Pseudoscalar-versus-pseudovector interactions in photo- and electroproduction of charged mesons

B. B. Deo and A. K. Bisoi
Physics Department, Utkal University, Bhubaneswar-4, Orissa, India
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The possibilities of distinguishing between pseudoscalar (ps) and pseudovector (pv) couplings by an analysis of experimental data on photoproduction and electroproduction processes are discussed. It is argued that the charged-K^+ electroproduction experimental results may be more reliable for the purpose. It is seen that "Born terms" can explain the qualitative features of some existing data on photo- and electroproduction of π and K mesons.

I. INTRODUCTION

Recently, Dombey and Read[1] studying the pion photo- and electroproduction processes have shown that the pseudovector Born approximation is the same as the amplitude obtained using PCAC (partially conserved axial-vector current) and current algebra. They have further shown that the pseudovector (pv) theory has a great advantage over the pseudoscalar (ps) theory in that it incorporates all the low-energy theorems for pion photo- and electroproduction. A long time ago it had been proved[2] that the two couplings give identical results to the first order in the coupling constant provided $g_{pv} = g_{ps}/2m$, where m is the nucleon mass. There have been many[2-8] theoretical attempts in the past to ascertain the degree to which pseudoscalar coupling is equivalent to pseudovector coupling. Recently Glück[9] has suggested and shown that electroproduction of charged pions at high energy may distinguish whether the pions are coupled to the nucleon in a pseudoscalar or a pseudovector interaction scheme whereas a photoproduction experiment may not bring out this distinction. If this distinction is really feasible experimentally, it will be of great theoretical interest. Before discussing the merits of such experimentation we wish to point out that as early as 1949, Case[3] not only had studied the equivalence of ps and pv couplings, but had remarked that even in photoproduction of charged mesons there will be a difference in the matrix elements for the two forms of interaction. This is, of course, due to the inclusion of the Pauli-type coupling, whereby the contribution of the magnetic moment of the nucleons exchanged in the s and u channels also comes into the picture. These have been neglected by Glück for considerations near forward angles. The distinction between couplings can also be studied in a more general SU(3) coupling scheme and in this paper we show that due to less interference, K^+ electroproduction may be a much better process to study to establish an experimental distinction.

This paper includes the contribution of the Pauli coupling to the photo- and electroproduction cross sections explicitly. The outline of the paper is as follows. In Sec. II we compare the π^+ and K^+ photoproduction data with ps and pv theory predictions. In Sec. III a similar comparison is made for π^+ and K^+ electroproduction. Section IV discusses the results of our comparison with experimental data. Some concluding comments are made in Sec. V. Two appendixes at the end give the formulas used for calculating the photo- and electroproduction cross sections.

II. PHOTOPRODUCTION

We have $\gamma(k)+p(p_1)\to K^+(q)+\Lambda(p_2)$, where the four-momenta of the interacting particles are given in the parentheses, and their masses are given by

$$k^2=0\,,\quad p_1^{\,2}=m^2\,,\quad q^2=m_K^{\,2}\,,\quad \text{and } p_2^{\,2}=M^2\,.$$

Neglecting the coupling due to (Λ,Σ) transition moments, the diagrams that contribute to the K^+ photoproduction Born amplitude in the ps theory are those given in Figs. 1(a), 1(b), and 1(c). Thus

$$T_{ps}=ieG_{ps}\bar{u}(p_2)\left[\gamma_5\frac{1}{\not p_1+\not k-m}\left(\not\epsilon+i\sigma_{\mu\nu}k^\nu\epsilon_\mu\frac{\mu_p}{2m}\right)+\frac{\gamma_5(2q-k)\cdot\epsilon}{(q-k)^2-m_K^{\,2}}+i\sigma_{\mu\nu}k^\nu\epsilon_\mu\frac{\mu_\Lambda}{2M}\frac{1}{\not p_2-\not k-M}\gamma_5\right]u(p_1), \tag{1}$$

where $\sigma_{\mu\nu}=\frac{1}{2}i(\gamma_\mu\gamma_\nu-\gamma_\nu\gamma_\mu)$; μ_p and μ_Λ are the anomalous magnetic moments of the proton and the Λ particle, respectively, in units of the nuclear magneton. G_{ps} is the dimensionless pseudoscalar coupling constant of the ΛpK vertex.

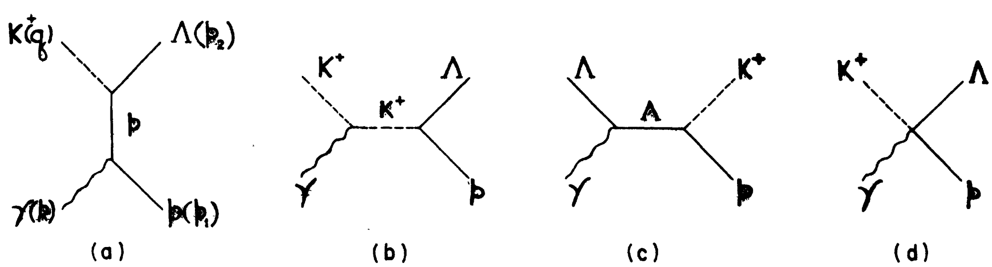

FIG. 1. Feynman diagrams for photoproduction.

In pseudovector theory

$$T_{pv} = ieG_{pv}\,\bar{u}(p_2)\left[\gamma_5 \not{q}\,\frac{1}{\not{p}_1+\not{k}-m}\left(\not{\epsilon}+i\sigma_{\mu\nu}k^\nu\epsilon_\mu\,\frac{\mu_p}{2m}\right)\right.$$
$$\left. +\gamma_5\,\frac{(\not{p}_1-\not{p}_2)(2q-k)\cdot\epsilon}{(q-k)^2-m_K^{\,2}} + i\sigma_{\mu\nu}k^\nu\epsilon_\mu\,\frac{\mu_\Lambda}{2M}\,\frac{1}{\not{p}_2-\not{k}-M}\gamma_5\not{q} - \gamma_5\not{\epsilon}\right]u(p_1), \tag{2}$$

where G_{pv} is the pseudovector coupling constant having the dimensions of length. The last term within the square brackets is from the contact diagram 1(d) characteristic of a derivative coupling. Using

$$G_{ps} = (m+M)G_{pv}\,, \tag{3}$$

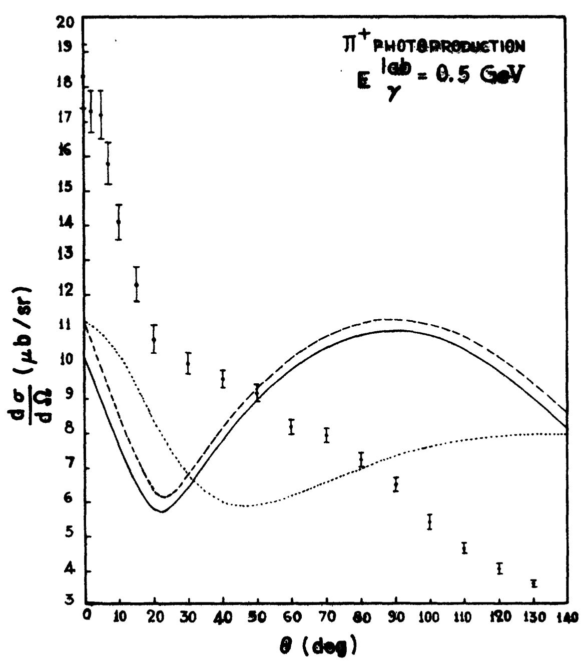

FIG. 2. $\pi^+ n$ photoproduction cross sections in ps (dashed curve) and pv (solid curve) theories. The dotted curve represents the electric Born approximation. The data are from Ref. 10.

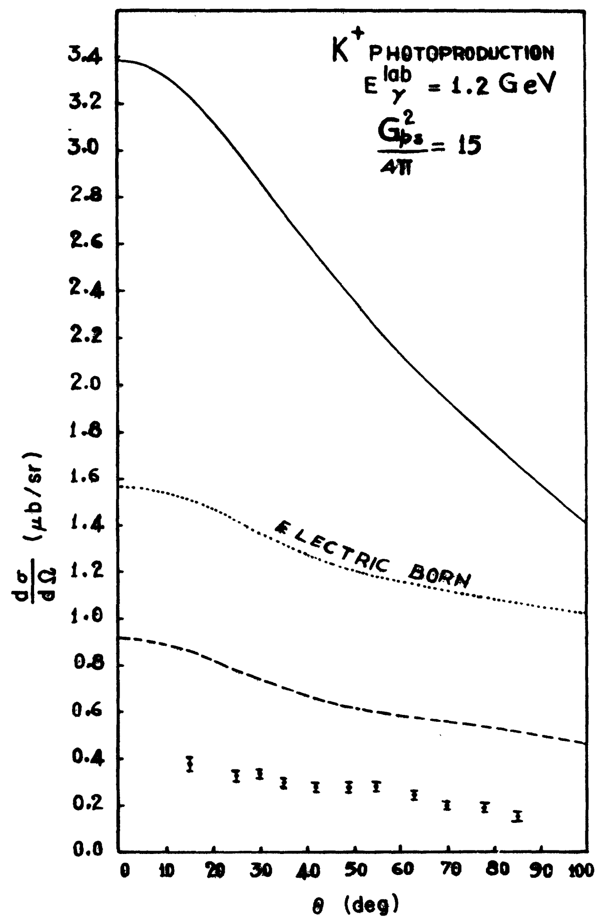

FIG. 3. $K^+\Lambda$ photoproduction cross sections in ps (dashed curve) and pv (solid curve) theories. The dotted curve represents the electric Born approximation. The data are from Ref. 11.

one obtains

$$T_{pv}=T_{ps}-\frac{ie}{2}G_{pv}\left(\frac{\mu_p}{m}+\frac{\mu_\Lambda}{M}\right)\bar{u}(p_2)\gamma_5\not{\epsilon}\not{k}u(p_1) \tag{4}$$

so that only on neglecting μ_p and μ_Λ one has $T_{pv}=T_{ps}$.

The matrix elements for the π^+ photoproduction can be obtained from the preceding equations by a replacement of quantities corresponding to the Λ particle and the kaon by those of the neutron and the pion, respectively, and the coupling constant G of the ΛpK vertex by $\sqrt{2}\,g$, where g is the coupling constant of the pion-nucleon vertex. Thus for π^+ photoproduction, one has

$$T_{pv}=T_{ps}-\frac{ie}{\sqrt{2}}g_{pv}\left(\frac{\mu_p+\mu_n}{m}\right)\bar{u}(p_2)\gamma_5\not{\epsilon}\not{k}u(p_1) \tag{5}$$

using $g_{ps}=2mg_{pv}$. Since

$$\left(\frac{\mu_p+\mu_n}{m}\right)\ll\left(\frac{\mu_p}{m}+\frac{\mu_\Lambda}{M}\right)$$

one may expect the difference between the ps and the pv theories to manifest itself more clearly in kaon photoproduction than in the corresponding case involving the pion. Figures 2 and 3 (see Refs. 10 and 11) show the comparison of the theoretical cross sections with typical data for π^+ and K^+ photoproduction, respectively. The theoretical cross sections have been calculated using the formulas given in Appendix A. From the figures it is evident that the magnetic moment coupling contributes considerably to the K^+ photoproduction cross section and one may justifiably hope that K^+ rather than π^+ photoproduction will serve as a better probe to investigate the difference between the ps and the pv theory predictions.

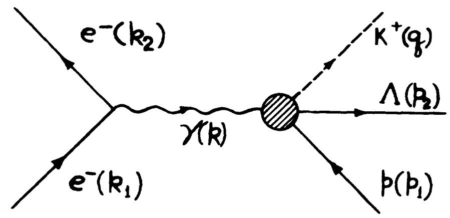

FIG. 4. Kinematics of single-kaon electroproduction.

III. ELECTROPRODUCTION

Figure 4 depicts the process

$$e^-(k_1)+p(p_1)\to e^-(k_2)+K^+(q)+\Lambda(p_2)$$

for which the T matrix may be written in the form

$$T=\epsilon_\mu J^\mu,$$

where

$$\epsilon_\mu=\bar{u}(k_2)\gamma_\mu u(k_1)/k^2.$$

From the properties of the Dirac spinors it follows that $k\cdot\epsilon=0$. Thus the matrix element may be considered as that of a virtual photoproduction process

$$\gamma(k)+p(p_1)\to K^+(q)+\Lambda(p_2)$$

in which $k^2=(k_1-k_2)^2$ is not equal to zero and ϵ_μ is the polarization four-vector of the spacelike photon.

The amplitude in the ps theory is

$$T^E_{ps}=ieG_{ps}\bar{u}(p_2)\Big[\gamma_5\frac{1}{\not{p}_1+\not{k}-m}\left(\not{\epsilon}F_1^p+i\sigma_{\mu\nu}k^\nu\epsilon_\mu\frac{\mu_p}{2m}F_2^p\right)+\frac{\gamma_5(2q-k)\cdot\epsilon}{(q-k)^2-m_K{}^2}F^K$$
$$+\left(\not{\epsilon}F_1^\Lambda+i\sigma_{\mu\nu}k^\nu\epsilon_\mu\frac{\mu_\Lambda}{2M}F_2^\Lambda\right)\frac{1}{\not{p}_2-\not{k}-M}\gamma_5+\gamma_5(F^K+F_1^\Lambda-F_1^p)\frac{k\cdot\epsilon}{k^2}\Big]u(p_1), \tag{6}$$

where the last term is the Fubini-Nambu-Wataghin[12] term added to guarantee gauge invariance, i.e., on replacing ϵ by k, the amplitude should vanish; $F_1(k^2)$ and $F_2(k^2)$ are the Dirac and Pauli form factors, and $F^K(k^2)$ is the kaon electromagnetic form factor. The amplitude in the pv theory is

$$T^E_{pv}=ieG_{pv}\bar{u}(p_2)\Big[\gamma_5\not{q}\frac{1}{\not{p}_1+\not{k}-m}\left(\not{\epsilon}F_1^p+i\sigma_{\mu\nu}k^\nu\epsilon_\mu\frac{\mu_p}{2m}F_2^p\right)+\frac{\gamma_5(\not{p}_1-\not{p}_2)(2q-k)\cdot\epsilon}{(q-k)^2-m_K{}^2}F^K$$
$$+\left(\not{\epsilon}F_1^\Lambda+i\sigma_{\mu\nu}k^\nu\epsilon_\mu\frac{\mu_\Lambda}{2M}F_2^\Lambda\right)\frac{1}{\not{p}_2-\not{k}-M}\gamma_5\not{q}-\gamma_5\not{\epsilon}F_c$$
$$+\gamma_5(m+M)(F^K+F_1^\Lambda-F_1^p)\frac{k\cdot\epsilon}{k^2}+\gamma_5\not{k}(F_1^\Lambda+F_c-F_1^p)\frac{k\cdot\epsilon}{k^2}\Big]u(p_1), \tag{7}$$

where $F_c(k^2)$ is the structure function coming from the contact term; $F_c(0)=1$.

Using relation (3), one can show that

$$T^E_{\text{pv}} = T^E_{\text{ps}} + ieG_{\text{pv}}\bar{u}(p_2)\gamma_5\left[(F^p_1 - F^\Lambda_1 - F_c)\left(\not{\epsilon} - \not{k}\,\frac{k\cdot\epsilon}{k^2}\right) - \frac{\not{\epsilon}\not{k}}{2}\left(\frac{\mu_p F^p_2}{m} + \frac{\mu_\Lambda F^\Lambda_2}{M}\right)\right]u(p_1)\,. \tag{8}$$

Thus on neglecting μ_p and μ_Λ, $T^E_{\text{pv}} = T^E_{\text{ps}}$ provided $F^p_1 = F^\Lambda_1 + F_c$.

Proceeding exactly along the same lines, one obtains in an obvious notation, for π^+ electroproduction,

$$T^E_{\text{pv}} = T^E_{\text{ps}} + ie\sqrt{2}\, g_{\text{pv}}\bar{u}(p_2)\gamma_5\left[(F^p_1 - F^n_1 - F_c)\left(\not{\epsilon} - \not{k}\,\frac{k\cdot\epsilon}{k^2}\right) - \frac{\not{\epsilon}\not{k}}{2m}(\mu_p F^p_2 + \mu_n F^n_2)\right]u(p_1)\,. \tag{9}$$

Appendix B gives the formulas used in the calculation of the electroproduction cross sections.

Figures 5 and 6 (see Refs. 13 and 14) compare the virtual photoproduction differential cross sections calculated in the ps and the pv theories with typical π^+ and K^+ data. Figure 7 and Figure 8 (see Refs. 13 and 14) are the plots of the longitudinal-transverse interference term $d\sigma^I/d\Omega$ in the two theories and their comparison with experimental data. Also shown are the pv curves with $F_c = F^p_1$. This particular choice brings the pv curve for π^+ electroproduction very near to the one obtained using the ps theory, thus making it difficult to choose between them. However, even in this limit the difference between the $d\sigma^I/d\Omega$ in the two theories for K^+ electroproduction is substantial, as is evident from Fig. 8. This again ascertains that K^+ processes are the better places to look for the difference in the two theories.

The K^+ electroproduction data for $d\sigma^I/d\Omega$ have been extracted from the cross-section data of Brown *et al.*[14] using the formula

$$\frac{d\sigma^I}{d\Omega} = \left[\left(\frac{d\sigma}{d\Omega}\right)_{\phi=0^\circ} - \left(\frac{d\sigma}{d\Omega}\right)_{\phi=180^\circ}\right]\frac{1}{[2\epsilon(1+\epsilon)]^{1/2}}$$

and the error in $d\sigma^I/d\Omega$ has been taken to be the rms value of the errors in $(d\sigma/d\Omega)_{\phi=0^\circ}$ and $(d\sigma/d\Omega)_{\phi=180^\circ}$. In the absence of data other than those of Ref. 14 for K^+ electroproduction, we have been obliged to make a crude approximation that the cross-section data averaged over $-36^\circ \leq \phi \leq 36^\circ$ represent $(d\sigma/d\Omega)_{\phi=0^\circ}$ and that those averaged over $-144^\circ \leq \phi \leq +144^\circ$ represent $(d\sigma/d\Omega)_{\phi=180^\circ}$.

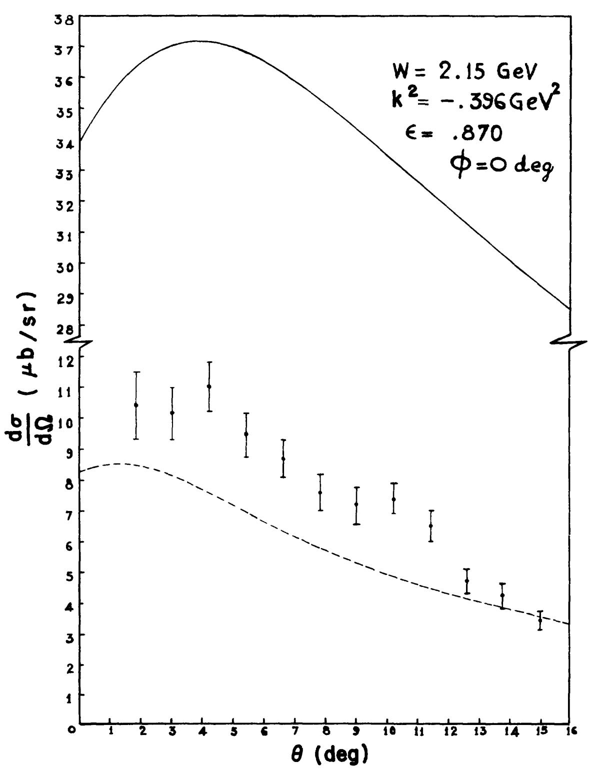

FIG. 5. Comparison of π^+n virtual-photoproduction differential-cross-section data (Ref. 13) with ps (dashed curve) and pv (solid curve) theory predictions.

IV. DISCUSSION

Our comparison of experimental data with the cross sections calculated in the two theories brings forth the following points:

1. Pion photoproduction cross sections calculated in the two theories lie close to each other and the

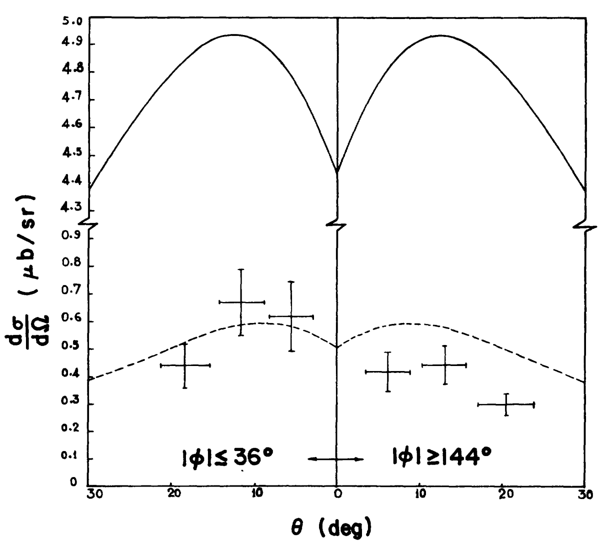

FIG. 6. The angular dependence and ϕ dependence of $K^+\Lambda$ virtual photoproduction cross section in ps (dashed curve) and pv (solid curve) theories. For these data $\langle W\rangle \approx 2.17$ GeV, $\langle k^2\rangle \approx -0.390$ GeV^2, and $\langle\epsilon\rangle \approx 0.86$. The data are from Ref. 14.

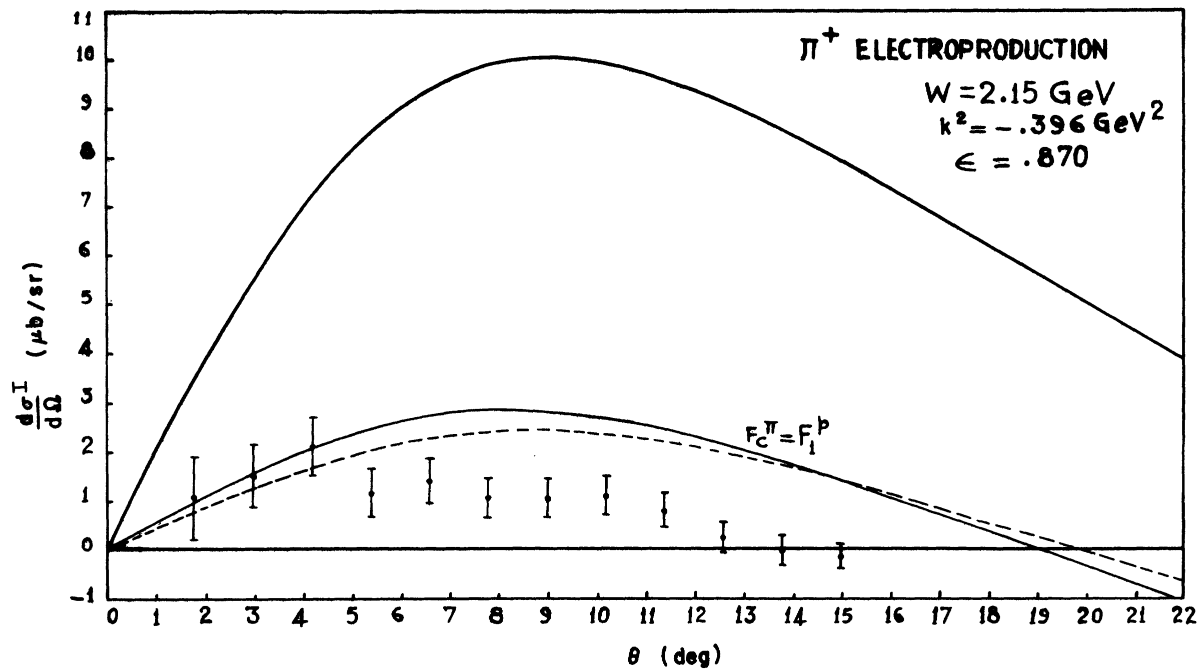

FIG. 7. The longitudinal-transverse interference term in the ps (dashed curve) and pv (solid curve) theories for π^+ electroproduction. The data are from Ref. 13.

experimental angular distribution is unable to distinguish between them. However, the kaon photoproduction data clearly favor the ps theory.

2. For electroproduction, the longitudinal-transverse interference term

$$\frac{d\sigma^I}{d\Omega}=\frac{(-16k^2)^{1/2}|\vec{q}\,|W}{(s-m^2)k_0}\times[f_5(-f_2+f_4+f_3\cos\theta)-f_6(f_1-f_3-f_4\cos\theta)]\sin\theta\,,$$

apart from being equal to zero at $\theta=0°$ and 180°, is also zero at an intermediate angle

$$\theta_0=\cos^{-1}\left[\frac{f_6(f_1-f_3)+f_5(f_2-f_4)}{f_5f_3+f_6f_4}\right].$$

At this angle $d\sigma^I/d\Omega$ changes sign from positive to negative. Such behavior is also shown by the experimental data. One can exploit this fact to distinguish between the ps and pv theories by noting which of them predicts a zero compatible with experimental observations. Figures 7 and 8 clearly exhibit that the experimental data favor the pseudoscalar coupling when $F_c \neq F_1^p$. On putting $F_c = F_1^p$ the π^+ data are no longer able to distinguish between the two theories whereas the difference is still evident in the K^+ case.

The longitudinal-transverse interference term is more reliable in bringing out this distinction

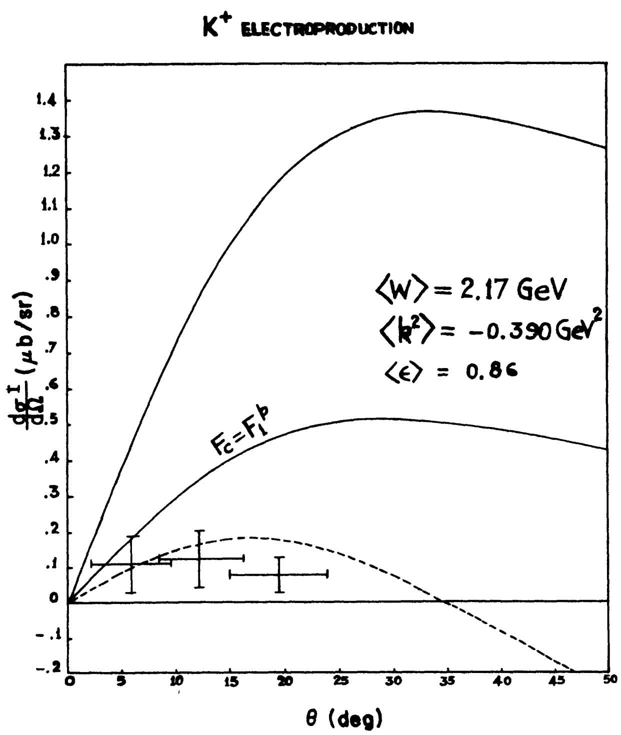

FIG. 8. The longitudinal-transverse interference term in the ps (dashed curve) and pv (solid curve) theories for K^+ electroproduction. The experimental points have been extracted from the cross-section data of Ref. 14.

than the virtual photoproduction differential cross section, since in the former one depends on a qualitative change in the behavior of the cross section, viz., that of its changing sign beyond a certain angle, while in the latter one depends on a relative change in magnitude of the cross sections, on which one cannot bank much due to the uncertainty surrounding the value of the ΛpK coupling constant.

V. CONCLUDING REMARKS

It has been conjectured[9] that an analysis of experiments[15] in electroproduction at high energy will be able to distinguish the additional terms of the pv theory. There are several difficulties; the main obstacle is the observation that theoretically the pionic exchanges give vanishing contributions to the cross sections. Experimentally, however, there is a forward spikelike enhancement and one thinks of a conspiring pion or Regge cuts to explain this phenomenon. If one has to use a theory where Born terms play a minor role, it will be difficult to estimate the correction term. Addition of s-channel resonances will make an analysis of the type suggested by Glück very unreliable.

However, in the case of K-meson photoproduction there is a tendency for a small forward dip. Experimentally the cross-section shape near the forward direction is more like one given by the Born terms and the direct-channel resonances are weaker and less interfering. The main hope would seem to lie in the analysis of the longitudinal-transverse interference term as indicated by our study.

We propose, therefore, that more accurate experiments on K^+ electroproduction be performed to facilitate an analysis of the type suggested in this paper in discriminating between the ps and the pv couplings in strong-interaction theory.

APPENDIX A

Invariant amplitudes

The T matrix for photoproduction may be written as

$$T=\sum_{j=1}^{4}A_j\bar{u}(p_2)M_j u(p_1), \tag{10}$$

where A_j are the four invariant amplitudes and the M_j are Lorentz- and gauge-invariant quantities. Following Thom,[16] we write

$$\begin{aligned}
M_1&=-\gamma_5\not{\epsilon}\not{k},\\
M_2&=2\gamma_5(p_1\cdot\epsilon\, p_2\cdot k-p_2\cdot\epsilon\, p_1\cdot k),\\
M_3&=\gamma_5(\not{\epsilon}\, p_1\cdot k-\not{k}\, p_1\cdot\epsilon),\\
M_4&=\gamma_5(\not{\epsilon}\, p_2\cdot k-\not{k}\, p_2\cdot\epsilon).
\end{aligned} \tag{11}$$

With the help of Eqs. (10) and (11), one extracts from Eq. (1)

$$\begin{aligned}
A_1^{\rm ps}&=eG_{\rm ps}\left(\frac{1+\mu_p}{s-m^2}+\frac{\mu_\Lambda}{u-M^2}\right),\\
A_2^{\rm ps}&=eG_{\rm ps}\frac{2}{(t-m_K{}^2)(s-m^2)},\\
A_3^{\rm ps}&=eG_{\rm ps}\frac{\mu_p}{m}\frac{1}{(s-m^2)},\\
A_4^{\rm ps}&=eG_{\rm ps}\frac{\mu_\Lambda}{M}\frac{1}{(u-M^2)}.
\end{aligned} \tag{12}$$

From Eq. (4), it follows that for the pseudovector case only the invariant amplitude A_1 gets modified, other amplitudes remaining unaltered. Thus

$$A_1^{\rm pv}=A_1^{\rm ps}+\frac{1}{2(m+M)}\left(\frac{\mu_p}{m}+\frac{\mu_\Lambda}{M}\right). \tag{13}$$

Cross sections

The differential cross sections are easily calculated using the following formulas given by Thom.

For unpolarized initial states

$$\frac{d\sigma}{d\Omega}=\frac{|\vec{q}|}{|\vec{k}|}[F_1{}^2+F_2{}^2-2\cos\theta F_1F_2+\sin^2\theta(\tfrac{1}{2}F_3{}^2+\tfrac{1}{2}F_4{}^2+F_1F_4+F_2F_3+\cos\theta F_3F_4)], \tag{14}$$

where θ is the c.m. scattering angle between the meson and the proton and

$$\begin{aligned}
F_1&=\frac{|\vec{k}|}{4\pi}\left(\frac{E_\Lambda+M}{2W}\right)^{1/2}\left[A_1-\frac{(W+m)}{2}A_3+\frac{(u-M^2)}{2(W-m)}A_4\right],\\
F_2&=\frac{|\vec{k}|}{4\pi}\left(\frac{E_\Lambda-M}{2W}\right)^{1/2}\left[-A_1-\frac{(W-m)}{2}A_3+\frac{(u-M^2)}{2(W+m)}A_4\right],\\
F_3&=\frac{|\vec{k}||\vec{q}|}{4\pi}\left(\frac{E_\Lambda+M}{2W}\right)^{1/2}[-(W-m)A_2+A_4],\\
F_4&=\frac{|\vec{k}||\vec{q}|}{4\pi}\left(\frac{E_\Lambda-M}{2W}\right)^{1/2}[(W+m)A_2+A_4].
\end{aligned} \tag{15}$$

E_Λ is the c.m. energy of the Λ particle and W is the total c.m. energy. The π^+ photoproduction cross section is calculated by replacing the quantities corresponding to the Λ particle and the K^+ by those of the neutron and the π^+, respectively. In our calculations we have used $G_{\rm ps}{}^2/4\pi=15$ and $\mu_\Lambda=-0.7$.

APPENDIX B

Invariant amplitudes

Using the six covariants suggested by Levy *et al.*[17] for kaon electroproduction,

$$\begin{aligned}
M_1 &= \tfrac{1}{2}\gamma_5(\not\epsilon \not k - \not k \not\epsilon)\,,\\
M_2 &= \gamma_5[(2q-k)\cdot \epsilon P\cdot k - (2q-k)\cdot k P\cdot\epsilon]\,,\\
M_3 &= \gamma_5(\not\epsilon q\cdot k - \not k q\cdot\epsilon)\,,\\
M_4 &= i\epsilon_{\alpha\beta\mu\nu}\gamma^\alpha q^\beta \epsilon^\mu k^\nu\,,\\
M_5 &= \gamma_5(q\cdot\epsilon k^2 - q\cdot k k\cdot\epsilon)\,,\\
M_6 &= \gamma_5(k\cdot\epsilon\not k - k^2\not\epsilon)\,,
\end{aligned}\tag{16}$$

where $P=\frac{1}{2}(p_1+p_2)$ and $\epsilon_{\alpha\beta\mu\nu}$ is the four-dimensional Levi-Civita tensor with $\epsilon_{0123}=+1$, one obtains from Eq. (6)

$$\begin{aligned}
A_1^{ps} &= -eG_{ps}\left\{\frac{1}{(s-m^2)}\left[F_1^p + \frac{\mu_p F_2^p}{2}\left(1-\frac{M}{m}\right)\right] + \frac{1}{(u-M^2)}\left[F_1^\Lambda + \frac{\mu_\Lambda F_2^\Lambda}{2}\left(1-\frac{m}{M}\right)\right]\right\},\\
A_2^{ps} &= eG_{ps}\left(\frac{F_1^p}{s-m^2} + \frac{F_1^\Lambda}{u-M^2}\right)\frac{1}{t-m_K^2}\,,\\
A_3^{ps} &= eG_{ps}\frac{\mu_p F_2^p}{m(s-m^2)}\,,\\
A_4^{ps} &= -eG_{ps}\left[\frac{\mu_p F_2^p}{2m(s-m^2)} + \frac{\mu_\Lambda F_2^\Lambda}{2M(u-M^2)}\right],\\
A_5^{ps} &= \frac{eG_{ps}}{k^2(t-m_K^2)}\left\{2F^K - [(s-m^2) - \tfrac{1}{2}(t-m_K^2) - \tfrac{1}{2}k^2] \times\left(\frac{F_1^p}{s-m^2} + \frac{F_1^\Lambda}{u-M^2}\right)\right\},\\
A_6^{ps} &= 0\,.
\end{aligned}\tag{17}$$

From Eqs. (8) and (16) it is easy to see that only two of the six invariant amplitudes get modified for the pseudovector case, the others remaining the same as in the pseudoscalar theory. Thus,

$$A_1^{pv} = A_1^{ps} - \frac{eG_{ps}}{2(m+M)}\left(\frac{\mu_p F_2^p}{m} + \frac{\mu_\Lambda F_2^\Lambda}{M}\right), \tag{18}$$

$$A_6^{pv} = -\frac{eG_{ps}}{k^2(m+M)}(F_1^p - F_1^\Lambda - F_c)\,. \tag{19}$$

As in the photoproduction case, the amplitudes for π^+ electroproduction can be derived from those of the K^+ by appropriate substitutions.

Form factors

In terms of the Sachs form factors, the Dirac and Pauli form factors are given by

$$F_1(k^2) = \left[G_E(k^2) - \frac{k^2}{4m^2}G_M(k^2)\right]\Big/\left(1-\frac{k^2}{4m^2}\right), \tag{20}$$

$$F_2(k^2) = [G_M(k^2) - G_E(k^2)]\Big/\mu\left(1-\frac{k^2}{4m^2}\right), \tag{21}$$

where μ is the anomalous magnetic moment and m is the mass of the particle under consideration. We assume that the Sachs form factors satisfy the scaling law and the dipole fit, viz.,

$$\begin{aligned}
G_E^p(k^2) &= \frac{G_M^p(k^2)}{1+\mu_p} = \frac{G_M^n(k^2)}{\mu_n} = \frac{4m^2}{k^2}\frac{G_E^n(k^2)}{\mu_n}\\
&= \frac{G_M^\Lambda(k^2)}{\mu_\Lambda} = \frac{4M^2}{k^2}\frac{G_E^\Lambda(k^2)}{\mu_\Lambda}\\
&= \left(1-\frac{k^2}{0.71\ \text{GeV}^2}\right)^{-2}.
\end{aligned}\tag{22}$$

This scaling law implies that

$$F_1^n(k^2) = F_1^\Lambda(k^2) = 0\,.$$

For the meson form factors, we use a vector-meson-exchange pole fit

$$F^\pi(k^2) = \left(1-\frac{k^2}{m_\rho{}^2}\right)^{-1},\quad m_\rho = 0.765\ \text{GeV} \tag{23}$$

and

$$F^K(k^2) = \left[1-\frac{k^2}{m_{K^*}{}^2}\right]^{-1},\quad m_{K^*} = 0.892\ \text{GeV}\,. \tag{24}$$

The form factor of the contact term $F_c(k^2)$ can be identified[18] with the axial-vector form factor $G_A(k^2)$ so that pv theory may agree with the current commutator point of view. Assuming the $G_A(k^2)$ to be dominated by axial-vector meson poles, we write

$$F_c^\pi(k^2) = \left(1-\frac{k^2}{m_{A_1}{}^2}\right)^{-1},\quad m_{A_1} = 1.07\ \text{GeV} \tag{25}$$

and

$$F_c^K(k^2) = \left(1-\frac{k^2}{m_{K_A}{}^2}\right)^{-1},\quad m_{K_A} = 1.24\ \text{GeV}\,. \tag{26}$$

Cross sections

The formulas are those of Levy *et al.*[17] The virtual photoproduction cross section $d\sigma/d\Omega$ can be written as a sum of four terms

$$\frac{d\sigma}{d\Omega}=\frac{d\sigma_U}{d\Omega}+\epsilon\frac{d\sigma_T}{d\Omega}\cos 2\phi+\epsilon\frac{d\sigma_L}{d\Omega}+\left[\tfrac{1}{2}\epsilon(1+\epsilon)\right]^{1/2}\frac{d\sigma_I}{d\Omega}\cos\phi\,, \tag{27}$$

where ϕ is the azimuthal angle between the electron and the hadron scattering planes, and the quantity ϵ expresses the transverse linear polarization of the virtual photon, and also its degree of longitudinal polarization.

The individual terms are given as follows:

$$\begin{aligned}
\frac{d\sigma_U}{d\Omega}&=\frac{|\vec{q}|W}{s-m^2}(H_1^2+H_2^2+H_3^2+H_4^2)\,,\\
\frac{d\sigma_T}{d\Omega}&=\frac{2|\vec{q}|W}{s-m^2}[H_3H_2-H_4H_1]\,,\\
\frac{d\sigma_L}{d\Omega}&=-\frac{2|\vec{q}|Wk^2}{(s-m^2)k_0^2}[H_5^2+H_6^2]\,,\\
\frac{d\sigma_I}{d\Omega}&=\frac{|\vec{q}|W}{(s-m^2)}\frac{(-8k^2)^{1/2}}{k_0}[(H_4-H_1)H_5+(H_3+H_2)H_6]\,,
\end{aligned} \tag{28}$$

where

$$\begin{aligned}
H_1&=-\frac{1}{\sqrt{2}}\sin\theta\cos\tfrac{1}{2}\theta(f_3+f_4)\,,\\
H_2&=-\sqrt{2}\cos\tfrac{1}{2}\theta(f_1-f_2)+H_3\,,\\
H_3&=\frac{1}{\sqrt{2}}\sin\theta\sin\tfrac{1}{2}\theta(f_3-f_4)\,,\\
H_4&=\sqrt{2}\sin\tfrac{1}{2}\theta(f_1+f_2)-H_1\,,\\
H_5&=-\cos\tfrac{1}{2}\theta(f_5+f_6)\,,\\
H_6&=-\sin\tfrac{1}{2}\theta(f_5-f_6)\,,
\end{aligned} \tag{29}$$

with

$$\begin{aligned}
f_{1,2}&=\frac{1}{8\pi W}[(E_p\pm m)(E_\Lambda\pm M)]^{1/2}[\pm(W\mp m)A_1+q\cdot k(A_3-A_4)+(W\mp m)(W\mp M)A_4-k^2A_6]\,,\\
f_{3,4}&=\frac{|\vec{q}||\vec{k}|}{8\pi W}\left(\frac{E_\Lambda\pm M}{E_p\pm m}\right)^{1/2}[\pm(s-m^2)A_2\mp\tfrac{1}{2}k^2(A_2-2A_5)+(W\pm m)(A_3-A_4)]\,,\\
f_{5,6}&=\frac{k_0}{8\pi W}[(E_p\pm m)(E_\Lambda\pm M)]^{1/2}\\
&\quad\times\Big[\pm A_1+(W\mp M)A_4-(W\mp m)A_6-\frac{1}{(E_p\pm m)}\{\pm W(|\vec{k}|^2-2q\cdot k)A_2\pm(q\cdot kk_0-q_0k^2)(A_5-\tfrac{3}{2}A_2)\\
&\qquad-[q_0(W\pm m)-q\cdot k](A_3-A_4)\}\Big]\,.
\end{aligned} \tag{30}$$

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