2$ for the scalar particle. Observe that in the $m^2 = 0$ limit the λ^2-dependent logarithm enters with a different coefficient from the corresponding Lorentz-gauge expression.[3,6] The reason for this is that in the Lorentz gauge there are two bosons with effective masses $\frac{1}{2}\lambda\hat\phi^2$ and $\frac{1}{6}\lambda\hat\phi^2$, while in the unitary Lagrangian there is only one boson, with mass $\frac{1}{2}\lambda\hat\rho^2$. It is gratifying that to $O(\hbar)$ the unitary effective potential has properties which simply correspond to the physical states.

C. Computation from the R_ξ Lagrangian

Although we have arrived at a renormalizable answer for V_U directly from $\mathcal{L}_U$, this was achieved in a rather singular fashion: A quartic divergence was eliminated by a cancellation of the ghost contribution against a portion of the photon contribution in (3.6a). It is desirable, therefore, to evaluate V_U with the help of regulators.

A convenient regularization of the unitary theory is provided by our R_ξ gauge. (We take the point of view that this procedure defines a regularization of the unitary Lagrangian, rather than a choice of gauge.) Conventionally[8] the unitary theory is obtained by rescaling v in (2.2) and (2.3) to $v\alpha$, and by letting $\alpha\to\infty$. However, this is by no means the only possible limiting procedure. One can rescale v to $v\alpha^{\epsilon+1/2}$; the unitary Lagrangian is obtained as $\alpha\to\infty$, provided $\epsilon>0$. Alternatively we may rescale v to $v\beta$ and let $\beta\to\infty$, with α fixed. In establishing that these limits yield the unitary theory, the following formulas are used:

$$\lim_{\alpha\to\infty}(1-i)\alpha^{\epsilon}(2\pi)^{-1/2}\exp\left[\frac{i}{\alpha}(y+\alpha^{\epsilon+1/2}x)^2\right]=\delta(x)\,,\qquad \epsilon>0 \quad (3.7)$$

$$\lim_{\beta\to\infty}(1-i)\beta(2\pi\alpha)^{-1/2}\exp\left[\frac{i}{\alpha}(y+\beta x)^2\right]=\delta(x).$$

We now show that when the above limiting procedures are applied to V calculated in the R_ξ gauge, then V_U as given in (3.6b) always emerges, provided the limit is *taken at fixed cutoff*, i.e., before the momentum integration is performed. When the limit is taken *after* the divergent integrals are evaluated, we find that V tends to $V_U + V_X$, where V_X is not well defined: It is infinite and the form of the infinity depends on the various possible limiting procedures.

To study the unitary limit, first note that $v\cdot\hat{\phi}$ should be set to zero, since field components parallel to v decouple from the theory. Thus, we may replace $v\times\hat{\phi}$ by $v\hat{\phi}$, and (2.12) becomes

$$V_1(\hat{\phi})=\frac{\hbar}{32\pi^2}\int_0^\infty dk^2k^2\{-2\ln(k^2+ev\hat{\phi})+3\ln(k^2+e^2\hat{\phi}^2)+\ln(k^2+m^2+\tfrac{1}{2}\lambda\hat{\phi}^2)[(k^2+ev\hat{\phi})^2+(k^2+\alpha e^2\hat{\phi}^2)(m^2+\tfrac{1}{6}\lambda\hat{\phi}^2)]\}\ . \quad (3.8)$$

Comparing this with (3.6), we see that the above may be written as

$$V_1(\hat{\phi})=V_{1U}(\hat{\phi})+V_X(\hat{\phi})\ , \quad (3.9)$$

where $V_{1U}(\hat{\phi})$ is the unrenormalized $O(\hbar)$ contribution to V_U of (3.6b) and

$$V_X(\hat{\phi})=\frac{\hbar}{32\pi^2}\int_0^\infty dk^2k^2\{-2\ln(k^2+ev\hat{\phi})+\ln[(k^2+ev\hat{\phi})^2+(k^2+\alpha e^2\hat{\phi}^2)(m^2+\tfrac{1}{6}\lambda\hat{\phi}^2)]\}$$
$$=\frac{\hbar}{32\pi^2}\int_0^\infty dk^2k^2\ln\left[1+\frac{(k^2+\alpha e^2\hat{\phi}^2)(m^2+\frac{1}{6}\lambda\hat{\phi}^2)}{(k^2+ev\hat{\phi})^2}\right]\ . \quad (3.10)$$

If we pass to the limit in (3.10) before the integral is evaluated, then V_X vanishes. This is seen by noting that, regardless of the precise limiting procedure, $e^2v^2\hat{\phi}^2$ always dominates $(k^2+\alpha e^2\hat{\phi}^2)\times(m^2+\lambda\hat{\phi}^2/b)$. Hence

$$\ln\left[1+\frac{(k^2+\alpha e^2\hat{\phi}^2)(m^2+\frac{1}{6}\lambda\hat{\phi}^2)}{(k^2+ev\hat{\phi})^2}\right]\to\ln 1=0\ .$$

Thus $V\to V_U$ at fixed cutoff.

However, if first the integration is performed with a cutoff and terms that vanish as the cutoff goes to infinity are dropped, we find, apart from a cutoff-dependent quartic polynomial in $\hat{\phi}$, that

$$V_X(\hat{\phi})=\frac{\hbar}{32\pi^2}\left[(ev\hat{\phi}\mu^2-\tfrac{1}{2}\alpha e^2\hat{\phi}^2\mu^2+\tfrac{1}{4}\mu^4)\ln\hat{\phi}^2+(\tfrac{1}{2}e^2v^2\hat{\phi}^2+ev\hat{\phi}\mu^2-\tfrac{1}{2}\alpha e^2\hat{\phi}^2\mu^2+\tfrac{1}{4}\mu^4)\ln\left(1+\frac{\alpha\mu^2}{v^2}\right)+AB\ln\frac{A+B}{A-B}\right],$$

$$A=ev\hat{\phi}+\tfrac{1}{2}\mu^2,\quad B=(ev\hat{\phi}\mu^2-\alpha e^2\hat{\phi}^2\mu^2+\tfrac{1}{4}\mu^4)^{1/2},\quad \mu^2=m^2+\tfrac{1}{6}\lambda\hat{\phi}^2\ . \quad (3.11)$$

This diverges as the unitary limit is taken; moreover, the form of the divergence depends on the nature of the limiting procedure. Nevertheless, there always exist contributions which are not polynomials in $\hat{\phi}$, e.g., the first term in (3.11).

D. Discussion

We have arrived at two expressions for the effective unitary potential: V_U calculated directly from $\mathcal{L}_U$, and $V_U + V_X$ obtained by regulating. V_U is renormalizable; V_X is ill-defined. There are two obstacles which prevent one from deciding in the present context which of the two results is more useful. First, our calculation is approximate; the effect of higher orders can make the two approaches compatible. Second, the effective potential is not directly related to a measurable quantity, and only measurable objects need be described by a finite and unambiguous formula in the unitary theory.

It must be remembered that $\mathcal{L}_U$ is not renormalizable, even though physical quantities are well defined. This has the consequence that a physically relevant calculation may require the inclusion of higher perturbative orders which are emphasized by the singular nature of the theory. On the other hand, in a calculation from the R_ξ Lagrangian, higher orders can be safely ignored, since the theory is renormalizable, but the unitary limit is in general ill-defined. For a physical quantity, however, once it is extracted from the complete effective potential, the limit should exist. In this connection it is reassuring that the parts of V_X which become infinite are all v- or α-dependent; such a dependence cannot occur in a measurable quantity.

Thus we conclude that in any practical calculation which is performed directly from V_U higher orders must be assessed to ensure that they do not affect the answer, while in a ξ-regulated calculation, the physically relevant portion must be extracted before regulators are sent to their limits. For example, in the Coleman-Weinberg model,[3] where m^2 is set to zero and λ is neglected compared to e^2, then to order e^4 V_X can be ignored and a unique effective potential is obtained. In a forthcoming paper devoted to finite temperature effects,[7] we shall show how a well-defined critical temperature can be extracted from $V_U + V_X$, even when $m^2 \neq 0$ and λ is not ignored compared to e^2.

IV. CONCLUSION

Spontaneous symmetry breaking in a gauge theory is presumably a gauge-invariant phenomenon, which can be observed in any gauge. Nevertheless, it is useful to develop a gauge-invariant procedure for establishing the existence of nonsymmetric solutions. We believe that we have given such a procedure by insisting that calculations of the effective potential be performed from the unitary Lagrangian. Additional advantages which emerge are (1) a clear physical interpretation can be given, and (2) the computation is simpler than in any of the conventional gauges. This latter point will be especially salient when the two-loop calculation is performed.[15] The need for this further approximation derives from our ignorance about the properties of the effective potential for the unitary Lagrangian in higher orders.

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APPENDIX: THE EFFECTIVE POTENTIAL IN QUANTUM MECHANICS

Ordinary quantum mechanics, i.e., potential theory, may be viewed as a field theory in one dimension—time. The "field" is just the position operator $q(t)$. (We consider quantum mechanics in only one spatial dimension; hence there is only one "field.") The formalism for the effective potential may be developed in complete analogy with the field-theoretic discussion. Everything is convergent and simple. All momentum integrals now involve a one-dimensional energy integration.

Let us consider a Lagrangian

$$L = \tfrac{1}{2} m \dot{q}^2(t) - U(q(t)) , \tag{A1}$$

with $U(q) = U(-q)$, and assume that a normalizable state $|0\rangle$ of lowest energy exists. Parity is a symmetry of the theory, and we may inquire whether this symmetry can be spontaneously violated, so that

$$\langle 0|q(0)|0\rangle \neq 0 . \tag{A2}$$

The answer is well known: No normalizable energy eigenstate is degenerate, hence spontaneous symmetry breaking does not occur. (Even non-normalizable states are not degenerate, provided the wave function vanishes when its position argument is $+\infty$ or $-\infty$.)

It is now clear that if U has minima away from the origin, the tree approximation to the effective potential gives erroneous results about symmetry breaking.[16] We have, to zero order in $\hbar$

$$V(\hat{q}) = U(\hat{q}) + O(\hbar) . \tag{A3}$$

When $U'(\hat{q}) = 0$ for $\hat{q} \neq 0$, it might be concluded wrongly that spontaneous parity violation occurs. Regardless of any minima of $U(\hat{q})$, one knows *a priori* that the correct solution is the symmetric one. [Since U is symmetric, $U'(0)$ always vanishes, provided it exists.] Even if the origin is a maximum of U, rather than a minimum, one must choose the symmetric theory.

Difficulties are compounded when the potential is computed to order $\hbar$:

$$V(\hat{q}) = U(\hat{q}) - \frac{i\hbar}{2} \int \frac{dE}{2\pi} \ln[mE^2 - U''(\hat{q}) + i\epsilon] + O(\hbar^2)$$

$$= U(\hat{q}) + \frac{\hbar}{2}\left(\frac{U''(\hat{q})}{m} - i\epsilon\right)^{1/2} + O(\hbar^2)\ . \tag{A4}$$

Let us suppose that $U(\pm\infty) = \infty$ so that all energy states are discrete and normalizable, and no unstable states exist in the theory. Further, let us assume that the origin is a maximum: $U''(0) < 0$. The anharmonic oscillator with imaginary frequency is an explicit example,

$$U(\hat{q}) = -\tfrac{1}{2} m\omega^2 \hat{q}^2 + \frac{\lambda}{4!}\hat{q}^4\ .$$

From the general theory it follows that $V''(0)$ is the inverse propagator (times $-i$) at zero energy. We now show that $V''(0) > 0$; hence $V''(0)$ cannot be approximated by $U''(0)$. In the present context the propagator is defined by

$$D(E) = \int_{-\infty}^{\infty} dt\, e^{iEt/\hbar} \langle 0|Tq(t)q(0)|0\rangle\ . \tag{A5a}$$

At zero energy this is also given by

$$D(0) = 2\int_0^{\infty} dt \langle 0|q(t)q(0)|0\rangle\ , \tag{A5b}$$

where time translation invariance is used to arrive at (A5b). Inserting a complete set of states in (A5b), we further find

$$D(0) = 2\sum_n \int_0^{\infty} dt \exp\left[\frac{i}{\hbar}(E_0 - E_n)t\right] |\langle 0|q(0)|n\rangle|^2$$

$$= 2i\sum_n \frac{\hbar}{E_0 - E_n + i\epsilon} |\langle 0|q(0)|n\rangle|^2\ . \tag{A6}$$

Since $\langle 0|q(0)|0\rangle = 0$, the ground state does not contribute to (A6). Also, the first excited state is separated from the ground state, hence there is no singularity in the denominator and the $i\epsilon$ may be dropped. We conclude therefore that

$$iD(0) = 2\sum_{n>0} \frac{\hbar}{E_n - E_0} |\langle 0|q(0)|n\rangle|^2 > 0\ . \tag{A7}$$

Hence $-i$ times the inverse propagator at zero energy must be real and positive. Consequently, $V''(0) > 0$, and tne correct physical theory corresponds to a minimum of V, though not necessarily to a minimum of U.[17]

Let us now compute $[iD(0)]^{-1}$ to order $\hbar$ from (A4). Upon differentiating (A4) and remembering that $U'''(0) = 0$, due to the symmetry of U, one finds

$$[iD(0)]^{-1} = V''(0)$$

$$= U''(0) + \frac{\hbar U''''(0)}{4[mU''(0) - i\epsilon]^{1/2}} + O(\hbar^2)\ . \tag{A8}$$

We have assumed the origin to be a maximum, $U''(0) < 0$. Equation (A8) is negative in lowest order and purely *imaginary* in order $\hbar$, in complete disagreement with the general result (A7).

It must be concluded that an expansion in powers of $\hbar$ for the effective potential is entirely unreliable in a wide class of quantum-mechanical examples. One may begin to understand the reason for the problems if the computation is carried out to order $\hbar^2$. Using the techniques previously described, as well as the explicit computation to order $\hbar^2$ in a $\lambda\phi^4$ field theory,[6] it is easy to arrive at the analogous answer in the present context. We find

$$V(\hat{q}) = U(\hat{q}) + \frac{\hbar}{2}\left(\frac{U''(q)}{m} - i\epsilon\right)^{1/2}$$

$$+ \frac{9}{224}\frac{\hbar^2}{m}[U''(\hat{q})]^{-7/9}\frac{d^2}{d\hat{q}^2}[U''(\hat{q})]^{7/9}$$

$$+ O(\hbar^3)\ . \tag{A9}$$

For the specific example of the anharmonic oscillator, this becomes

$$V(\hat{q}) = -\frac{m\omega^2\hat{q}^2}{2} + \frac{\lambda}{4!}\hat{q}^4 + \frac{\hbar}{2}\left(-\omega^2 + \frac{\lambda}{2m}\hat{q}^2 - i\epsilon\right)^{1/2}$$

$$+ \frac{\hbar^2\lambda}{32m^2}\frac{1}{[-\omega^2 + (\lambda/2m)\hat{q}^2]^2}\left(-\omega^2 + \frac{5}{18}\frac{\lambda}{m}\hat{q}^2\right)$$

$$+ O(\hbar^3)\ . \tag{A10}$$

According to Symanzik,[10] $V(\hat{q})$ has a direct physical significance: $V(\hat{q})$ is given by the stationary value of the Hamiltonian in a state $|\psi\rangle$ which is normalized to unity and for which $\langle\psi|q(0)|\psi\rangle$ is fixed at $\hat{q}$:

$$\begin{aligned} &V(\hat{q}) = \langle\psi|H|\psi\rangle\ ,\\ &\delta\langle\psi|H|\psi\rangle = 0\ ,\\ &\langle\psi|\psi\rangle = 1\ ,\\ &\langle\psi|q(0)|\psi\rangle = \hat{q}\ . \end{aligned} \tag{A11}$$

Clearly $V(\hat{q})$ is real, according to (A11). Examining (A10), we see that reality is obtained only for $\hat{q}^2 > 2m\omega^2/\lambda$. Consequently, if the series (A10) is an accurate representation for $V(\hat{q})$, this can only be true for $\hat{q}^2 > 2m\omega^2/\lambda$. As $\hat{q}$ approaches $(2m\omega^2/\lambda)^{1/2}$ from above, the third term in the series becomes infinite, and obviously the series cannot be a correct representation for $V(\hat{q})$. Therefore the loop expansion can only be valid for large $\hat{q}$. [Note that when U is $O(q^n)$ for large q, the ratio of successive terms in (A9) is $O(q^{(-n/2)-1})$. This makes it plausible that the series correctly represents $V(\hat{q})$ for large q.] The small-q behavior is improperly represented by the series. Not only does one obtain a complex value for $V(\hat{q})$, but also the series indicates that the origin is a maximum, whereas general principles force it to be a minimum. [When all bubble graphs[6] contributing to $V(\hat{q})$ are summed, one finds that the series (A10) converges for $(\lambda/2m)\hat{q}^2 - \omega^2 > \frac{3}{4}(\hbar^{2/3}\lambda^{2/3}/m)$.]

The purpose of this discussion is to exhibit the inapplicability of the loop expansion for the effective potential in a familiar context. No definite lesson for quantum field theory in four dimensions can be drawn from this exercise, other than a general *caveat*. The field theory, because of its dependence on continuous spatial parameters, as well as time, is radically different from potential theory. In particular one expects that a symmetry can be violated spontaneously. Whether or not the loop expansion for the effective potential is a reliable tool for the study of this phenomenon in field theory is an open question.

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[9]This point of view is common in early papers on spontaneous mass generation for vector mesons, see, e.g., P. W. Higgs [Phys. Rev. 145, 1156 (1966)] or T. W. B. Kibble [*ibid.* 155, 1554 (1967)]. However, in the contemporary literature, the gauge aspect has been dominant. Our approach, given below, involves a change of variables in a functional integral. This was first discussed by D. J. Gross and R. Jackiw [Phys. Rev. D 6, 477 (1972)].

[10]K. Symanzik, Commun. Math. Phys. 16, 48 (1970); see also S. Coleman, in Lectures given at the 1973 International Summer School of Physics, "Ettore Majorana," Erice, Italy (unpublished).

[11]While this investigation was readied for publication, we learned that S. Weinberg, Ref. 7, has also computed the effective potential from the unitary Lagrangian.

[12]B. W. Lee, NAL Report No. PUB-73/71-THY (unpublished).

[13]It is well known that a redefinition of generalized coordinates, where the transformation to new coordinates involves only the old coordinates and not the old momenta, is a canonical transformation; see, e.g., H. Goldstein, *Classical Mechanics* (Addison-Wesley, Reading, Massachusetts, 1950), p. 244.

[14]Note that a direct evaluation of V_U by summing Feynman graphs for the Green's functions at zero momentum—a method used in Ref. 3—appears to be impossible to carry out for the unitary Lagrangian. The difficulty lies in the fact that V_U must be computed in the symmetric theory, while Feynman rules for the unitary Lagrangian are not available in the symmetric case (i.e., when ρ is not shifted by its vacuum expectation value). On the other hand, the functional method, described in Ref. 6, may be readily applied to this problem.

[15]L. Dolan, unpublished work.

[16]This point has been stressed by F. Low, and the considerations in this appendix arose from conversations with him.

[17]In a four-dimensional field theory there appear several obstacles which prevent an analogous proof that V'' must be positive for the physical solution. One might argue that the spectral representation for the propagator

$$iD(q^2) = \int_0^\infty \frac{dm^2 \rho(m^2)}{m^2 - q^2 - i\epsilon}$$

implies that

$$iD(0) = \int_0^\infty \frac{dm^2 \rho(m^2)}{m^2} > 0,$$

hence $V'' > 0$. However, the following considerations may invalidate the proof. (1) There is no guarantee that the unsubtracted spectral representation is valid. (2) If there are massless particles present, $\rho(m^2)/m^2$ may not be integrable. (3) In a gauge theory $\rho(m^2)$ need not be positive. The last two difficulties presumably do not arise for the unitary Lagrangian.