does not have Regge behavior. In addition, the model lacks an odd-G-parity ground state, and hence resembles a world in which G parities have been reversed. As the model stands, therefore, it is not suitable for building a realistic model of hadrons. What is interesting, however, is the mechanism by which the zero-mass vector particle apparently either does not exist or decouples from the model. Perhaps a more realistic model will incorporate the features which allow for the elimination of the zero-mass vector particle.

ACKNOWLEDGMENTS

The author is happy to acknowledge many fruitful and informative discussions with Dr. A. Neveu concerning zero-intercept models and his work with Dr. Gervais.[8]

---

*Work supported in part by N.S.F. Grant No. GP-38097X and also in part by N.S.F. Grant No. AT(11-1)-3072 (Princeton University).

[1]See S. Mandelstam, in *Lectures on Elementary Particles and Quantum Field Theory*, 1970 Brandeis Summer Institute in Theoretical Physics, edited by S. Deser, M. Grisaru, and H. Pendleton (MIT Press, Cambridge, Mass., 1970), Vol. I.

[2]J. Goldstone, P. Goddard, C. Rebbi, and C. Thorn, Nucl. Phys. B56, 109 (1973).

[3]A. Neveu and J. H. Schwarz, Phys. Rev. D 4, 1109 (1971).

[4]R. C. Brower and K. A. Friedman, Phys. Rev. D 7, 535 (1973); J. H. Schwarz, Nucl. Phys. B46, 61 (1972).

[5]J. L. Gervais and A. Neveu, Nucl. Phys. B63, 127 (1973).

[6]M. A. Virasoro, Phys. Rev. D 1, 2933 (1970).

[7]J. H. Schwarz, Nucl. Phys. B48, 525 (1972).

[8]After this work was done, we understood that L. Clavelli, S. Fenster, and J. L. Uretsky also derived a form manifestly lacking Regge behavior for the original model of Gervais and Neveu.

PHYSICAL REVIEW D VOLUME 9, NUMBER 10 15 MAY 1974

Nature of broken SU(4) × SU(4) symmetry

T. K. Kuo*

Department of Physics, Purdue University, West Lafayette, Indiana 47907

(Received 17 January 1974)

If there is an additional conserved quantum number, the charm, then the relative parity between the charmed and uncharmed states is not measurable. This leads to a symmetry operator $\tilde{W}$, which turns out to be a finite rotation in SU(4) ×SU(4). The existence of $\tilde{W}$ restrains the pattern of symmetry breaking. Among a number of possibilities, the most plausible is one in which SU(4) ×SU(4) is first broken spontaneously down to SU(3) × SU(3) × U(1). The latter is then broken both explicitly and spontaneously to U(2) × U(1).

I. INTRODUCTION

For a long time neutral currents have presented an outstanding question mark in weak-interaction theories. The situation took a dramatic turn recently on two fronts. Experimentally,[1] even though the $\Delta S=1$ neutral currents are known to be very severely suppressed, first indications of the $\Delta S=0$ neutral currents seem to have been found. Theoretically,[2] developments of a renormalizable gauge theory of the weak interactions also call for the existence of neutral currents. The absence of the $\Delta S=1$ part, however, sets very stringent boundary conditions on theoretical models. To date the simplest model which can account for these features appears to be the SU(4) model of Glashow, Iliopoulos, and Maiani.[3]

In the SU(4) model, the charged weak currents take the form

$$J_\mu = \bar{q}\gamma_\mu \tfrac{1}{2}(1+\gamma_5)\mathcal{M}_+ q, \tag{1}$$

where the quartet quarks are

$$q = \begin{pmatrix} \mathcal{P} \\ \mathcal{N} \\ \lambda \\ \mathcal{P}' \end{pmatrix} \tag{2}$$

and the matrix $\mathcal{M}_+$ is

$$\mathfrak{M}_+ = \begin{pmatrix} 0 & \cos\theta & \sin\theta & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & -\sin\theta & \cos\theta & 0 \end{pmatrix}. \tag{3}$$

The neutral current is diagonal, given by

$$J_\mu^0 = \bar{q}\gamma_\mu \tfrac{1}{2}(1+\gamma_5)\mathfrak{M}_0 q, \tag{4}$$

$$\mathfrak{M}_0 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \tag{5}$$

If we take the quartet model seriously, then it is natural to consider SU(4)×SU(4) as the symmetry group of the hadrons.

Let us first establish our notations.[4] The generators of SU(4)×SU(4) will be denoted as

$$F_i \text{ and } F_i^5, \quad i = 1, \ldots, 15. \tag{6}$$

The combinations

$$(F_i)_\pm = \tfrac{1}{2}(F_i \pm F_i^5) \tag{7}$$

satisfy the SU(4) algebra separately, with

$$[(F_i)_+, (F_j)_-] = 0. \tag{8}$$

In the 4×4 representation, the diagonal generators are given by

$$I_3 = F_3 = \begin{pmatrix} \frac{1}{2} & 0 & 0 & 0 \\ 0 & -\frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \tag{9}$$

$$Y = \frac{2}{\sqrt{3}} F_8 = \begin{pmatrix} \frac{1}{3} & 0 & 0 & 0 \\ 0 & \frac{1}{3} & 0 & 0 \\ 0 & 0 & -\frac{2}{3} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \tag{10}$$

and

$$Z = \left(\frac{3}{2}\right)^{1/2} F_{15} = \begin{pmatrix} \frac{1}{4} & 0 & 0 & 0 \\ 0 & \frac{1}{4} & 0 & 0 \\ 0 & 0 & \frac{1}{4} & 0 \\ 0 & 0 & 0 & -\frac{3}{4} \end{pmatrix}. \tag{11}$$

Z is related to the charm quantum number $\mathfrak{C}$ and the baryon number B by

$$\mathfrak{C} = \tfrac{2}{3}Z + \tfrac{1}{2}B. \tag{12}$$

We will often denote by $U_Z(1)$ the one-parameter group generated by Z. U(2) shall signify the group generated by the isospin and hypercharge operators, with the global relation $(-1)^{2I} = (-1)^Y$.

The SU(4)×SU(4) symmetry shall be broken down to isospin, hypercharge, and charm conservation, given by the group $U(2)\times U_Z(1)$. The important question is: How is the symmetry broken? If one follows the corresponding considerations in SU(3)×SU(3), one would assume that there is an explicit symmetry-breaking term which transforms, under SU(4)×SU(4), predominantly as $(4, \bar{4}) + (\bar{4}, 4)$. One writes

$$H = H_0 + \epsilon H', \tag{13}$$

$$H' = u_0 + c u_8 + d u_{15}, \tag{14}$$

where ϵ, c, and d are real constants, and u_i $(i = 0, \ldots, 15)$, together with v_i $(i = 0, \ldots, 15)$, form the $(4, \bar{4})$ and $(\bar{4}, 4)$ representation of SU(4)×SU(4). One might also assume, just as in the SU(3)×SU(3) case, that as $\epsilon \to 0$ (i.e., no explicit symmetry breaking), the vacuum is invariant, not under SU(4)×SU(4), but under SU(4). This means that physical states would form SU(4) multiplets, even if SU(4)×SU(4) were exact. A major difficulty with Eq. (13), interpreted in this way, is that it predicts an abundance of charmed hadronic states around 1 GeV. A way out was the suggestion[5] that SU(4) is actually spontaneously broken down to SU(3), so that, as $\epsilon \to 0$ in Eq. (13), the vacuum state is invariant, not under SU(4), but only under its subgroup SU(3). There will now be 21 Goldstone bosons, 15 of which are responsible for the breakdown SU(4)×SU(4)→SU(4), and the other six (charm carrying) are responsible for SU(4)→SU(3).

A model along these lines was suggested earlier.[6] In fact, choosing c and d in Eq. (14) so that we may write H' as a quark mass term of the form

$$H' = m_{\mathcal{P}'}\bar{\mathcal{P}}'\mathcal{P}' + m_\lambda \bar{\lambda}\lambda, \tag{15}$$

with

$$m_{\mathcal{P}'}/m_\lambda \approx 200, \tag{16}$$

then shows this H' giving rise to charmed Goldstone boson masses of the order of 5 GeV. These bosons will most probably interact with the existing uncharmed states to form massive charmed states. The resulting hadronic spectra can then be visualized as follows. In the range 0.5–2.5 GeV, we have the known hundreds of uncharmed states. After a gap, there will again be hundreds of charmed states, with masses in the range 5–7 GeV. As was already suggested, pair production of these massive states could provide a natural explanation of the rise of $\sigma_T(pp)$ at the CERN Intersecting Storage Rings (ISR) energies.[7] It should be emphasized that if the rise in $\sigma_T(pp)$ is due indeed to the copious pair production of the charmed particles, then we expect σ_T to have a step-function behavior. This is to be contrasted with theories in which σ_T rises without bound, such as a $\sigma_T \sim \ln s$

behavior. We might also add that the existence of massive charmed states gives a new mass scale ~10 GeV. This would lead to the breakdown of scaling behavior at such energies.[8]

It turns out that assuming an explicit symmetry-breaking term H' to be of the form in Eq. (14) is by no means an arbitrary matter.[9] As was pointed out[10] earlier in the case of SU(3)×SU(3), there is the general consideration of whether H' is well defined. In fact, since *a priori* we cannot fix the coordinates in SU(4)×SU(4), H' can suffer an arbitrary rotation R in the SU(4)×SU(4) space. Provided that there exists an R such that $RH'R^{-1}$ is of the same form as H', but not equal to H', then H' is not well defined. Mathematically, one is certainly free to choose H' or $RH'R^{-1}$ and build a model accordingly. The important thing is that the physical results from H' or $RH'R^{-1}$ are different. To say that one can choose H' over $RH'R^{-1}$ (or vice versa) amounts to assuming our *a priori* ability of fixing the coordinates in the SU(4)× SU(4) space.

In Sec. II we establish the existence of a finite rotation $R=\tilde{W}=\exp(i4\pi Z_-)$ which meets the above requirements. The physical meaning of this result is discussed in Sec. III. It is found that $\tilde{W}$ is the symmetry operator which corresponds to the physical equivalence of the two parity operators P and $(-1)^{4Z}P$. In theories for which there is an explicit symmetry-breaking term $H'\neq \tilde{W}H'\tilde{W}^{-1}$, P and $(-1)^{4Z}P$ would no longer be physically equivalent. Finally, in Sec. IV we discuss the implications of our results. We find that the pattern of SU(4) ×SU(4)-symmetry breaking is a very complicated one. Excluding the possibilities of "unlikely" theories, the conclusion is reached that SU(4)×SU(4) is broken in two stages. First, SU(4)×SU(4) is spontaneously broken to $SU(3)\times SU(3)\times U_Z(1)$. Next, in a different fashion, $SU(3)\times SU(3)\times U_Z(1)$ is broken down to $U(2)\times U_Z(1)$. This time the symmetry breaking is in part explicit and in part spontaneous.

Before we go on, we wish to add a few remarks concerning the nature of spontaneously broken symmetries, in contrast with explicitly broken symmetries. As has been emphasized,[11] spontaneously broken symmetries are not broken, but are bona fide symmetries. (For this reason, "hidden symmetries" is probably better terminology.) It merely corresponds to symmetry realizations for which the physical states do not exhibit multiplet structures, so that they also do not form irreducible representations of the symmetry group. We should emphasize that the association of multiplets to symmetry groups is based on the application of the superposition principle, which enables one to construct (by superposition) physical states which are also irreducible representations of the symmetry groups. In general, in situations for which the superposition principle is inapplicable, symmetries will be hidden. Examples abound in classical physics. For instance, rotational symmetry does not give rise to spherical bodies, nor does parity produce mirror-symmetric objects.

II. EXISTENCE OF $\tilde{W}$ AND ITS PROPERTIES

As far as the transformation properties are concerned, the operators u_i and v_i may be written as $\bar{q}\lambda_i q$ and $i\bar{q}\lambda_i\gamma_5 q$, where λ_i are the generalization to SU(4) of the λ matrices in SU(3). An expression of the form $\sum_i(a_i u_i + b_i v_i)$ may be written as $\bar{q}_R \mathfrak{M} q_L + \bar{q}_L \mathfrak{M}^\dagger q_R$, where

$$q_L \equiv \tfrac{1}{2}(1+\gamma_5)q\,,\quad q_R\equiv \tfrac{1}{2}(1-\gamma_5)q\,,$$

and $\mathfrak{M}=\sum_i(a_i+ib_i)\lambda_i$. A rotation in SU(4)×SU(4) may be represented by a pair of 4×4 matrices U_R and U_L. Under the rotation, the transformation of $\sum_i(a_i u_i + b_i v_i)$ is given by

$$\mathfrak{M}\to\mathfrak{M}'=U_R^{-1}\mathfrak{M}U_L\,. \tag{17}$$

Now, the operator

$$Z_-\equiv\tfrac{1}{2}(Z-Z^5) \tag{18}$$

has the 4×4 representation in $SU(4)_-$:

$$Z_-=\begin{pmatrix}\frac14 & 0 & 0 & 0\\ 0 & \frac14 & 0 & 0\\ 0 & 0 & \frac14 & 0\\ 0 & 0 & 0 & -\frac34\end{pmatrix}. \tag{19}$$

With respect to $SU(4)_+$, it is the identity. Thus, the finite rotation

$$\tilde{W}=\exp(i4\pi Z_-) \tag{20}$$

has the 4×4 representation

$$\tilde{W}_R=\begin{pmatrix}-1 & 0 & 0 & 0\\ 0 & -1 & 0 & 0\\ 0 & 0 & -1 & 0\\ 0 & 0 & 0 & -1\end{pmatrix}, \tag{21}$$

$$\tilde{W}_L=I\,. \tag{22}$$

Also, since $\exp(i8\pi Z_-)=1$,

$$\tilde{W}^2=I\,. \tag{23}$$

It follows immediately that

$$\tilde{W}u_i\tilde{W}^{-1}=\tilde{W}u_i\tilde{W}=-u_i \tag{24}$$

and

$$\tilde{W}v_i\tilde{W}=-v_i\,. \tag{25}$$

Returning to Eq. (13), we find that if

$$H'=u_0+cu_8+du_{15}\,, \tag{26}$$

then

$$\tilde{W}H'\tilde{W} = -H'. \quad (27)$$

It is readily seen that $\tilde{W}$ is in the center of $SU(4)_-$, and hence in $SU(4)\times SU(4)$. $\tilde{W}$ thus commutes with the whole $SU(4)\times SU(4)$ group:

$$[\tilde{W}, SU(4)\times SU(4)] = 0. \quad (28)$$

In particular, $\tilde{W}$ commutes with the electromagnetic as well as the weak currents:

$$[\tilde{W}, J_\mu^{em}] = [\tilde{W}, J_\mu^{wk}] = 0. \quad (29)$$

However, $\tilde{W}$ does not commute with the parity operator. In fact, noting that

$$P\tilde{W}P = P\exp[i2\pi(Z - Z^5)]P = \exp[i2\pi(Z + Z^5)], \quad (30)$$

we see that

$$\tilde{W}P\tilde{W}^{-1} = \tilde{W}P\tilde{W} = \tilde{W}P\tilde{W}PP = (-1)^{4Z}P. \quad (31)$$

The existence of $\tilde{W}$ and its uniqueness may now be readily understood by referring to a corresponding analysis[10,11] of $SU(3)\times SU(3)$. When we have the symmetry group $U(2)\times Z_2$, or isospin, hypercharge, and parity invariance, there are two physically equivalent parity operators—P and $(-1)^{2I}P = (-1)^Y P$. The symmetry operator connecting them happens to be a rotation in the $SU(3)\times SU(3)$ space:

$$W = \exp(i2\pi I_-), \quad (32)$$

$$WPW^{-1} = (-1)^{2I}P. \quad (33)$$

It is unique up to arbitrary (and uninteresting) rotations around the I_3 and Y axes. Going over to $SU(4)\times SU(4)$, one is dealing with the residual symmetry $U(2)\times U(1)$, or $\vec{I}$, Y, and Z conservation. The additional Z conservation induces yet a new ambiguity in the parity operator, namely, P and $(-1)^{4Z}P$. [Note that Z is normalized so that $\exp(i8\pi Z) = 1$.] The symmetry operator effecting this equivalence is now $\tilde{W} = \exp(i4\pi Z_-)$, a finite rotation in $SU(4)\times SU(4)$. It is unique up to rotations around the I_3, Y, and Z axes. Mathematically, $\tilde{W}$ corresponds to the unique outer automorphism of the group $U_Z(1)\times Z_2$, for Z and parity conservation.

III. PHYSICAL INTERPRETATIONS

In the previous section we have found a finite $SU(4)\times SU(4)$ rotation $\tilde{W} = \exp(i4\pi Z_-)$ with the following properties:

$$\tilde{W}^2 = 1, \quad (34)$$

$$\tilde{W}P\tilde{W} = (-1)^{4Z}P, \quad (35)$$

$$\tilde{W}u_i\tilde{W} = -u_i, \quad (36)$$

$$[\tilde{W}, F_i] = [\tilde{W}, F_i^5] = 0. \quad (37)$$

Since, under the assumption of Z and P conservation, there are two physically equivalent parity operators P and $(-1)^{4Z}P$, $\tilde{W}$ must be a symmetry operator:

$$[\tilde{W}, H] = 0. \quad (38)$$

On the other hand, according to Eq. (36), if the symmetry-breaking term $H' \sim (4, \bar{4}) + (\bar{4}, 4)$, then

$$[\tilde{W}, H'] = -H', \quad (39)$$

$$H' = u_0 + cu_8 + du_{15}. \quad (40)$$

The existence of $\tilde{W}$ as a finite $SU(4)\times SU(4)$ rotation thus rules out an explicit symmetry-breaking term which transforms like $(4, \bar{4}) + (\bar{4}, 4)$.

If, however, there are explicit symmetry-breaking terms transforming differently, such as

$$H' \sim (6, \bar{6}) + \bar{6}, 6) \text{ or } (10, \overline{10}) + (\overline{10}, 10) \text{ or } (15, 15), \quad (41)$$

then we can readily establish that $[\tilde{W}, H'] = 0$. Therefore, $\tilde{W}$ does not have any restrictive power for these possibilities.

Because a $(4, \bar{4}) + \bar{4}, 4)$ term can be written as a quark mass term, Eq. (38) may be said to imply that quarks must be massless.

Equation (38) might appear surprising, all the more so because it proves to be highly restrictive. As was emphasized earlier, while it is very easy to construct mathematical models violating Eq. (38), such models have definite physical implications. Namely, any such model, in which there is an explicit symmetry-breaking term H' with $\tilde{W}H'\tilde{W} \neq H'$, entails the physical inequivalence of the parity operators P and $(-1)^{4Z}P$.

On the other hand, even though $[H, \tilde{W}] = 0$, it certainly does not follow that all physical states must necessarily exhibit the $\tilde{W}$ symmetry. Indeed, if $|q\rangle$ denotes the quartet quark states, and if we assign "positive" parity to them,

$$P|q\rangle = +|q\rangle, \quad (42)$$

then

$$P(\tilde{W}|q\rangle) = \tilde{W}^2 P(\tilde{W}|q\rangle) = \tilde{W}[(-1)^{4Z}P]|q\rangle = -(\tilde{W}|q\rangle). \quad (43)$$

This means that quarks must have parity doublets ($|q\rangle$ and $W|q\rangle$) if it were to exhibit the $\tilde{W}$ symmetry. However, to the extent that only SU(3) multiplets are presumed to exist in the limit of $SU(4)\times SU(4)$ symmetry, and since $\tilde{W}$ is a chiral rotation, it is clear that $\tilde{W}$ itself should be realized as a spontaneously broken, discrete, symmetry. That is, while

$$[\tilde{W},H]=0 \tag{44}$$

physical states are not eigenstates of $\tilde{W}$. In particular, the physical vacuum is doubly degenerate,

$$\tilde{W}|0\rangle=|\tilde{0}\rangle\neq|0\rangle. \tag{45}$$

As long as we have Z and P conservation, $\tilde{W}$ is a symmetry operator and there can be no physical distinction between $|0\rangle$ and $|\tilde{0}\rangle$.

So far we have been considering the case when there is an explicit symmetry-breaking term in SU(4)×SU(4). It is found that the quarks must be massless, according to Eq. (38). Let us now turn to the case when there is no explicit symmetry-breaking term. It is well known that the quarks may acquire masses by coupling to a scalar-meson multiplet which has nonvanishing vacuum expectation values. The coupling itself is fully SU(4)×SU(4)-symmetric. Thus the theory is also fully symmetric. The asymmetry is introduced by picking a particular vacuum state and a set of nonvanishing vacuum expectation values. Explicitly, let us consider a $(4,\bar{4})+(\bar{4},4)$ scalar-meson multiplet φ_i, $i=0,\ldots,15$. They will be coupled to the quarks in an SU(4)×SU(4)-symmetric way:

$$\sum_i \bar{q}\lambda_i q\varphi_i\,. \tag{46}$$

If φ_i develops the vacuum expectation values

$$\left\langle\sum_i \lambda_i\varphi_i\right\rangle_0=\begin{pmatrix}\alpha&0&0&0\\0&\alpha&0&0\\0&0&\beta&0\\0&0&0&\gamma\end{pmatrix}, \tag{47}$$

then the term $\sum_i \bar{q}\lambda_i q\varphi_i$ would give rise to an effective symmetry-breaking term of the form

$$H'\sim u_0+cu_8+du_{15}\,. \tag{48}$$

Summarizing, under the assumption of no explicit symmetry-breaking term in SU(4)×SU(4), the quarks may acquire mass through the spontaneous breaking mechanism. In this case, $\tilde{W}$, as part of SU(4)×SU(4), is itself spontaneously broken.

IV. DISCUSSION

In this paper we considered the breaking of SU(4)×SU(4). It is assumed that the symmetry breaking is dominated by a $(4,\bar{4})+(\bar{4},4)$ term or the quark mass term. The existence of a finite rotation $\tilde{W}=\exp(i4\pi Z_-)$ with the properties

$$\tilde{W}u_i\tilde{W}^{-1}=-u_i\,, \tag{49}$$

however, severely restricts the options of the model. We found the following:

(1) If SU(4)×SU(4) is broken down to U(2)×U(1) *explicitly*, then the symmetry breaking cannot be due to a term transforming like $(4,\bar{4})+(\bar{4},4)$. In this case, the quarks are massless.

(2) If SU(4)×SU(4) is broken spontaneously in total, and hence also the usual SU(3), then the quark mass term is arbitrary. The "ur-Hamiltonian" (original Hamiltonian) is entirely SU(4)×SU(4)-symmetric. Symmetry breaking is induced through the nonvanishing vacuum expectation values of a $(4,\bar{4})+(\bar{4},4)$ scalar-meson multiplet. The operator $\tilde{W}$, while a symmetry of the Hamiltonian, is spontaneously broken. To these two possibilities we now wish to add a third.

(3) SU(4)×SU(4) is broken in part spontaneously and in part explicitly. Specifically, the symmetry breaking is considered to occur in two stages. First SU(4)×SU(4) is broken spontaneously down to SU(3)×SU(3)×U(1). This will be induced by a $(4,\bar{4})+(\bar{4},4)$ scalar-meson multiplet, with the vacuum expectation value

$$\left\langle\sum \lambda_i\varphi_i\right\rangle_0=\begin{pmatrix}0&0&0&0\\0&0&0&0\\0&0&0&0\\0&0&0&a\end{pmatrix}. \tag{50}$$

It gives the $\mathcal{P}'$ quark a mass proportional to a. Next, SU(3)×SU(3) is broken down to U(2), in the usual fashion. That is, there is an explicit symmetry-breaking term $\epsilon H'$. When $\epsilon\to 0$, the vacuum is only SU(3)-invariant. We have now the chain of symmetries:

$$\mathrm{SU}(4)\times\mathrm{SU}(4)\to\mathrm{SU}(3)\times\mathrm{SU}(3)\times\mathrm{U}(1)\to\mathrm{U}(2)\times\mathrm{U}(1). \tag{51}$$

The symmetry operator $\tilde{W}$ is then spontaneously broken, since it is contained in that part of SU(4)×SU(4) which is spontaneously broken. Also, since SU(3) is explicitly broken, the symmetry operator W, described[10,11] and discussed in earlier works, restrains the explicit symmetry-breaking term $\epsilon H'$ of SU(3)×SU(3). However, W itself is spontaneously broken.

Of the three possibilities listed above, which one is the most plausible? It seems to us that case (1), in which all four quarks are massless, is a rather unlikely choice. We also believe that the explicit breaking of SU(3) to SU(2) is called for. It is true that a clear distinction between spontaneously broken and explicitly broken symmetry is lacking. However, the success of SU(3) as a broken symmetry is based on perturbation calculations, using an explicit, small, symmetry-breaking term. These considerations make case (2) also unattractive.

In summary, we believe that the most plausible picture of the broken SU(4) $\times$SU(4) symmetry is one in which the symmetry breaking occurs in two stages. In the first stage SU(4) $\times$SU(4) is broken down spontaneously to SU(3) $\times$SU(3) $\times$U(1). The latter is then broken, both spontaneously and with an explicit symmetry-breaking term, dominated by $(3, \bar{3}) + (\bar{3}, 3)$, while maintaining the symmetry W, which is itself spontaneously broken.

Even though this picture is rather complicated, it does have a few nice properties. The physical constraints that charmed states, if they exist, must be very massive could be reflected in our model by the quark mass relation $m_{\mathcal{P}'} \gg (m_{\lambda}, m_{\mathcal{P}}, m_{\mathfrak{N}})$. This is easily accommodated since $m_{\mathcal{P}'}$ is generated in the first stage of the symmetry breaking. From a group-theoretical viewpoint, our analysis brings out the difference between SU(4) and SU(3) when combined with parity. It renders less arbitrary the assumption that SU(4) is spontaneously broken to SU(3), but SU(3) is explicitly broken down to U(2). Finally, the suppression of quark masses by the $\tilde{W}$ operator has another interesting feature. It may offer a rationale for hadronic quarks to have a different mass scale as compared with ordinary hadrons. In deep-inelastic lepton-hadron scattering, very light quarks or partons, rather than massive ones, seem to be called for by the existing experimental data. The existence of $\tilde{W}$ could provide the mechanism for understanding these phenomena.

---

*Work supported in part by the U. S. Atomic Energy Commission.

[1]F. J. Hasert *et al.*, Phys. Lett. 46B, 138 (1973).

[2]S. Weinberg, Phys. Rev. Lett. 19, 1264 (1967); A. Salam, in *Elementary Particle Theory: Relativistic Groups and Analyticity (Nobel Symposium No. 8)*, edited by N. Svartholm (Almqvist, Stockholm, 1968).

[3]S. L. Glashow, J. Iliopoulos, and L. Maiani, Phys. Rev. D 2, 1285 (1970).

[4]See, for instance, D. Amati, H. Bacry, J. Nuyts, and J. Prentki, Nuovo Cimento 39, 1732 (1964).

[5]P. Dittner and S. Eliezer, Phys. Rev. D 8, 1929 (1973).

[6]P. Dittner, S. Eliezer, and T. K. Kuo, Phys. Rev. Lett. 30, 1274 (1973).

[7]U. Amaldi *et al.*, Phys. Lett. 44B, 112 (1973); S. R. Amendolia *et al.*, *ibid.* 44B, 119 (1973).

[8]Compare with M. S. Chanowitz and S. Drell, Phys. Rev. Lett. 30, 807 (1973).

[9]From a different direction, restrictions on H' were also obtained by D. A. Dicus and V. S. Mathur [Phys. Rev. D 9, 1003 (1974)] and by S. C. Prasad [Phys. Rev. D 9, 1017 (1974)].

[10]T. K. Kuo, Phys. Rev. D 2, 342 (1970).

[11]T. K. Kuo, Phys. Rev. D 4, 3620 (1971); 5, 1033 (1972).