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η-X mixing and chiral-symmetry breaking. II

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An earlier argument suggesting that the η-X mixing angle vanishes is generalized. This theoretical conclusion is supported by a recent measurement of the decay rate $\eta \to 2\gamma$.

In a recent paper,[1] hereafter referred to as I, the question of η-X mixing was discussed within the framework of the chiral-symmetry-breaking model of Glashow and Weinberg[2] and Gell-Mann, Oakes, and Renner.[3] In particular, the approach of Glashow and Weinberg[2] was followed. The Hamiltonian density was assumed to be

$$\mathcal{H} = \mathcal{H}_0 - \epsilon_0\sigma_0 - \epsilon_8\sigma_8 \,. \tag{1}$$

$\mathcal{H}_0$ is chiral-invariant, ϵ_0 and ϵ_8 are the symmetry breaking parameters, and σ_0 and σ_8 are local scalar fields belonging to the eighteen-dimensional $(3,\bar{3})+(\bar{3},3)$ representation of $SU(3)\times SU(3)$. It was further assumed that the various scalar σ_i and pseudoscalar π_i ($i=0,\ldots,8$) were good interpolating fields for the scalar and pseudoscalar meson nonets.

In I, it was shown that if all octet wave-function renormalization constants were equal, but different from the singlet wave-function renormalization constant, then the η-X mixing angle vanished.

It is the purpose of this note to show that it is possible to demonstrate that the η-X mixing angle vanishes without making any restrictive assumptions about the equality of certain wave-function renormalization constants. In order to simplify the notation, we introduce *unrenormalized* weak decay constants F_i and *unrenormalized* masses M_i, which are related to the physical weak decay constants f_i and physical masses m_i through the renormalization constants Z_i. For example,

$$F_\pi = Z_\pi^{1/2} f_\pi \tag{2}$$

and

$$M_\pi = Z_\pi^{-1/2} m_\pi \,. \tag{3}$$

In this notation, we have the usual relations[1,2]

$$F_\pi = \frac{2}{\sqrt{3}}\langle\sigma_0\rangle + (\tfrac{2}{3})^{1/2}\langle\sigma_8\rangle \,, \tag{4a}$$

$$F_K = \frac{2}{\sqrt{3}}\langle\sigma_0\rangle - \frac{1}{\sqrt{6}}\langle\sigma_8\rangle \,, \tag{4b}$$

$$F_\kappa = -(\tfrac{3}{2})^{1/2}\langle\sigma_8\rangle \,, \tag{4c}$$

and

$$F_\pi M_\pi^2 = \frac{2}{\sqrt{3}}\epsilon_0 + (\tfrac{2}{3})^{1/2}\epsilon_8 \,, \tag{5a}$$

$$F_K M_K^2 = \frac{2}{\sqrt{3}}\epsilon_0 - \frac{1}{\sqrt{6}}\epsilon_8 \,, \tag{5b}$$

$$F_\kappa M_\kappa^2 = -(\tfrac{3}{2})^{1/2}\epsilon_8 \,, \tag{5c}$$

between F_i, M_i ($i=\pi, K, \kappa$) and the symmetry breaking quantities ϵ_0, ϵ_8, $\langle\sigma_0\rangle$, and $\langle\sigma_8\rangle$.

If we now consider the η and assume that there is no η-X mixing, we have the additional relations

$$F_\eta = \frac{2}{\sqrt{3}}\langle\sigma_0\rangle - (\tfrac{2}{3})^{1/2}\langle\sigma_8\rangle \,, \tag{4d}$$

$$F_\eta M_\eta^2 = \frac{2}{\sqrt{3}}\epsilon_0 - (\tfrac{2}{3})^{1/2}\epsilon_8 \,. \tag{5d}$$

Upon eliminating $\langle\sigma_0\rangle$, $\langle\sigma_8\rangle$, ϵ_0, and ϵ_8 from Eqs. (4) and (5), we have the usual relations[1,2]

$$F_K = F_\pi + F_\kappa \tag{6a}$$

and

$$F_K M_K^2 = F_\pi M_\pi^2 + F_\kappa M_\kappa^2 \,. \tag{7a}$$

In addition we have[1,2]

$$3F_\eta = 4F_K - F_\pi \tag{6b}$$

and

$$3F_\eta M_\eta^2 = 4F_K M_K^2 - F_\pi M_\pi^2 \,. \tag{7b}$$

From Eqs. (6) and (7) we see that

$$(4F_K - F_\pi)M_\eta^2 = 4F_K M_K^2 - F_\pi M_\pi^2 \tag{8}$$

or

$$(4F_K - F_\pi)m_0^2 Z_8^{-1} = 4F_K M_K^2 - F_\pi M_\pi^2 \,, \tag{9}$$

where m_0 is the unmixed mass of the η.

Extending this to the more general case where we allow for η-X mixing, we find[1] that the unrenormalized η-X propagator matrix (at zero momentum transfer) satisfies

$$\Delta^{-1}(0)\begin{pmatrix} (\tfrac{2}{3})^{1/2}\langle\sigma_0\rangle - (\tfrac{1}{3})^{1/2}\langle\sigma_8\rangle \\ (\tfrac{2}{3})^{1/2}\langle\sigma_8\rangle \end{pmatrix} = \begin{pmatrix} (\tfrac{2}{3})^{1/2}\epsilon_0 - (\tfrac{1}{3})^{1/2}\epsilon_8 \\ (\tfrac{2}{3})^{1/2}\epsilon_8 \end{pmatrix} . \tag{10}$$

$\Delta^{-1}(0)$ is related to the η-X mass squared matrix by

$$\Delta^{-1}(0)=\begin{pmatrix} Z_8^{-1}(m_\eta^2\cos^2\theta+m_X^2\sin^2\theta) & Z_8^{-1/2}Z_0^{-1/2}(m_X^2-m_\eta^2)\sin\theta\cos\theta \\ Z_0^{-1/2}Z_8^{-1/2}(m_X^2-m_\eta^2)\sin\theta\cos\theta & Z_0^{-1}(m_\eta^2\sin^2\theta+m_X^2\cos^2\theta)\end{pmatrix}, \tag{11}$$

where θ is the η-X mixing angle.

From Eqs. (10) and (11) we find that the mixing angle satisfies

$$\tan\theta=-\left(\frac{Z_8}{Z_0}\right)^{1/2}\frac{\alpha-cm_\eta^2Z_8^{-1}}{\beta-dm_\eta^2Z_0^{-1}} \tag{12}$$

and

$$\tan\theta=\left(\frac{Z_0}{Z_8}\right)^{1/2}\frac{\beta-dm_X^2Z_0^{-1}}{\alpha-cm_X^2Z_8^{-1}}, \tag{13}$$

where c, d, α, and β are given by

$$c=\frac{1}{3\sqrt{2}}(4F_K-F_\pi),$$

$$d=-\tfrac{2}{3}(F_K-F_\pi),$$

$$\alpha=\frac{1}{3\sqrt{2}}(4F_KM_K^2-F_\pi M_\pi^2),$$

$$\beta=-\tfrac{2}{3}(F_KM_K^2-F_\pi M_\pi^2). \tag{14}$$

From Eqs. (12) and (13) we see that $\tan^2\theta$ satisfies

$$\begin{aligned}\tan^2\theta&=-\frac{\alpha-cm_\eta^2Z_8^{-1}}{\alpha-cm_X^2Z_8^{-1}}\,\frac{\beta-dm_X^2Z_0^{-1}}{\beta-dm_\eta^2Z_0^{-1}}\\&=-\frac{(4F_KM_K^2-F_\pi M_\pi^2)-(4F_K-F_\pi)m_\eta^2Z_8^{-1}}{(4F_KM_K^2-F_\pi M_\pi^2)-(4F_K-F_\pi)m_X^2Z_8^{-1}}\\&\quad\times\frac{M_K^2-m_X^2Z_0^{-1}}{M_K^2-m_\eta^2Z_0^{-1}},\end{aligned} \tag{15}$$

where we have also made use of Eqs. (6a) and (7a).

In addition, the mixing angle θ satisfies

$$\tan^2\theta=\frac{m_0^2-m_\eta^2}{m_X^2-m_0^2}, \tag{16}$$

where m_0 is the unmixed η mass. Assuming[4] that m_0 satisfies Eq. (9) we may rewrite Eq. (16) as

$$\tan^2\theta=-\frac{(4F_KM_K^2-F_\pi M_\pi^2)-(4F_K-F_\pi)m_\eta^2Z_8^{-1}}{(4F_KM_K^2-F_\pi M_\pi^2)-(4F_K-F_\pi)m_X^2Z_8^{-1}}. \tag{17}$$

Equating Eqs. (15) and (17), we find that we must have

$$4F_KM_K^2-F_\pi M_\pi^2=(4F_K-F_\pi)m_\eta^2Z_8^{-1}, \tag{18}$$

provided that the wave-function renormalization constants Z_i are relatively finite. From Eqs. (17) and (18) we conclude that the mixing angle θ vanishes. This is the same conclusion arrived at in I under more restrictive assumptions.

In I, it was suggested that the main problem confronting a theoretical prediction of zero η-X mixing was the experimental fact[5] that the decay rate for $\eta\to2\gamma$ appeared to be enhanced well above the SU(3) prediction of this rate from the $\pi^0\to2\gamma$ decay rate. Consequently various mechanisms other than η-X mixing were conjectured for the enhancement of the $\eta\to2\gamma$ decay rate.

Since that paper[1] was written, Browman *et al.*[6] have reported a second measurement of this decay rate and found

$$\Gamma(\eta\to2\gamma)=(302\pm67)\text{ eV}. \tag{19}$$

This second result is not much larger than the SU(3) prediction

$$\Gamma(\eta\to2\gamma)=(175\pm20)\text{ eV}, \tag{20}$$

based on the decay rate[7]

$$\Gamma(\pi^0\to2\gamma)=(7.8\pm0.9)\text{ eV}, \tag{21}$$

which suggests that the enhancement mechanisms conjectured in I are not necessary.

Making use of the vector-dominance model of Gell-Mann, Sharp, and Wagner,[8] the decay rate $\Gamma(\eta\to2\gamma)$ has been calculated elsewhere[9] using the asymptotic nonet symmetry predictions[10] for the dimensionless photon–vector-meson coupling constants. Assuming the $\pi^0\to2\gamma$ decay rate quoted above, it was found[9] that

$$\Gamma(\eta\to2\gamma)=(284\pm33)\text{ eV}, \tag{22}$$

which agrees with the most recent measurement[6] of this decay rate within experimental error.

In summary, within the framework of the $(3,\bar{3})+(\bar{3},3)$ model[2,3] of chiral-symmetry breaking, a more general argument for the vanishing of the η-X mixing angle has been presented than in I. The most recent measurement[6] of the decay rate $\eta\to2\gamma$ tends to support this argument. A resolution of the large discrepancy between the two experimental measurements[5,6] of the decay rate $\eta\to2\gamma$ seems to be a matter of considerable urgency.

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