Vector anomaly and the magnetic moment of the W boson*

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In a unified gauge theory without heavy leptons, a possible vector anomaly (for the lepton contribution) can be precluded by an appropriate choice of basis tensors. This in turn gives a definite prediction for the weak leptonic contribution to the magnetic and quadrupole moments of the W boson. Furthermore, self-consistency (rather than convergence considerations) requires that the spectral form for the magnetic form factor in terms of the photon momentum has at least one subtraction, while that in terms of the W momentum is unsubtracted.

In a recent paper[1] we used causal methods to consider the axial-vector anomaly[2] that appears in unified (weak + electromagnetic) gauge theories which do not involve heavy leptons.[3,4] In such theories there is also the possibility of an anomaly in the vector current.[5] The purpose of this paper is to point out that, although from a source-theoretical viewpoint there seems to be no *a priori* reason why this latter anomaly should vanish, it may be eliminated by an appropriate choice of basis tensors, which in turn can lead to a definite prediction for the magnetic moment of the W boson.

For simplicity, we restrict our attention to the lowest-order lepton contribution to the W form factors in Weinberg's[3] $SU_2 \times U_1$ model or Schwinger's U_2 model. One technical point should be noted at the outset: When the lepton mass (m) is less than the W-boson mass (m_W), an unstable anomalous threshold appears.[6,7] We have handled this problem by evaluating all the integrals under the assumption $m > m_W$, and then performing mass extrapolation on the integrated results.

The vector anomaly refers to the divergence of the vector current coupled to one of the W's. As noted in connection with the axial-vector anomaly,[1] one can only obtain this from a process in which that W is not restricted to being a free particle. Therefore, we consider a "sidewise" causal process in which a virtual W creates a real charged lepton and a neutrino which subsequently recombine to form a real W and a photon (Fig. 1). The corresponding vacuum (persistence) amplitude is[8]

$$\langle 0_+ | 0_- \rangle = e\lambda_3{}^2 \frac{1}{8\pi} \times \int \frac{(dp)}{(2\pi)^4} \frac{(dk)}{(2\pi)^4} W_1^\mu(-p)\hat{q}\tilde{I}_{\mu\nu\lambda} W_2^\nu(Q) A_1^\lambda(-k), \tag{1}$$

where

$$\tilde{I}^{\mu\nu\lambda} = 4\pi \int d\omega_q \, d\omega_{q'} (2\pi)^4 \delta(Q-q-q') \times \Big\{ \mathrm{Tr}\Big[(m-\gamma\cdot q')\gamma^\nu(1+i\gamma_5\hat{q})\gamma\cdot q\gamma^\mu \frac{1}{m+\gamma\cdot(p-q)}\gamma^\lambda \Big] + \mathrm{Tr}[(m-\gamma\cdot q')\gamma^\nu\gamma\cdot q\gamma^\sigma] \frac{1}{Q^2+m_W{}^2}\Big(g_{\sigma\kappa} + \frac{Q_\sigma Q_\kappa}{m_W{}^2}\Big)[2p^\lambda g^{\kappa\mu} + (k-p)^\kappa g^{\mu\lambda} - (k+Q)^\mu g^{\kappa\lambda}] \Big\}, \tag{2}$$

$\hat{q}$ is the charge matrix, $Q = p + k$, and $\lambda_3 = (2)^{-1/2} g$ or e in Weinberg's[3] or Schwinger's[4] model, respectively.

The vector part of Eq. (2) can be expressed in terms of three (electromagnetically) gauge-invariant tensors:

$$\tilde{I}^{\mu\nu\lambda} = \tilde{A}_1(k^\mu g^{\nu\lambda} - k^\nu g^{\mu\lambda}) + \tilde{A}_2 Q^\nu[p^\lambda k^\mu - (p\cdot k)g^{\mu\lambda}] + \tilde{A}_3\{k^\mu k^\nu p^\lambda - (p\cdot k)[\alpha k^\nu g^{\mu\lambda} + (1-\alpha)k^\mu g^{\nu\lambda}]\}. \tag{3}$$

Here α is an arbitrary parameter which characterizes the possible bases. The evaluation of Eq. (2) leads to ($M^2 = -Q^2$)

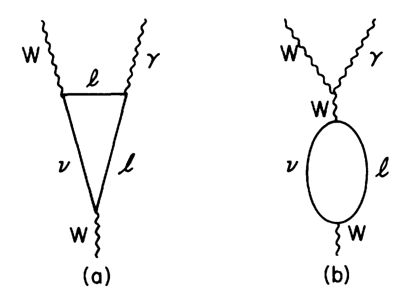

FIG. 1. Lepton contributions to the "sidewise" W form factors.

$$\tilde{A}_1 = -\frac{m^2}{M^2 - m_W^2}\frac{(M^2-m^2)^2}{M^4} + \tfrac{1}{4}(2\alpha - 1)(M^2 - m_W^2)\tilde{A}_3,$$

$$\tilde{A}_2 = -\frac{2}{3}\frac{1}{M^6(M^2-m_W^2)^2}\left\{(M^2-m^2)[M^4(m_W^2+12m^2) - M^2m^2(5m_W^2+6m^2)+4m_W^2m^4] - 6m^2M^6\ln\frac{M^2}{m^2}\right\}, \quad (4)$$

$$\tilde{A}_3 = \frac{4}{3}\frac{1}{M^4(M^2-m_W^2)^3}\left\{(M^2-m^2)[M^4(2m_W^2+15m^2)+M^2m^2(5m_W^2+3m^2) - m_W^2m^4] - 6M^4m^2(M^2+m_W^2+2m^2)\ln\frac{M^2}{m^2}\right\}.$$

Assuming no contact terms[9] we can space-time generalize the individual amplitudes as follows[8]:

$$A_i = \int_{m^2}^{\infty}\frac{dM^2}{2\pi i}\,\frac{\tilde{A}_i}{Q^2+M^2-i\epsilon}. \quad (5)$$

The anomaly is the difference between the naive divergence and the actual divergence.[1,10] We find for the latter

$$\begin{aligned} iQ_\nu I^{\mu\nu\lambda} &= i[k^\mu Q^\lambda - (k\cdot Q)g^{\mu\lambda}][A_1 + Q^2A_2 + \alpha(k\cdot Q)A_3] \\ &= [k^\mu Q^\lambda - (k\cdot Q)g^{\mu\lambda}]\left\{\int_{m^2}^{\infty}\frac{dM^2}{2\pi}\,\frac{1}{Q^2+M^2-i\epsilon}[\tilde{A}_1 - M^2\tilde{A}_2 - \tfrac{1}{2}\alpha(M^2-m_W^2)\tilde{A}_3] + \int_{m^2}^{\infty}\frac{dM^2}{2\pi}(\tilde{A}_2+\tfrac{1}{2}\alpha\tilde{A}_3)\right\}, \end{aligned} \quad (6)$$

where $I^{\mu\nu\lambda}$ is the generalized form of Eq. (3). The second integral here, a constant, is the vector anomaly, and the requirement that it vanish,[11]

$$\int_{m^2}^{\infty} dM^2(\tilde{A}_2 + \tfrac{1}{2}\alpha\tilde{A}_3) = 0, \quad (7)$$

determines

$$\alpha = 1. \quad (8)$$

With this choice of α we calculate, from Eq. (4), the leptonic contributions to the magnetic and quadrupole moments, respectively:

$$\begin{aligned} \Upsilon &= \frac{\lambda_3^2}{16\pi^2}\int_{m^2}^{\infty}\frac{dM^2}{M^2-m_W^2}\tilde{A}_1(M^2) \\ &= -\frac{G}{4\sqrt{2}\,\pi^2}\left\{\tfrac{1}{3}m_W^2 + m^2\left[\frac{2m^2}{m_W^2} - 1 - \frac{2m^2}{m_W^2}\left(\frac{m^2}{m_W^2}-1\right)\ln\frac{m^2}{m^2-m_W^2}\right]\right\} \end{aligned} \quad (9)$$

and

$$\begin{aligned} \mathfrak{Q} &= -\frac{\lambda_3^2}{16\pi^2}m_W^2\int_{m^2}^{\infty}\frac{dM^2}{M^2-m_W^2}\tilde{A}_3(M^2) \\ &= -\frac{G}{4\sqrt{2}\,\pi^2}\left\{\tfrac{4}{9}m_W^2 + \tfrac{4}{3}m^2\left[\frac{m^2}{m_W^2} - \frac{3}{2} - \left(\frac{m^2}{m_W^2}-1\right)^2\ln\frac{m^2}{m^2-m_W^2}\right]\right\}. \end{aligned} \quad (10)$$

The naively mass-extrapolated values for Υ and $\mathfrak{Q}$ at $m=0$ are in agreement with Bardeen *et al.*[12] (except for the sign of $\mathfrak{Q}$). These authors use the n-dimensional regularization scheme which, because of the absence of surface terms, precludes the possibility of vector anomalies.

In order to examine questions of self-consistency, let us compare these results with those obtained from the "direct channel" causal processes of Fig. 2, wherein a virtual photon produces two real leptons which subsequently annihilate to form a $W^\pm$ pair. The causal vacuum amplitude has the general form

$$\langle 0_+|0_-\rangle = e\lambda_3^2\,\frac{1}{16\pi^2}\frac{1}{2} \times\int\frac{(dp)}{(2\pi)^4}\frac{(dp')}{(2\pi)^4}W_1^\mu(-p)\hat{q}I_{\mu\nu\lambda}W_1^\nu(-p_1')A_2^\lambda(Q), \quad (11)$$

where

$$
\begin{aligned}
I_{\mu\nu\lambda} = {} & \mathcal{F}[g_{\mu\nu}(p-p')_{\lambda'} + 2Q_\mu g_{\nu\lambda'} - 2Q_\nu g_{\mu\lambda'}] \\
& \times (Q^2 g^{\lambda'}{}_{\lambda} - Q^{\lambda'} Q_\lambda) \\
& + \mathcal{G} Q_\mu Q_\nu (p-p')_\lambda + \mathcal{H}(Q_\mu g_{\nu\lambda} - Q_\nu g_{\mu\lambda}) \\
& + i\hat{q}\,\mathcal{D}(Q^2 g_\lambda{}^{\lambda'} - Q_\lambda Q^{\lambda'})\epsilon_{\mu\nu\lambda'\sigma}(p-p')^\sigma ,
\end{aligned}
\tag{12}
$$

and $Q = p + p'$. Here we have written the basis tensors in explicitly gauge-invariant form, so that the scalar functions $\mathcal{F}$ (electric, $g = 2$), $\mathcal{G}$ (quadrupole), $\mathcal{H}$ (magnetic), and $\mathcal{D}$ (pseudoelectric) may be space-time extrapolated while maintaining gauge invariance. The processes we are considering here correspond to

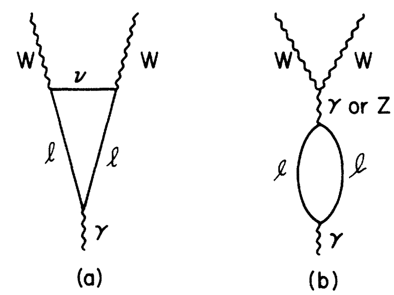

FIG. 2. Lepton contributions to the "direct channel" W form factors.

$$
\begin{aligned}
I^{\mu\nu\lambda} = 8\pi \int & d\omega_q \, d\omega_{q'} (2\pi)^4 \delta(Q - q - q') \\
& \times \left\{ \mathrm{Tr}\left[(m + \gamma\cdot q)\gamma^\mu \frac{1}{\gamma\cdot(p-q)} \gamma^\nu(-1 + i\gamma_5 \hat{q})(m - \gamma\cdot q')\gamma^\lambda \right] \right. \\
& \left. + [g^{\mu\nu}(p-p')^{\lambda'} + 2Q^\mu g^{\nu\lambda'} - 2Q^\nu g^{\mu\lambda'}]\left(\frac{2}{Q^2} - \frac{1}{Q^2 + m_Z{}^2}\right) \mathrm{Tr}[(m + \gamma\cdot q)\gamma_{\lambda'}(m - \gamma\cdot q')\gamma^\lambda] \right\} .
\end{aligned}
\tag{13}
$$

Upon performing the phase-space integration we find

$$
\mathcal{F} = \frac{1}{2}\frac{v}{M^2}\frac{1}{\zeta^4}\{(4\zeta^2 + \zeta^4 - \tfrac{4}{3}\zeta^2 v^2 - v^4) + (\zeta^2 - v^2)[4\zeta^2 - (\zeta^2 + v^2)^2]L\} - \frac{2}{3}\frac{v}{M^2}(3 - v^2)\left(2 - \frac{M^2}{M^2 - m_Z{}^2}\right), \tag{14}
$$

$$
\mathcal{G} = -\frac{1}{2}\frac{v}{M^2}\frac{1}{\zeta^6}\{(-\zeta^4 + \tfrac{22}{3}\zeta^2 v^2 - 5v^4) - \zeta^2(\zeta^4 - \tfrac{2}{3}\zeta^2 v^2 + v^4) + (\zeta^2 - v^2)^2[\zeta^2 + 5v^2 + \zeta^2(\zeta^2 + v^2)]L\}, \tag{15}
$$

$$
\mathcal{H} = \frac{v}{2\zeta^4}\{-\zeta^4 - 3v^4 + 4\zeta^2 + (\zeta^2 - v^2)[4\zeta^2 + (\zeta^2 + v^2)(\zeta^2 - 3v^2)]L\}, \tag{16}
$$

where

$$
v^2 = 1 - \frac{4m^2}{M^2}, \quad \zeta^2 = 1 - \frac{4m_W{}^2}{M^2}, \quad M^2 = -Q^2, \tag{17}
$$

and

$$
L = \frac{1}{2\zeta v}\ln\frac{v + \zeta}{v - \zeta}. \tag{18}
$$

We have not presented the pseudoelectric weight function $\mathcal{D}$, since it is not of interest for our consideration here, and it has appeared elsewhere [see Ref. 1, Eq. (22)].

After performing space-time extrapolation [see Eq. (5)], we find for the leptonic contributions to the magnetic and quadrupole moments, respectively, assuming no contact terms,

$$
\begin{aligned}
\Upsilon' &= -\frac{\lambda_3{}^2}{32\pi^2}\int_{4m^2}^{\infty}\frac{dM^2}{M^2}\mathcal{H} \\
&= \Upsilon - \frac{Gm_W{}^2}{6\sqrt{2}\,\pi^2}
\end{aligned}
\tag{19}
$$

and

$$
\mathcal{Q}' = -\frac{2\lambda_3{}^2 m_W{}^2}{32\pi^2}\int_{4m^2}^{\infty}\frac{dM^2}{M^2}\mathcal{G} = \mathcal{Q}. \tag{20}
$$

We see that although the quadrupole moments calculated in this way are the same in the "sidewise" and "direct" channels, the magnetic moments are not. Thus it is necessary, in the direct channel, to add a contact term to the magnetic form factor[13] (since contact terms cannot be added to the sidewise spectral forms arbitrarily without reintroducing vector anomalies). It is important to recognize that this contact term is a consequence of the self-consistency of the theory, rather than any requirement that the spectral integrals be convergent.

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[6]R. J. Ivanetich, Phys. Rev. D 6, 2805 (1972), and private communication.

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[8]We use the notation and methodology of J. Schwinger, *Particles, Sources, and Fields* (Addison-Wesley, Reading, Mass., 1970), Vol. I; Vol. II (1973).

[9]The requirement that there be no vector anomaly strongly restricts the contact terms.

[10]L. L. DeRaad, K. A. Milton, and W.-y. Tsai, Phys. Rev. D 6, 1766 (1972). Earlier references to the "triangle" anomaly are cited there.

[11]As mentioned above, there seems to be no *a priori* reason for this requirement to be true; however, it seems to be the simplest possible way to remove the ambiguity in the basis, and hence in the magnetic moment. We note that although it is possible to choose a basis in which the vector anomaly vanishes, it is not possible to do so for the axial-vector anomaly.

[12]W. A. Bardeen, R. Gastmans, and B. Lautrup, Nucl. Phys. B46, 319 (1972).

[13]That is, the single-spectral form in terms of the photon momentum requires at least one subtraction, while that in terms of the W momentum is unsubtracted.

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Dual pion model with zero intercept and nine dimensions*

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The recent work of Gervais and Neveu is a promising way of building dual models with intercept not equal to one, yet preserving the gauges of Virasoro. In this work, we generalize their work to include the dual pion model. We give an explicit form for the 4-point function. We find that, as in the original model, Regge behavior is broken. Furthermore, we find that the lowest-mass meson has even G parity and is therefore not a likely candidate for the pion.

I. INTRODUCTION

One of the principal reasons the dual resonance model[1] (DRM) does not provide an accurate description of hadron physics is that the intercept of the leading trajectory is 1. Previous attempts to alter the intercept have always introduced unwanted ghost states or have broken duality.

In the work of Goldstone, Goddard, Rebbi, and Thorn[2] the DRM was shown to be Lorentz-invariant and manifestly ghost-free only if the dimension of space-time was 26 and the intercept was 1. By choosing a suitable Coulomb gauge, they found that two of the dimensions of space-time could be eliminated, corresponding to zero-norm states and ghosts states. They speculated, moreover, about the existence of a model in less than 26 dimensions, with intercept less than 1, which still remains ghost-free but introduces zero-norm states. Their work could be extended to the dual pion model,[3] where the critical dimension[4] is 10.

Recently, Gervais and Neveu[5] have constructed ghost-free dual models with intercept 0 and with dimension 25, which satisfy the ghost-eliminating gauges of Virasoro[6] and which contain zero-norm states.

Their method was surprisingly simple: Construct the most general Hermitian vertex out of Virasoro's L_n's and then force this new vertex to have conformal spin 1 but arbitrary mass. Imposing conformal spin 1 on the vertex is almost sufficient to allow one to uniquely determine the model. They found that the model is solvable and ghost-free in 25 dimensions if we have intercept 0.

In this paper, we extend their work to the dual pion model, which turns out to be ghost-free in nine dimensions with intercept 0. In Sec. II we apply the techniques of Gervais and Neveu to obtain the conditions on the model. In particular, we show that the lowest meson state, with mass 0, has G parity +1 and hence is not the candidate for the pion. In Sec. III we solve explicitly for the 4-point function for scattering of the lowest