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<sup>3</sup>We include both  $N_{1/2}$  resonances and  $N_{3/2}(\Delta)$  resonances under the general term " $N^*$  resonances."

<sup>4</sup>Previously the fixed- $t$  dispersion integrand whose energy was above the data region was approximated by a  $\delta$ -function representation of the imaginary parts of the high-energy resonances. We now take an exact treatment of these resonances in the integrand.

<sup>5</sup>J. R. Holt *et al.*, contribution of the International Symposium on Electron and Photon Interactions at High Energies, Bonn, 1973 (unpublished).

<sup>6</sup>This number does not include the special parameters (Ref. 2) we used as a precaution against the nonconvergence of the partial-wave series and which seem to

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## Threshold connection between semi-inclusive deep electroproduction and annihilation

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A unified discussion is given of the scaling limit for one-particle inclusive electroproduction,  $e + p \rightarrow e + \text{hadron} + \text{anything}$ , and for its crossed reactions such as  $e^+ + e^- \rightarrow \bar{p} + \text{hadron} + \text{anything}$ ,  $p + \text{hadron} \rightarrow e^+ + e^- + \text{anything}$ , etc. General threshold relations, which hold also in absence of analytic continuation across the kinematical thresholds, are derived, generalizing previous results for the scaling functions of deep-inelastic scattering and annihilation.

It was pointed out by Drell, Levy, and Yan<sup>1</sup> that for a class of graphs in the cutoff Yukawa theory the scaling functions for  $e^+e^-$  annihilation are analytic continuations of those for inelastic electron scattering. Detailed investigations<sup>2</sup> have shown, however, that this is not true in general, because of certain "double discontinuity" graphs. Nevertheless, it was possible to show that a threshold ( $\omega=1$ ) connection is still expected to hold between the two processes, independently of analytical continuation.<sup>3</sup> Experimentally, two-particle inclusive  $e^+e^-$  annihilation is more directly related to the triggering system required for colliding-beam experiments, whereas detector limitations may render more difficult the verification of the threshold relation for one-particle inclusive annihilation. For this reason, among

others, we have undertaken the effort to provide the threshold relations among the inclusive processes with two observed hadrons (initial or final). Similarly, as for deep-inelastic scattering and annihilation and on the same assumptions, we have derived such threshold relations which again hold independently of analytic continuation.

We consider the processes ( $l$  stands for lepton and  $h$  for hadron)

$$(A) \quad l + \bar{l} \rightarrow h_1 + h_2 + \text{anything},$$

$$(B) \quad h_1 + h_2 \rightarrow l + \bar{l} + \text{anything},$$

$$(C) \quad l + \bar{l} + h_1 \rightarrow h_2 + \text{anything},$$

$$(D) \quad l + h_1 \rightarrow l + h_2 + \text{anything},$$

$$(E) \quad l + \bar{l} + h_2 \rightarrow h_1 + \text{anything},$$

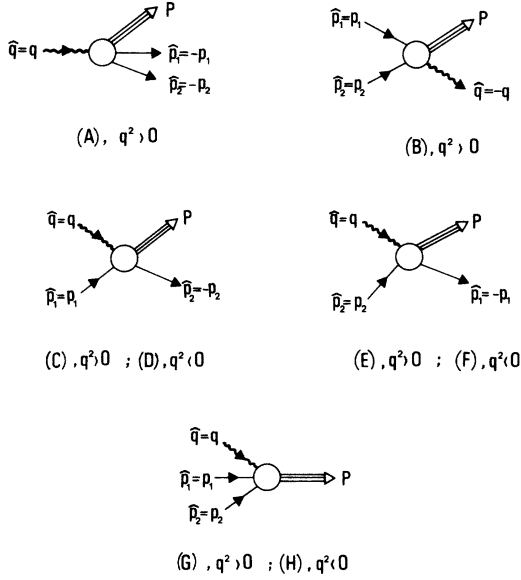


FIG. 1. Feynman graphs of (A)  $l + \bar{l} \rightarrow h_1 + h_2 + \text{anything}$ ; (B)  $h_1 + h_2 \rightarrow l + \bar{l} + \text{anything}$ ; (C)  $l + \bar{l} + h_1 \rightarrow h_2 + \text{anything}$ ; (D)  $l + h_1 \rightarrow l + h_2 + \text{anything}$ ; (E)  $l + \bar{l} + h_2 \rightarrow h_1 + \text{anything}$ ; (F)  $l + h_2 \rightarrow l + h_1 + \text{anything}$ ; (G)  $l + \bar{l} + h_1 + h_2 \rightarrow \text{anything}$ ; (H)  $l + h_1 + h_2 \rightarrow l + \text{anything}$ ; where  $l = \text{lepton}$ ,  $h = \text{hadron}$ . For each graph two related set of momenta,  $\hat{p}_1, \hat{p}_2$ , and  $\hat{q}$  (of positive time component) and  $p_1, p_2$ , and  $q$ , are introduced. The Bjorken limit is defined as  $q^2 \rightarrow +\infty$  for (A), (B), (C), (E), (G), and  $q^2 \rightarrow -\infty$  for (D), (F), (H), keeping  $\omega_1 = -(2p_1 \cdot q)/q^2$ ,  $\omega_2 = -(2p_2 \cdot q)/q^2$ , and  $\eta = (2p_1 \cdot p_2)/q^2$  fixed.

(F)  $l + h_2 \rightarrow l + h_1 + \text{anything}$ ,

(G)  $l + \bar{l} + h_1 + h_2 \rightarrow \text{anything}$ ,

(H)  $l + h_1 + h_2 \rightarrow l + \text{anything}$ ,

corresponding to the graphs of Fig. 1, where the momenta  $\hat{p}_{1,2}$  and  $\hat{q}$  (of positive time component) are indicated together with the definitions of  $p_{1,2}$  and  $q$ . We shall study the limit  $q^2 \rightarrow \pm\infty$  [(+) for (A), (B), (C), (E), (G), (-) for (D), (F), (H)], but keeping

$$\omega_1 = -\frac{2p_1 \cdot q}{q^2}, \quad \omega_2 = -\frac{2p_2 \cdot q}{q^2}, \quad \eta = \frac{2p_1 \cdot p_2}{q^2} \quad (1)$$

finite. Detailed study of the limiting kinematics for (A), (B), . . . , (H) presents two solutions,  $\eta \rightarrow 0$  or  $\eta \rightarrow \omega_1 \omega_2$ , for each case, except for (B), for which only  $\eta \rightarrow \omega_1 \omega_2$  is allowed.

We call  $\eta \rightarrow 0$  the target region (t.r.) and  $\eta \rightarrow \omega_1 \omega_2$  the photon region (ph. r.). Figure 2 shows the kinematical domains for (A), (B), . . . , (H) in the ph. r. The threshold regions of interest, connecting the only measurable processes (A), (B), (D), (F), are those around  $\omega_1 = 1$ ,  $\omega_2 = 1$  for positive  $\omega_2, \omega_1$ . No such common thresholds exist for (A), (B), (D),

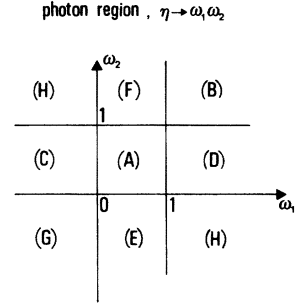


FIG. 2. Kinematical domains in the  $\omega_1$ - $\omega_2$  plane for processes (A)–(H) in the photon region ( $\eta \rightarrow \omega_1 \omega_2$ ).

(F) in the t.r. For each process  $\lambda = (A), (B), \dots, (H)$  one defines a structure tensor

$$W_{\mu\nu}^{(\lambda)}(q, p_1, p_2) = (2\pi)^4 \sum_n \delta^{(4)}(q + p_1 + p_2 - P) \times \langle i^{(\lambda)} | j_\mu(0) | n^{(\lambda)} \rangle \times \langle n^{(\lambda)} | j_\nu(0) | i^{(\lambda)} \rangle, \quad (2)$$

where

$$|i^{(A)}\rangle = 0, \quad |n^{(A)}\rangle = | -p_1, -p_2, n \rangle;$$

$$|i^{(B,G,H)}\rangle = | p_1, p_2 \rangle, \quad |n^{(B,G,H)}\rangle = | n \rangle;$$

$$|i^{(C,D)}\rangle = | p_1 \rangle, \quad |n^{(C,D)}\rangle = | -p_2, n \rangle;$$

$$|i^{(E,F)}\rangle = | p_2 \rangle, \quad |n^{(E,F)}\rangle = | -p_1, n \rangle.$$

In the limit  $q^2 \rightarrow \pm\infty$ ,  $\omega_1, \omega_2$  fixed, and limited transverse momenta one has, on the assumption of scaling,

$$W_{\mu\nu}^{(\lambda)}(q, p_1, p_2) \rightarrow - \left( g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) F_1^{(\lambda)}(\omega_1, \omega_2, \eta) - \frac{2}{M_1^2} \left( p_{1,\mu} - \frac{p_1 \cdot q}{q^2} q_\mu \right) \left( p_{1,\nu} - \frac{p_1 \cdot q}{q^2} q_\nu \right) \times \frac{1}{\omega_1 q^2} F_2^{(\lambda)}(\omega_1, \omega_2, \eta),$$

where  $\eta \rightarrow 0$  or  $\eta \rightarrow \omega_1 \omega_2$ . The dominant graphs contributing in the photon region in a parton-model description<sup>4</sup> are shown in Fig. 3. In addition, for (B) the “connected” graph of Fig. 4, where the vertical line denotes Pomeron exchange, is expected to also contribute in the limit.<sup>4</sup> It is suggested, however, that its contribution is numerically smaller.<sup>5</sup> One can show from the graphs in Fig. 3 that for vector currents and spin- $\frac{1}{2}$  partons, in the ph. r.  $\eta \rightarrow \omega_1 \omega_2 + |q^2|^{-1} f(\omega_1, \omega_2, \bar{t})$ ,

$$\begin{aligned}
 F_2^{(A,B)}(\omega_1, \omega_2, \vec{t}) &= \frac{M_1^2}{\omega_1 |1 - \omega_1| (1 - \omega_2)} \int ds_1 ds_2 d\vec{t} \left[ \text{Im}T_{(\rho)} \left( s_2, \frac{s_2 + \omega_2 (\vec{t} + \omega_2^{-1} \vec{t})^2}{1 - \omega_2} + \frac{M_2^2}{\omega_2} \right) \text{Im}T_{(ap)} \left( s_1, \frac{s_1 + \omega_1 \vec{t}^2 + M_1^2}{1 - \omega_1 + \omega_1} \right) \right. \\
 &\quad \left. + \text{Im}T_{(ap)} \left( s_2, \frac{s_2 + \omega_2 (\vec{t}^2 + \omega_2^{-1} \vec{t})^2}{1 - \omega_2} + \frac{M_2^2}{\omega_2} \right) \text{Im}T_{(\rho)} \left( s_1, \frac{s_1 + \omega_1 \vec{t} + M_1^2}{1 - \omega_1 + \omega_1} \right) \right], \tag{3a}
 \end{aligned}$$

$$\begin{aligned}
 F_2^{(D,F)}(\omega_1, \omega_2, \vec{t}) &= \frac{M_1^2}{\omega_1 |1 - \omega_1| (1 - \omega_2)} \int ds_1 ds_2 d\vec{t} \left[ \text{Im}T_{(\rho)} \left( s_2, \frac{s_2 + \omega_2 (\vec{t} + \omega_2^{-1} \vec{t})^2}{1 - \omega_2} + \frac{M_2^2}{\omega_2} \right) \text{Im}T_{(\rho)} \left( s_1, \frac{s_1 + \omega_1 \vec{t}^2 + M_1^2}{1 - \omega_1 + \omega_1} \right) \right. \\
 &\quad \left. + \text{Im}T_{(ap)} \left( s_2, \frac{s_2 + \omega_2 (\vec{t} + \omega_2^{-1} \vec{t})^2}{1 - \omega_2} + \frac{M_2^2}{\omega_2} \right) \text{Im}T_{(ap)} \left( s_1, \frac{s_1 + \omega_1 \vec{t}^2 + M_1^2}{1 - \omega_1 + \omega_1} \right) \right], \tag{3b}
 \end{aligned}$$

where  $T_{(\rho)}$  ( $T_{(ap)}$ ) is the nonamputated amplitude for two partons into two hadrons with incoming parton (antiparton), and  $\vec{t}$  is the (limited) transverse momentum of  $h_2$ . We also report the expression for the scaling functions of deep-inelastic scattering (dis),  $l+h \rightarrow l$  + anything, and deep-

inelastic annihilation (dia),  $l+T \rightarrow h$  + anything:

$$\begin{aligned}
 F_2^{(\pm)}(\omega) &= \frac{M^2}{\omega |1 - \omega|} \\
 &\quad \times \int ds d^2t \left[ \text{Im}T_{(\rho)} \left( s, \frac{s + \omega \vec{t}^2 + M^2}{1 - \omega + \omega} \right) \right. \\
 &\quad \left. + \text{Im}T_{(ap)} \left( s, \frac{s + \omega \vec{t}^2 + M^2}{1 - \omega + \omega} \right) \right] \tag{4}
 \end{aligned}$$

[(+) for dis, (-) for dia],  $\omega > 1$  for dis,  $0 < \omega < 1$  for dia. It will be convenient to define  $\text{Im}T_{(\rho)} = R + P$  and  $\text{Im}T_{(ap)} = P$  (the idea is to distinguish between resonant and diffractive contributions), and corresponding integrals [see Eq. (4)]

$$\mathcal{R}^{(\pm)}(\omega) = \frac{M^2}{\omega |1 - \omega|} \int ds d^2t R^{(\pm)} \left( s, \frac{s + \omega \vec{t}^2 + M^2}{1 - \omega + \omega} \right) \tag{5}$$

[(+) for dis, (-) for dia]. Similarly, one defines a quantity  $\mathcal{P}^{(\pm)}(\omega)$  as the corresponding integral over  $P^{(\pm)}$ . Equation (4) becomes

$$F_2^{(\pm)}(\omega) = \mathcal{R}^{(\pm)}(\omega) + 2\mathcal{P}^{(\pm)}(\omega)$$

and one recovers interesting factorization properties for the "integrated scaling function" (over transverse momentum) in the photon region:

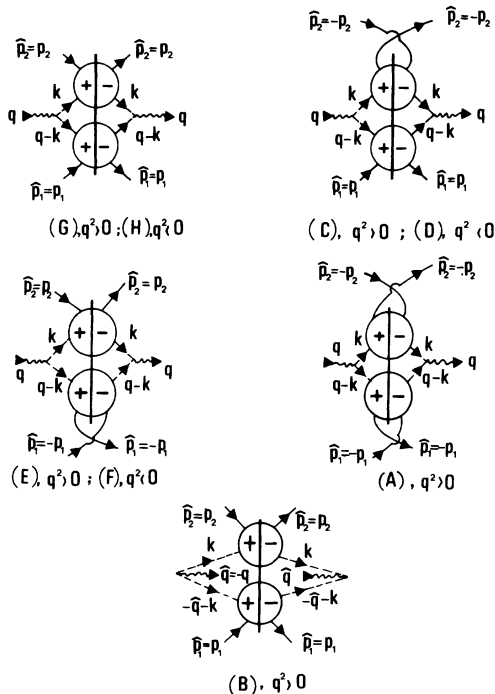


FIG. 3. Dominant parton-model contributions to processes (A)–(H) in the photon region. Partons are represented by broken lines. Summation over the different parton species is understood. The bubbles denote nonamputated amplitudes for two partons into two hadrons. The vertical heavy lines denote unitary cuts. The (+), (–) signs identify the limit convention of energies to their real values.

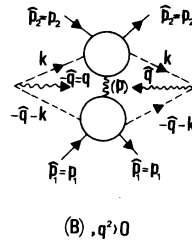


FIG. 4. Pomeron-exchange term contributing to process (B).

$$\begin{aligned}\mathfrak{F}_2^{(A,B)}(\omega_1, \omega_2) &= \int d^2t F_2^{(A,B)}(\omega_1, \omega_2, \vec{t}) \\ &= \frac{\omega_2^3}{M_2^2} [\mathcal{R}^{(-,+)}(\omega_2) \mathcal{O}^{(-,+)}(\omega_1) \\ &\quad + \mathcal{O}^{(-,+)}(\omega_2) \mathcal{R}^{(-,+)}(\omega_1) \\ &\quad + 2\mathcal{O}^{(-,+)}(\omega_2) \mathcal{O}^{(-,+)}(\omega_1)], \quad (6a)\end{aligned}$$

$$\begin{aligned}\mathfrak{F}_2^{(D,F)}(\omega_1, \omega_2) &= \int d^2t F_2^{(D,F)}(\omega_1, \omega_2, \vec{t}) \\ &= \frac{\omega_2^3}{M_2^2} [\mathcal{R}^{(-,+)}(\omega_2) \mathcal{R}^{(+,-)}(\omega_1) \\ &\quad + \mathcal{R}^{(+,-)}(\omega_2) \mathcal{O}^{(+,-)}(\omega_1) \\ &\quad + \mathcal{O}^{(+,-)}(\omega_2) \mathcal{R}^{(+,-)}(\omega_1) \\ &\quad + 2\mathcal{O}^{(+,-)}(\omega_2) \mathcal{O}^{(+,-)}(\omega_1)]. \quad (6b)\end{aligned}$$

We recall that a threshold relation<sup>3</sup>

$$\lim_{\omega \rightarrow 1^+} F_2^{(+)}(\omega) = \lim_{\omega \rightarrow 1^-} F_2^{(-)}(\omega) \quad (7)$$

can be derived between dis and dia from Eq. (4) on the assumption that  $\text{Im}T_{(p)}(s, k^2)$  and  $\text{Im}T_{(ap)}(s, k^2)$  are analytic functions of  $k^2$  with singularities only on the real positive axis. This implies the coin-

cidence of the two limits  $k^2 \rightarrow \pm\infty$  in both  $\text{Im}T_{(p)}(s, k^2)$  and  $\text{Im}T_{(ap)}(s, k^2)$ , and, from Eq. (4), leads to Eq. (7). A more general assumption (which only leads to threshold proportionality of  $F_2^{(\text{dis})}$  and  $F_2^{(\text{dia})}$ ) is

$$\lim_{k^2 \rightarrow \mp\infty} \text{Im}T_{(p)}^{(\pm)}(s, k^2) = (\mp k^2)^{-\gamma(p)} f_{(p)}^{(\pm)}(s), \quad (8)$$

$$\lim_{k^2 \rightarrow \mp\infty} \text{Im}T_{(ap)}^{(\pm)}(s, k^2) = (\mp k^2)^{-\gamma(ap)} f_{(ap)}^{(\pm)}(s).$$

Feynman-graph calculations<sup>2</sup> in  $\phi^3$  theory show that the stronger assumption is presumably valid (i.e., one expects  $f^+ = f^-$ ). In renormalizable theories with softening of large momenta, Feynman-graph calculations<sup>6</sup> indicate that Eq. (8) at least should hold. Keeping the more general form in Eq. (8), we notice that for  $\mathcal{R}^{(\pm)}(\omega)$  in Eq. (5), it leads to ( $\lambda_R$  and  $\gamma_R$  are constants)

$$\lim_{\omega \rightarrow 1^\pm} \mathcal{R}^{(\pm)}(\omega) = M^2 \lambda_R^{(\pm)} |1 - \omega|^{\gamma_R - 1} \quad (9)$$

and to a similar equation for the analogous quantity  $\mathcal{O}^{(\pm)}(\omega)$  (in terms of corresponding  $\lambda_P, \gamma_P$ ). For the integrated scaling functions of Eqs. (6a) and 6(b) one then obtains

$$\lim_{\omega_1 \rightarrow 1^-, \omega_2 \rightarrow 1^+} \mathfrak{F}_2^{(A,B)}(\omega_1, \omega_2) = \frac{M^2}{M_2^2} \omega_2^3 \lambda_R^{(-,+)} |1 - \omega_1|^{\gamma_R - 1} \{ \mathcal{O}^{(-,+)}(\omega_2) + N^{(-,+)}(\omega_2) [\mathcal{R}^{(-,+)}(\omega_2) + 2\mathcal{O}^{(-,+)}(\omega_2)] \}, \quad (10a)$$

$$\lim_{\omega_2 \rightarrow 1^-, \omega_1 \rightarrow 1^+} \mathfrak{F}_2^{(A,B)}(\omega_1, \omega_2) = \lambda_R^{(-,+)} |1 - \omega_2|^{\gamma_R - 1} \{ \mathcal{O}^{(-,+)}(\omega_1) + N^{(-,+)}(\omega_1) [\mathcal{R}^{(-,+)}(\omega_1) + 2\mathcal{O}^{(-,+)}(\omega_1)] \}, \quad (11a)$$

$$\begin{aligned}\lim_{\omega_1 \rightarrow 1^+, \omega_2 \rightarrow 1^-} \mathfrak{F}_2^{(D,F)}(\omega_1, \omega_2) &= \frac{M^2}{M_2^2} \omega_2^3 \lambda_R^{(+,-)} |1 - \omega_1|^{\gamma_R - 1} \{ \mathcal{R}^{(+,-)}(\omega_2) + \mathcal{O}^{(+,-)}(\omega_2) + N^{(+,-)}(\omega_2) [\mathcal{R}^{(+,-)}(\omega_2) + 2\mathcal{O}^{(+,-)}(\omega_2)] \} \\ &\simeq \frac{M^2}{M_2^2} \omega_2^3 \lambda_R^{(+,-)} |1 - \omega_1|^{\gamma_R - 1} [\mathcal{R}^{(+,-)}(\omega_2) + \mathcal{O}^{(+,-)}(\omega_2)], \quad (10b)\end{aligned}$$

$$\begin{aligned}\lim_{\omega_2 \rightarrow 1^-, \omega_1 \rightarrow 1^+} \mathfrak{F}_2^{(D,F)}(\omega_1, \omega_2) &= \lambda_R^{(+,-)} |1 - \omega_2|^{\gamma_R - 1} \{ \mathcal{R}^{(+,-)}(\omega_1) + \mathcal{O}^{(+,-)}(\omega_1) + N^{(+,-)}(\omega_1) [\mathcal{R}^{(+,-)}(\omega_1) + 2\mathcal{O}^{(+,-)}(\omega_1)] \} \\ &\simeq \lambda_R^{(+,-)} |1 - \omega_2|^{\gamma_R - 1} [\mathcal{R}^{(+,-)}(\omega_1) + \mathcal{O}^{(+,-)}(\omega_1)], \quad (11b)\end{aligned}$$

where

$$N^{(\pm)}(\omega) = \frac{\lambda^{(\pm)}}{\lambda_R^{(\pm)}} |1 - \omega|^{\gamma_P - \gamma_R} \quad (12)$$

is expected to be very small. Finally,

$$\begin{aligned}\lim_{\substack{\omega_1 \rightarrow 1^-, \omega_2 \rightarrow 1^+ \\ \omega_2 \rightarrow 1^-, \omega_1 \rightarrow 1^+}} \mathfrak{F}_2^{(A,B)}(\omega_1, \omega_2) &= \frac{M^2}{M_2^2} \lambda_R^{(-,+)} \lambda_P^{(-,+)} (|1 - \omega_1|^{\gamma_R - 1} |1 - \omega_2|^{\gamma_P - 1} \\ &\quad + |1 - \omega_1|^{\gamma_P - 1} |1 - \omega_2|^{\gamma_R - 1}), \quad (13a)\end{aligned}$$

$$\begin{aligned}\lim_{\substack{\omega_1 \rightarrow 1^+, \omega_2 \rightarrow 1^- \\ \omega_2 \rightarrow 1^-, \omega_1 \rightarrow 1^+}} \mathfrak{F}_2^{(D,F)}(\omega_1, \omega_2) &= \frac{M^2}{M_2^2} \lambda_R^{(+,-)} \lambda_R^{(-)} |1 - \omega_2|^{\gamma_R - 1} |1 - \omega_1|^{\gamma_R - 1}. \quad (13b)\end{aligned}$$

Therefore, we see that:

(i) In the "double threshold" region  $\omega_1, \omega_2 \rightarrow 1^\pm$ ,  $\mathfrak{F}_2^{(A)}$  and  $\mathfrak{F}_2^{(B)}$  are proportional (equal, if  $\lambda^+ = \lambda^-$ ) to the same function of  $\omega_1, \omega_2$ ; so also for  $\mathfrak{F}_2^{(D)}$  and  $\mathfrak{F}_2^{(F)}$ .

We notice that under the condition<sup>7</sup>  $\omega_1 \rightarrow 1^\pm$ , in a

fixed range, and  $\omega_2$  such that

$$\frac{\mathcal{R}^{(-,+)}(\omega_2)}{\mathcal{P}^{(-,+)}(\omega_2)} \ll \frac{1}{N^{(-,+)}(\omega_1)}, \quad (14)$$

Eq. (10a) becomes

$$\lim_{\omega_1 \rightarrow 1^-} \mathcal{F}_2^{(A,B)}(\omega_1, \omega_2) = \frac{M_1^2}{M_2^2} \omega_2^3 \lambda_R^{(-,+)} |1 - \omega_1|^{\gamma_R - 1} \times \mathcal{P}^{(-,+)}(\omega_2), \quad (15)$$

and thus, under such conditions, we have:

(ii) The integrated scaling functions for A (B) and D (F) have the same power behavior in  $|1 - \omega_1|$  for  $\omega_1 \rightarrow 1^\pm$ , but are proportional to different functions of  $\omega_2$  [essentially  $\omega_2^3$  times the Pomeron term of dia (dis) for A (B) and  $\omega_2^3$  times the sum of resonance and Pomeron terms of dia (dis) for D (F)].

Similarly, for  $\omega_2 \rightarrow 1^\pm$ , in a fixed range and  $\omega_1$

such that

$$\frac{\mathcal{R}^{(-,+)}(\omega_1)}{\mathcal{P}^{(-,+)}(\omega_1)} \ll \frac{1}{N^{(-,+)}(\omega_2)}, \quad (14')$$

we have:

(iii) The integrated scaling functions for A (B) and F (D) have the same power behavior in  $|1 - \omega_2|$ , for  $\omega_2 \rightarrow 1^\pm$ , but are proportional to different functions of  $\omega_1$  [essentially the Pomeron term of dia (dis) for A (B) and the sum of resonance and Pomeron terms of dia (dis) for F (D)].

We stress, however, that conclusions regarding  $\mathcal{F}_2^{(B)}$  do not hold whenever B has significant contributions from the diagram in Fig. 4, and possible deviations from our conclusions might be taken as indications of those terms (so far, seemingly smaller).

<sup>1</sup>S. D. Drell, D. J. Levy, and T.-M. Yan, Phys. Rev. 187, 2159 (1969); Phys. Rev. D 1, 1035 (1970); 1, 1617 (1970).

<sup>2</sup>R. Gatto, P. Menotti, and I. Vendramin, Nuovo Cimento Lett. 4, 79 (1971); Ann. Phys. (N.Y.) 79, 1 (1973); P. V. Landshoff and I. C. Polkinghorne, Phys. Rev. D 6, 3708 (1972); G. Altarelli and L. Maiani, Nucl. Phys. B51, 509 (1973).

<sup>3</sup>R. Gatto, P. Menotti, and I. Vendramin, Nuovo Cimento Lett. 5, 754 (1972); P. V. Landshoff and I. C. Polkinghorne, Nucl. Phys. B53, 473 (1972); Z. F. Ezawa, Tokyo report (unpublished).

<sup>4</sup>P. V. Landshoff and J. C. Polkinghorne, Nucl. Phys. B33, 221 (1971).

<sup>5</sup>P. V. Landshoff and J. C. Polkinghorne, Nucl. Phys. B36, 642(E) (1972); P. V. Landshoff, DAMTP Report No. 72/35, 1972 (unpublished).

<sup>6</sup>Landshoff and Polkinghorne, Ref. 3.

<sup>7</sup>To gain some quantitative ideas, we use some available models. Assuming (Ref. 8)  $\mathcal{R}(\omega) = B^{-1}(m, \alpha_0 - 1) \times \omega^{-m + \alpha_0} (\omega - 1)^{m-1}$  (with  $m = 4$ ,  $\alpha_0 = 0.5$ ), and (Ref. 9)  $\mathcal{P}(\omega) = (0.115)[(\omega - 1)/\omega]^{\mu-1}$  (with  $\mu = 5$ ), then for, e.g.,  $1 < \omega_1 < 1.1$ ,  $N^{(+)}(\omega_1) \lesssim 0.02$ , one has  $[\mathcal{R}(\omega_2)/\mathcal{P}(\omega_2)] \lesssim (0.05)[N^{(+)}(\omega_1)]^{-1}$  already for  $\omega_2 \gtrsim 6$ .

<sup>8</sup>P. V. Landshoff and J. C. Polkinghorne, Nucl. Phys. B28, 240 (1971).

<sup>9</sup>D. Atkinson and A. P. Contogouris, Nucl. Phys. B31, 429 (1971).