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[1]For a complete bibliography as well as a thorough analysis of the current data, see M. Afzal *et al.*, Nuovo Cimento 15A, 61 (1973). In particular see Fig. 8 of this paper, with its striking evidence for decoupling of the longitudinal decay model.

[2]References to modern SU(6) phenomenology: E. Colglazier and J. Rosner, Nucl. Phys. B27, 349 (1971); W. Peterson and J. Rosner, Phys. Rev. D 6, 820 (1972); D. Faiman and D. Plane, Nucl. Phys. B50, 379 (1972). In fact, it was our conversation with Dr. Faiman and Dr. Rosner, in conjunction with the new data (Ref. 1), which prompted the present comment.

[3]This work is contained in the following series: R. Brout, F. Englert, and C. Truffin, Phys. Lett. 29B, 590 (1969); F. Englert, R. Brout, and C. Truffin, *ibid.* 29B, 686 (1969), F. Englert, H. Stern, and R. Brout, Nuovo Cimento 66A, 845 (1970); C. Truffin, R. Brout, and F. Englert, *ibid.* 2A, 169 (1971).

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Shielding of the Pomeron*

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A Pomeron trajectory describing rising cross sections is generally a hard branch-point surface. Shielding cuts are introduced in order to make the Pomeron compatible with two-particle unitarity in the t channel. These shielding singularities may well be important phenomenologically.

The asymptotic expansion of amplitudes describing high-energy scattering is not only controlled by the familiar s-channel bounds, but must also comply with t-channel restrictions. Within the framework of local relativistic field theory and dispersion relations, many of these t-channel constraints are well established. They are based upon the existence of a unique complex angular momentum interpolation, which, in turn, follows from dispersion relations and polynomial boundedness in one variable. Of special relevance is t-channel unitarity. Ideally, we would like to make the asymptotic s-channel expansion explicitly compatible with multiparticle t-channel equations, but it is perhaps mainly the lowest two-particle threshold (e.g., $t=4m_\pi^2$) which is relevant for the properties of the diffraction peak.

It is the purpose of this paper to show how the Pomeron can be made compatible with two-particle t-channel unitarity in case it is *not* a simple Regge-pole trajectory $j=\alpha(t)$ with the appropriate branch point at the threshold $t=t_0$. In particular, we are concerned with situations where the Pomeron is a branch-point surface leading to a rising total cross section.[1] A special example[2] is the amplitude which saturates the Froissart bound,[3] but our method is completely general. It makes use of shielding surfaces $\alpha_s(t)$, which may be relevant phenomenologically at medium high energies.

We assume that the continued partial-wave amplitude $F(t,j)$ satisfies the elastic unitarity condition for $t_0 \le t < t_i$. We write this condition in the analytic form

$$F_{\rm II}^{-1}(t,j)=F^{-1}(t,j)+2i\rho(t), \qquad (1)$$

where $\rho(t)=[(t-t_0)/t]^{1/2}$, and where the subscript II indicates the continuation into the second sheet associated with the elastic square-root cut. If $j=\alpha(t)$ is a singular surface of F such that $F(t,j\to\alpha(t))\to\infty$, then Eq. (1) implies

$$F(t,j\to\alpha(t))\to 1/[2i\rho(t)].$$

A priori, this behavior is incompatible with the continuity theorem of functions with two or more complex variables.[4] But there are several ways out. We mention the following possibilities:

(1) The trajectory $\alpha(t)$ has an appropriate branch point at $t=t_0$. Then $\alpha^{\rm II}(t)\ne\alpha(t)$ and $F(t,j\to\alpha^{\rm II}(t))\to\infty$; hence there is no difficulty. This possibility is familiar for ordinary Regge poles.[4,5]

(2) There is a shielding surface $\alpha_s(t)$ so that the limit $j\to\alpha(t)$ and the continuation from sheet I to sheet II are not interchangeable.[6] Similar shielding cuts also protect nonsense wrong-signature fixed poles.[7]

(3) The character of the singular surface $\alpha(t)$ of $F(t,j)$ has a specific t-dependent character[8]; for example $F(t,j)\to[j-\alpha(t)]^{\beta(t)}$ for $j\to\alpha(t)$.

Of course, these possibilities must be considered within the framework of other known properties of the amplitude $F(t,j)$.

In this paper, we are interested in the shielding-surface method, which appears to be rather natural if $\alpha(t)$ is a branch-point trajectory. Explicit examples for shielding cuts of pole trajectories with $\alpha=\alpha^{II}$ have been discussed in previous publications.[9,10] Here we are interested in branch-point surfaces. As an example, let us consider an amplitude which has a singular surface of the form

$$F(t,j)\propto\{[j-\alpha_0(t)]^2-at\}^{-\beta} \tag{2}$$

for

$$j\to\alpha_\pm(t)=\alpha_0(t)\pm(at)^{1/2}. \tag{3}$$

We assume $\beta>0$, $a\geqslant 0$, $\alpha^{II}(t)=\alpha_0(t)$, and that $\alpha_0(t)$ is regular at $t=0$. The case $\alpha_0(t)\equiv 1$, $a=t_0^{-1}$, and $\beta=\frac{3}{2}$ corresponds to the positive-signature amplitude which gives

$$F(s,t)\propto i(4t_0)^{-1}s(\ln s)^2\frac{2J_1(\sqrt{\tau})}{\sqrt{\tau}}+O(s\ln s) \tag{4}$$

for $s\to\infty$, $\tau=-(t/t_0)(\ln s)^2$ fixed. In Eqs. (2) and (3) the s-channel bounds require $\alpha_0(t)+(at)^{1/2}\leqslant 1+(t/t_0)^{1/2}$ for $0\leqslant t\leqslant t_0$, and $\alpha_0(t)\leqslant 1$ for $t\leqslant 0$.

For the purpose of shielding the singularities (3), we need a pair of singular surfaces $\alpha_{s\pm}(t)$ so that $\alpha_{s\pm}(t_0)=\alpha_\pm(t_0)$. We use

$$\alpha_{s\pm}(t)=\alpha_\pm(t)+c(t-t_0), \tag{5}$$

where $c>0$. In order to see how the shielding works and to learn about the character of the surfaces (5), we make the rather general ansatz

$$(t-t_0)^jF^{-1}(t,j)=\{[j-\alpha_0(t)]^2-at\}^{\beta}\times\left(\Phi(t,j)-\frac{1}{\pi}\{[j-\alpha_0(t)]^2-at\}^{\nu}\int_{-\infty}^{\alpha_1(t)}d\lambda\,\rho(t,\lambda)\frac{\chi(t,j;\lambda)}{[\lambda-\alpha_0(t)-i\epsilon][(j-\lambda)^2-at]^{\beta+\nu}}\right), \tag{6}$$

where $\alpha_1(t)=\alpha_0(t)+c(t-t_0)$. The functions ρ and χ satisfy the conditions

$$\rho(t,\alpha_0(t))=\rho(t),\qquad \chi(t,j;\alpha_0(t))=(t-t_0)^j. \tag{7}$$

Otherwise, these functions, as well as $\Phi(t,j)$, are analytic as required. For example, they may be chosen as

$$\rho(t,\lambda)=\left[\frac{\alpha_1(t)-\lambda}{\alpha_1(t)+ct_0-\lambda}\right]^{1/2}$$

and

$$\chi(t,j;\lambda)=\left[\frac{\alpha_1(t)-\lambda}{c}\right]^j, \tag{8}$$

at least near the relevant limits.

Let us continue the amplitude (6) into the second Riemann sheet of the elastic branch point at $t=t_0$. As we move with t around t_0, the end point of the integral encircles the pole at $\lambda=\alpha_0(t)$ in the integrand, and we just obtain Eq. (1) with the factor $(t-t_0)^j$. The hard branch points at $j=\alpha_\pm(t)$ are present in the physical sheet of $F(t,j)$, but now they do not suddenly disappear as we continue into sheet II, because the shielding surfaces $j=\alpha_{s\pm}(t)$ intervene and provide a refuge in their second sheets. The branch points at $j=\alpha_{s\pm}(t)$ are present in Eq. (6) as end-point singularities of the integral at $\lambda=\alpha_1(t)$. With the expressions (8) for ρ and χ, they have the character

$$[j-\alpha_{s\pm}(t)]^{3/2-\beta-\nu+\alpha_{s\pm}(t)}. \tag{9}$$

We can take the limit $j\to\alpha_{s\pm}(t)$ in Eq. (6) and subsequently let $t\to t_0$. Then we find $F^{-1}(t,j)\propto(t-t_0)^{1/2}$, as required for masking the elastic unitarity cut. Note also that for $\beta=1$, $\nu=$ integer, we have simple poles at $j=\alpha_\pm(t)$, and Eq. (9) reflects the square-root shielding cut obtained in Ref. 6. There is, of course, the superimposed winding point due to the factor $(t-t_0)^j$ in Eq. (6). Although we can remove this "centrifugal cut" of $F(t,j)$ for $t\leqslant t_0$ in the physical sheet of the t plane with the help of the factor $(t-t_0)^{-j}$, this is not possible in the second sheet. Therefore, this branch point appears in models like Eq. (6) which make the continuation into sheet II explicit.

In order to see the possible phenomenological implications of shielding surfaces like $j=\alpha_s(t)$, we consider now the amplitude (6) for the case $\beta=\frac{3}{2}$, $\alpha_0(t)\equiv 1$. With $a=t_0^{-1}$, the Sommerfeld-Watson transform can then give saturation of the Froissart bound. For $t=0$, we obtain

$$F(0,j)\propto(j-1)^{-3}\left[\Phi(0,j)+\frac{1}{\pi}(j-1)^{2\nu}\int_{-\infty}^{1-ct_0}d\lambda\frac{(1-ct_0-\lambda)^{j+1/2}}{(1-\lambda)^{3/2}(j-\lambda)^{3+2\nu}}\right]^{-1}. \tag{10}$$

There is the leading third-order pole for $j\to 1$, which gives rise to $\sigma\sim(\ln s)^2$. Expanding the second factor in powers of $(j-1)$, we may obtain terms proportional to $(j-1)^{-2}$ and/or $(j-1)^{-1}$, depending upon the choice of the parameter ν and the properties of the function $\Phi(0,j)$. However, these terms may just as well be absent if we assume that $\Phi(0, j) = $ constant and that ν is sufficiently large.

On the other hand, there is definitely a contribution from the shielding surfaces $j = \alpha_{s\pm}(t)$, which coincide at $t = 0$, where they have the intercept $\alpha_s(0) = 1 - ct_0$. Therefore the amplitude $F(0, j)$ has a singularity of the type

$$(j - 1 + ct_0)^{1/2 + ct_0 + 2\nu}. \qquad (11)$$

Again, the square-root factor is in evidence. The branch point (11) gives a contribution to the asymptotic expansion in the s channel which is proportional to

$$s^{1 - ct_0}(\ln s)^{-3/2 - ct_0 - 2\nu}. \qquad (12)$$

Assuming, for example, that $c \simeq 1\ \mathrm{GeV}^{-2}$ and $t_0 = 4m_\pi^2$, the intercept of $\alpha_{s\pm}(t)$ is not much below $j = 1$.

We see that the requirements of t-channel two-particle unitarity may well have an important influence upon the phenomenology of diffraction scattering.[11] Although the shielding mechanism does not disturb the leading asymptotic term for $\ln s$ large and $\tau = -at(\ln s)^2$ fixed, like the one in Eq. (4), it nevertheless introduces corrections which can be very important at presently accessible energies.

In another paper we will discuss the applications of our shielding mechanism to nucleon-nucleon and meson-nucleon amplitudes and to other many-channel systems.

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