Inelastic electron scattering in the symmetric quark model

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The electroproduction process $ep \to e N^*$ is studied, using the nonrelativistic symmetric quark model with various binding potentials. The following conclusions are obtained: (i) A modified Woods-Saxon potential $-V_0[(r/b)+1]/\{(r/b)+\exp[(r-R)/a]\}$ gives the best fit among various quark-quark potentials. (ii) This potential gives values correct for the elastic cross section and the cross section corresponding to the production of the first peak. However, this gives values a little low for the cross sections corresponding to the production of the second and third peaks.

I. INTRODUCTION

It has been known[1] for some time that the nonrelativistic quark model with a harmonic-oscillator potential gives Gaussian-type form factors for the processes $ep \to eN^*$. Therefore, the predicted magnitude is too small for large q^2. If a $1/r$ potential is used, then the elastic form factors are improved, but it gives values too small for the higher resonances.[2]

The cross sections calculated nonrelativistically depend on the frame in which the form factors are calculated. Thornber[1-3] calculated them in the N^* rest frame. Krammer[4] calculated the form factors in the Breit frame using an antisymmetric spatial wave function for the S state. Le Yaouanc *et al.*[5] pointed out that if the nonrelativistic form factors are calculated in the Breit frame or in the frame in which the nucleon and isobar have equal but opposite velocities, a harmonic-oscillator potential gives correct values for $d\sigma/d\Omega(N^*)/d\sigma/d\Omega(\text{elastic})$. However, $d\sigma/d\Omega(N^*)$ and $d\sigma/d\Omega(\text{elastic})$ themselves still strongly disagree with the experimental data. Abdullah *et al.*[6] applied this method to the neutrino reaction and predicted the ratio $d\sigma/d\Omega(\nu N \to N^* \mu^-)/d\sigma/d\Omega(\nu N \to p\mu^-)$.

Recently we have investigated [7,8] the electroproduction processes using a quark-diquark model. [In the previous papers[7-11] we have called our model a two-body baryon model. However, this nomenclature is not adequate since there are many kinds of "two-body baryon models" (e.g., Ref. 12). We feel that it is better to call our model a quark-diquark model using the terminology of Lichtenberg although the quark-diquark model examined extensively by Lichtenberg *et al.*[13] is not the same as our model.]

This model is closely related to the quark model. Relations of the magnetic moments among members of the lowest $\frac{1}{2}^+$ baryon octet in this model are completely the same as those predicted in the quark model if a relation $g_q = 4g_d$ holds[11] (g_q and g_d are the gyromagnetic ratios of quarks and diquarks, respectively). A quark-diquark model gives similar results to those of the quark model for the cross sections of the processes $ep \to eN^*$.[7,8]

Recently, we have found by a numerical method[7,8] that a modified Woods-Saxon potential, which has merits of both a harmonic-oscillator and a $1/r$ potential, gives good agreement with the experimental data except for the $\Delta_{33}(1236)$-production cross section, for which this model gives slightly higher values. In the quark-diquark model the masses of the quark and diquark are assumed to be very large.

In this paper we will show that also in the quark model the use of a modified Woods-Saxon potential improves the theoretical prediction appreciably.

In this case the cross section for the $\Delta_{33}(1236)$ production is also predicted correctly, and this potential gives reasonable energy levels for the baryon resonances.

In Sec. II, the basic formalism is briefly reviewed. In Sec. III, the cross sections are examined using various potentials and the results are compared with the experimental data. In Sec. IV, we present concluding remarks.

II. FORMALISM

The formalism to be used here has been described previously,[1,2,5,8] and only a brief summary will be given here. The generalization of the Bjorken-Walecka formula[5] is

$$\left.\frac{d\sigma}{d\Omega}\right|_{\rm lab}=\left(\frac{d\sigma}{d\Omega}\right)_{\rm NS}\frac{mM}{(\alpha E+\beta E')^2}\times\left[\frac{q^4}{\vec{q}^4}|f_c|^2+\left(\frac{q^2}{2\vec{q}^2}+\frac{(\alpha E+\beta E')^2}{m^2}\tan^2\frac{\theta}{2}\right)\times(|f_+|^2+|f_-|^2)\right],$$

$$\left(\frac{d\sigma}{d\Omega}\right)_{\rm NS}=\frac{e^4\cos^2(\frac{1}{2}\theta)}{4\epsilon^2\sin^4(\frac{1}{2}\theta)}\frac{1}{1+(2\epsilon/m)\sin^2(\frac{1}{2}\theta)}, \quad (1)$$

$$e^2=1/137.03,$$

where θ is the scattering angle of the electron, ϵ is the incident electron energy in the laboratory system, m is the proton mass, and M is the isobar mass. $(d\sigma/d\Omega)_{\rm NS}$ is the cross section for the elastic scattering of an electron by a spinless point proton. The frame in which nonrelativistic form factors are calculated is determined by the parameters α and β. $\vec{q}$, $\vec{P}$, $\vec{P}'$, E, and E' refer to this frame.[5]

$$\vec{P}'-\vec{P}=\vec{q},\quad \alpha+\beta=1,\quad \alpha\vec{P}+\beta\vec{P}'=0, \quad (2)$$

$$E=(m^2+\beta^2\vec{q}^2)^{1/2},\quad E'=(M^2+\alpha^2\vec{q}^2)^{1/2}, \quad (3)$$

$$\vec{q}^2=q^2+\frac{[\beta M^2-\alpha m^2+\frac{1}{2}(\alpha-\beta)(q^2+m^2+M^2)]^2}{\alpha^2m^2+\beta^2M^2+\alpha\beta(q^2+m^2+M^2)}. \quad (4)$$

We have the N^* rest frame for $\alpha=0$, $\beta=1$, the Breit frame (BF) for $\alpha=\beta=\frac{1}{2}$, and the least-velocity frame[14] (LVF) for $\alpha=M/(m+M)$, $\beta=m/(m+M)$. For the elastic nucleon form factors the scaling law holds only for the BF and LVF.[8] In the BF and LVF the quark and hadron velocities are smaller than those in the N^* rest frame and in the laboratory frame, so that the nonrelativistic approximation seems to be better in the former than the latter. Therefore, we consider only the BF and LVF hereafter.

Expressions for the form factors $|f_c|^2$ and $|f_+|^2+|f_-|^2$ are

$$|f_c|^2=2\pi\sum_{J=0}^{\infty}|\langle J_f\|\hat{M}_J^{\rm Coul}(|\vec{q}|)\|J_i\rangle|^2,$$

$$|f_+|^2+|f_-|^2=2\pi\sum_{J=1}^{\infty}[|\langle J_f\|\hat{T}_J^{\rm el}(|\vec{q}|)\|J_i\rangle|^2+|\langle J_f\|\hat{T}_J^{\rm mag}(|\vec{q}|)\|J_i\rangle|^2], \quad (5)$$

where

$$\hat{M}_{JM}^{\rm Coul}(|\vec{q}|)=\int d^3x\, j_J(|\vec{q}|x)Y_{JM}(\Omega_x)\hat{\rho}(\vec{x}),$$

$$\hat{T}_{JM}^{\rm el}(|\vec{q}|)=\frac{1}{|\vec{q}|}\int d^3x[\vec{\nabla}j_J(|\vec{q}|x)\vec{Y}_{JJ1}^{M}(\Omega_x)]\cdot\vec{\hat{J}}(\vec{x}), \quad (6)$$

$$\hat{T}_{JM}^{\rm mag}(|\vec{q}|)=\int d^3x\, j_J(|\vec{q}|x)\vec{Y}_{JJ1}^{M}(\Omega_x)\cdot\vec{\hat{J}}(\vec{x}).$$

Here $\vec{Y}_{JJ1}^{M}(\Omega_x)$ is a vector spherical harmonic and

$$\hat{\rho}(\vec{x})=\sum_{i=1}^{3}\hat{Q}(i)\delta(\vec{x}-\vec{r}_i),$$

$$\vec{\hat{J}}(\vec{x})=\sum_{j=1}^{3}\frac{\hat{Q}(j)}{2im_q}\{\delta(\vec{x}-\vec{r}_j)\vec{\nabla}+\vec{\nabla}\delta(\vec{x}-\vec{r}_j)\}+\vec{\nabla}\times\sum_{i=1}^{3}\hat{Q}(i)\mu_q\delta(\vec{x}-\vec{r}_i)\vec{\sigma}(i). \quad (7)$$

It is well-known that $\mu_q=\mu_p\approx-\frac{3}{2}\mu_n$.

Using the conventional quantum-number assignments shown in Table I, the shell-model wave function for each resonance is obtained. The c.m. motion is not separated out, but we will discuss this problem later. This assumption is the same as that used in Ref. 2.

Then the spatial wave functions are given by

$$\{56\}:\ \Psi|_0^0=Y_0^0(\Omega_1)Y_0^0(\Omega_2)Y_0^0(\Omega_3)R_0(r_1)R_0(r_2)R_0(r_3)\ \text{for } S \text{ state}, \quad (8)$$

$$\Psi|_L^M=(\tfrac{1}{3})^{1/2}[Y_L^M(\Omega_1)R_L(r_1)Y_0^0(\Omega_2)R_0(r_2)Y_0^0(\Omega_3)R_0(r_3)+Y_0^0(\Omega_1)R_0(r_1)Y_L^M(\Omega_2)R_L(r_2)Y_0^0(\Omega_3)R_0(r_3)$$
$$+Y_0^0(\Omega_1)R_0(r_1)Y_0^0(\Omega_2)R_0(r_2)Y_L^M(\Omega_3)R_L(r_3)]\ \text{for even } L(\neq 0)\ \text{states}; \quad (9)$$

$$\{70\}:\ \Psi'|_L^M=(\tfrac{1}{2})^{1/2}[Y_L^M(\Omega_1)R_L(r_1)Y_0^0(\Omega_2)R_0(r_2)Y_0^0(\Omega_3)R_0(r_3)-Y_0^0(\Omega_1)R_0(r_1)Y_L^M(\Omega_2)R_L(r_2)Y_0^0(\Omega_3)R_0(r_3)],$$

$$\Psi''|_L^M=(\tfrac{1}{6})^{1/2}[Y_L^M(\Omega_1)R_L(r_1)Y_0^0(\Omega_2)R_0(r_2)Y_0^0(\Omega_3)R_0(r_3)+Y_0^0(\Omega_1)R_0(r_1)Y_L^M(\Omega_2)R_L(r_2)Y_0^0(\Omega_3)R_0(r_3)$$
$$-2Y_0^0(\Omega_1)R_0(r_1)Y_0^0(\Omega_2)R_0(r_2)Y_L^M(\Omega_3)R_L(r_3)]\ \text{for odd } L\ \text{states}, \quad (10)$$

The form factor for each resonance is listed in Table I. We use the same notations as in Ref. 8:

$$I_{fi}=\int r^2R_{L_f}(r)j_{L_f}(|\vec{q}|r)R_S(r)dr, \quad (11)$$

$$\mu=\frac{g_q}{2m_q}, \quad (12)$$

$$A_{L_f}=\int r^2R_{L_f}(r)[j_{L_f+1}(|\vec{q}|r)+j_{L_f-1}(|\vec{q}|r)]\times\frac{\partial}{\partial r}R_S(r)dr, \quad (13)$$

TABLE I. Assignment of baryon resonances and form factors in the quark model.

Resonances	S	Excitation	SU(6)	$\lvert f_C\rvert^2$	$\lvert f_M\rvert^2$	$\lvert f_E\rvert^2$
$P_{11}(940)$	$\frac{1}{2}$	$1S$	{56}	I_{SS}^2	$2\vec{q}^2\mu_q^2 I_{SS}^2$	0
$P_{11}(1470)$	$\frac{1}{2}$	$2S$	{56}	$I_{2S,S}^2$	$2\vec{q}^2\mu_q^2 I_{2S,S}^2$	0
$D_{13}(1520)$	$\frac{1}{2}$	$1P$	{70}	$\frac{2}{3}I_{PS}^2$	$\vec{q}^2\mu_q^2 I_{PS}^2$	$\frac{1}{3}\vec{q}\mu_q^2\left(I_{SP}+\frac{4A_1}{3g_q\lvert\vec{q}\rvert}\right)^2$
$S_{11}(1535)$	$\frac{1}{2}$	$1P$	{70}	$\frac{1}{3}I_{PS}^2$	0	$\frac{2}{3}\vec{q}^2\mu_q^2\left(I_{SP}-\frac{2A_1}{3g_q\lvert\vec{q}\rvert}\right)^2$
$D_{15}(1680)$	$\frac{3}{2}$	$1P$	{70}	0	0	0
$D_{13}(1675)$	$\frac{3}{2}$	$1P$	{70}	0	0	0
$F_{15}(1690)$	$\frac{1}{2}$	$1D$	{56}	I_{DS}^2	$\frac{4}{3}\vec{q}^2\mu_q^2 I_{DS}^2$	$\frac{2}{3}\vec{q}^2\mu_q^2\left(I_{SD}+\frac{6A_2}{5g_q\lvert\vec{q}\rvert}\right)^2$
$S_{11}(1710)$	$\frac{3}{2}$	$1P$	{70}	0	0	0
$P_{11}(1751)$	$\frac{1}{2}$	$3S$	{56}	$I_{3S,S}^2$	$2\vec{q}^2\mu_q^2 I_{3S,S}^2$	0
$P_{13}(1860)$	$\frac{1}{2}$	$1D$	{56}	$\frac{2}{3}I_{DS}^2$	$\frac{1}{3}\vec{q}^2\mu_q^2 I_{DS}^2$	$\vec{q}^2\mu_q^2\left(I_{SD}-\frac{4A_2}{5g_q\lvert\vec{q}\rvert}\right)^2$
$G_{17}(2190)$	$\frac{1}{2}$	$1F$	{70}	$\frac{4}{3}I_{FS}^2$	$\frac{5}{3}\vec{q}^2\mu_q^2 I_{FS}^2$	$\vec{q}^2\mu_q^2\left(I_{SF}+\frac{8A_3}{7g_q\lvert\vec{q}\rvert}\right)^2$
$P_{33}(1236)$	$\frac{3}{2}$	$1S$	{56}	0	$\frac{16}{9}\vec{q}^2\mu_q^2 I_{SS}^2$	0
$S_{31}(1640)$	$\frac{1}{2}$	$1P$	{70}	$\frac{1}{3}I_{PS}^2$	0	$\frac{2}{27}\vec{q}^2\mu_q^2\left(I_{SP}+\frac{2A_1}{g_q\lvert\vec{q}\rvert}\right)^2$
$D_{33}(1691)$	$\frac{1}{2}$	$1P$	{70}	$\frac{2}{3}I_{PS}^2$	$\frac{1}{9}\vec{q}^2\mu_q^2 I_{PS}^2$	$\frac{1}{27}\vec{q}^2\mu_q^2\left(I_{SP}-\frac{4A_1}{g_q\lvert\vec{q}\rvert}\right)^2$
$F_{35}(1913)$	$\frac{3}{2}$	$1D$	{56}	0	$\frac{32}{189}\vec{q}^2\mu_q^2 I_{DS}^2$	$\frac{28}{27}\vec{q}^2\mu_q^2\left(I_{SD}-\frac{18A_2}{5g_q\lvert\vec{q}\rvert}\right)^2$
$P_{31}(1934)$	$\frac{3}{2}$	$1D$	{56}	0	$\frac{4}{27}\vec{q}^2\mu_q^2 I_{DS}^2$	0
$F_{37}(1950)$	$\frac{3}{2}$	$1D$	{56}	0	$\frac{64}{63}\vec{q}^2\mu_q^2 I_{DS}^2$	0

$$|f_M|^2 = 2\pi\sum_{J=0}^{\infty} |\langle J_f \| \hat{T}_J^{\text{mag}} \| J_i\rangle|^2\,, \tag{14}$$

$$|f_E|^2 = 2\pi\sum_{J=0}^{\infty} |\langle J_f \| \hat{T}_J^{\text{el}} \| J_i\rangle|^2\,, \tag{15}$$

$$|f_+|^2 + |f_-|^2 = |f_M|^2 + |f_E|^2\,. \tag{16}$$

III. RESULTS AND DISCUSSIONS

A. Potentials

First we calculate the cross sections using the following approximations: (1) The mass of the quark is so large that we can omit the terms of order $1/m_q$. (2) We do not separate out the c.m. motion.

Then the results of the quark model are closely related to those of the quark-diquark model.[8] From Table I we can show, using approximation (1),

$$|f_C|^2 \propto I_{fi}^{\ 2}\,,$$
$$|f_+|^2 + |f_-|^2 \propto I_{fi}^{\ 2}\,. \tag{17}$$

Let us review the properties of I_{fi} for various potentials.

(a) Harmonic-oscillator potential (HOP). This potential gives a Gaussian form factor and gives values too low for large q^2. This result is the same for all nonsingular potentials.

(b) 1/r potential. The potential

$$V(r) = -\frac{V_0}{r} \tag{18}$$

gives for the elastic form factors

$$\frac{G_M}{\mu_p} = G_E = I_{SS} = \frac{1}{(1+q^2/4V_0^{\,2}m_q^{\,2})^2}\,. \tag{19}$$

This formula is the famous dipole formula and by comparison with the experimental data we obtain $4m_q^{\,2}V_0^{\,2} = 0.71\ (\text{GeV}/c)^2$. However, this potential gives values too low for the cross sections corresponding to the production of resonances with $L_f \geq 1$, because a $1/r$ potential is too shallow in the outer region.[8]

(c) Modified Woods-Saxon potential (MWP). The potential possesses the merits of both HOP and of a $1/r$ potential:

$$V(r) = -V_0\,\frac{(r/b)+1}{(r/b)+e^{(r-R)/a}}\,. \tag{20}$$

For small values of r this potential approaches a $1/r$ potential and for $r\sim R$ it approaches a Woods-Saxon potential. We set the parameters as follows:

Set (I): $R = 1.4$ fm , $a = 0.05R$, $b = 0.15R$,

$$V_0 = 2 \times 9.38/(m_q R^2).$$

This set of parameters [referred to as (I)] corresponds to the set (B) in Ref. 8.

B. Comparison with experimental results

Now let us compare these results with experimental data. The quantities that we are interested in here are the elastic form factors and the ratio

$$\left.\frac{d\sigma}{d\Omega}\right|_{\text{in}} \bigg/ \left.\frac{d\sigma}{d\Omega}\right|_{\text{el}} \text{ for } \theta = 6^\circ \,, \tag{21}$$

where $d\sigma/d\Omega|_{\text{in}}$ is the cross section for the process $ep \to eN^*$, and $d\sigma/d\Omega|_{\text{el}}$ is the elastic cross section for the same q^2. Although $d\sigma/d\Omega|_{\text{in}}$ drops steeply with q^2, the ratio $d\sigma_{\text{in}}/d\sigma_{\text{el}}$ is nearly constant in a wide range $1 \lesssim q^2 \lesssim 7$ $(\text{GeV}/c)^2$.

We use, for the elastic cross section,

$$\left.\frac{d\sigma}{d\Omega}\right|_{\text{el}} = \left(\frac{d\sigma}{d\Omega}\right)_{\text{NS}} \left(\frac{G_E{}^2 + \tau G_M{}^2}{1+\tau} + 2\tau G_M{}^2 \tan^2(\tfrac{1}{2}\theta)\right),$$

$$G_E = \frac{G_M}{\mu_p} = \left(\frac{1}{1+q^2/0.71}\right)^2, \quad \tau = \frac{q^2}{4m^2} \tag{22}$$

for both the theoretical and experimental values. Experimental data are taken from Breidenbach,[15] Drees,[16] and Mo.[17] All of these data are based on the SLAC-MIT experiments but the fitting procedures are slightly different. The plots of the theoretical predictions are given by HOP, a $1/r$ potential, and MWP(I). The nonrelativistic form factors are calculated in the BF and LVF for each case.

(a) Elastic scattering (Fig. 1). The results are completely the same as those of the quark-diquark model. HOP gives values too low for large q^2, a $1/r$ potential gives the dipole formula, and MWP gives values very close to the dipole formula.

(b) First peak (Fig. 2). This peak corresponds to $\Delta_{33}(1236)$ resonance. HOP gives values too small for large q^2, but a $1/r$ potential and MWP give a good agreement. In the quark-diquark model the latter potentials give values which are slightly too large.[8]

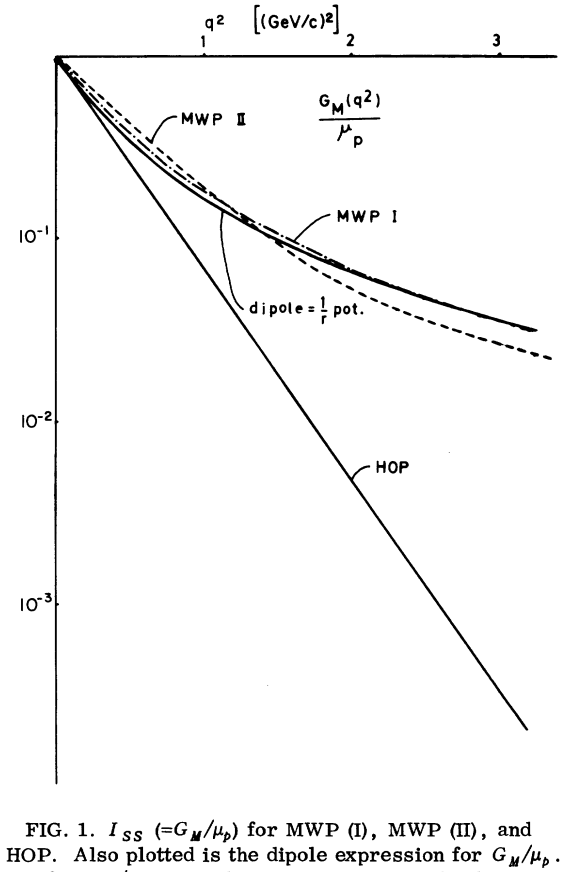

FIG. 1. I_{SS} $(=G_M/\mu_p)$ for MWP (I), MWP (II), and HOP. Also plotted is the dipole expression for G_M/μ_p. I_{SS} for a $1/r$ potential is the same as the dipole expression.

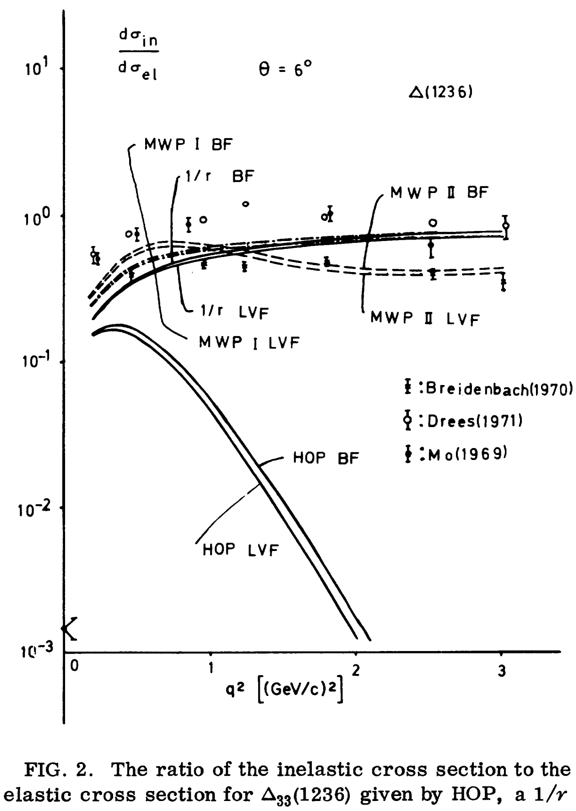

FIG. 2. The ratio of the inelastic cross section to the elastic cross section for $\Delta_{33}(1236)$ given by HOP, a $1/r$ potential, MWP(I), and MWP(II) in the BF and LVF.

(c) The second and third peaks (Figs. 3, 4). The second peak is composed of $D_{13}(1520)$ and $S_{11}(1535)$. $D_{13}(1520)$ dominates this bump in the quark model as well as in the quark-diquark model. The third peak is composed of $D_{15}(1680)$, $D_{13}(1675)$, $F_{15}(1690)$, $S_{11}(1710)$, $S_{31}(1640)$, and $D_{33}(1691)$. Owing to the Moorhouse selection rule[18] (this selection rule does not hold in the quark-diquark model), the matrix elements of $D_{15}(1680)$, $D_{13}(1675)$, and $S_{11}(1710)$ vanish. Among the remaining resonances, $F_{15}(1690)$ dominates this peak in the quark model as well as in the quark-diquark model. For these two peaks HOP gives values too small for large q^2 and a $1/r$ potential gives values too low for all q^2. The theoretically predicted values are greatly improved by MWP(I) but they are still too small.

In the quark-diquark model MWP gives a satisfactory agreement with the experimental data, but in the quark model we cannot obtain such a satisfactory agreement, especially for the second and third peaks. One might think that the set of parameters (I) is not suitable.

In order to increase the cross sections for the second and third peaks we must decrease the radius R where the potential is cut off (see Ref. 8). Therefore, let us choose another set of parameters.

Set (II): $R = 1.1$ fm, $a = 0.03R$, $b = 0.07R$,

$$V_0 = 23.5/(m_q R^2).$$

Since the "range" of the MWP(II) is smaller than that of MWP(I), even the S state is distorted and the elastic form factor deviates from the dipole formula considerably.

The results given by MWP(II) are plotted in Fig. 1 (elastic form factor), Fig. 2 (the first peak), Fig. 5 (the second peak), and Fig. 6 (the third peak). Although the cross sections for the second and third peaks increase, their shapes are badly distorted and disagree with the experimental data. Therefore, it is not helpful to change R for this purpose.

We have further investigated changing the bottom shape of the potential; for example,

$$V(r) = -V_0 \frac{(r/b)^\gamma + 1}{(r/b)^\gamma + e^{(r-R)/a}}. \qquad (23)$$

For $\gamma = 1$ the potential (23) reduces to (20). The agreement with experiment is not greatly im-

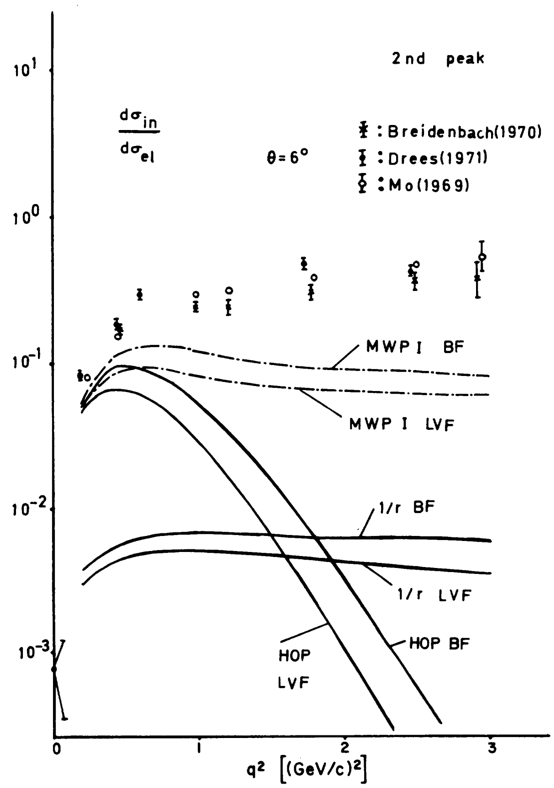

FIG. 3. The ratio of the inelastic cross section to the elastic cross section for the second peak given by HOP, a $1/r$ potential, and MWP(I) in the BF and LVF with $m_q = \infty$.

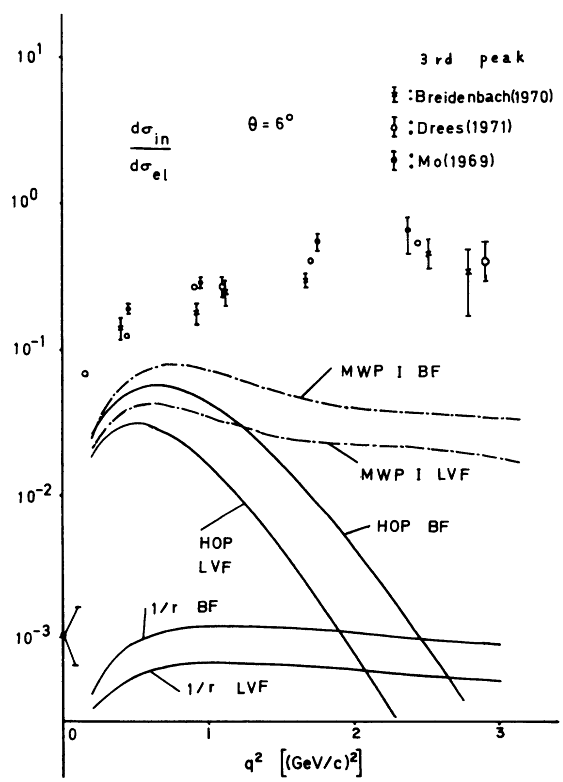

FIG. 4. The same as Fig. 3 but with a different peak, with $m_q = \infty$.

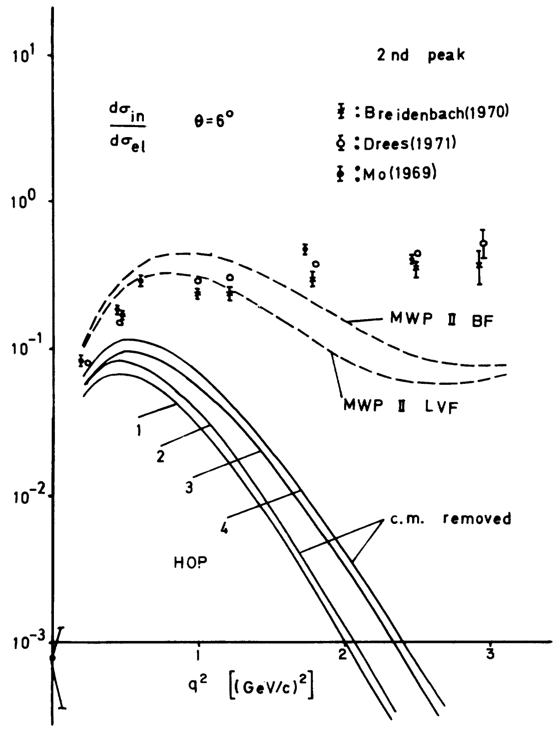

FIG. 5. The ratio of the inelastic cross section to the elastic cross section for the second peak. The plotted theoretical curves are curve 1, given by HOP in the LVF (the c.m. motion is not separated); curve 2, given by HOP in the LVF (the c.m. motion is separated); curve 3, given by HOP in the BF (the c.m. motion is not separated); and curve 4, given by HOP in the BF (the c.m. motion is separated). The other curves are given by MWP(II) in the BF and LVF with $m_q=\infty$.

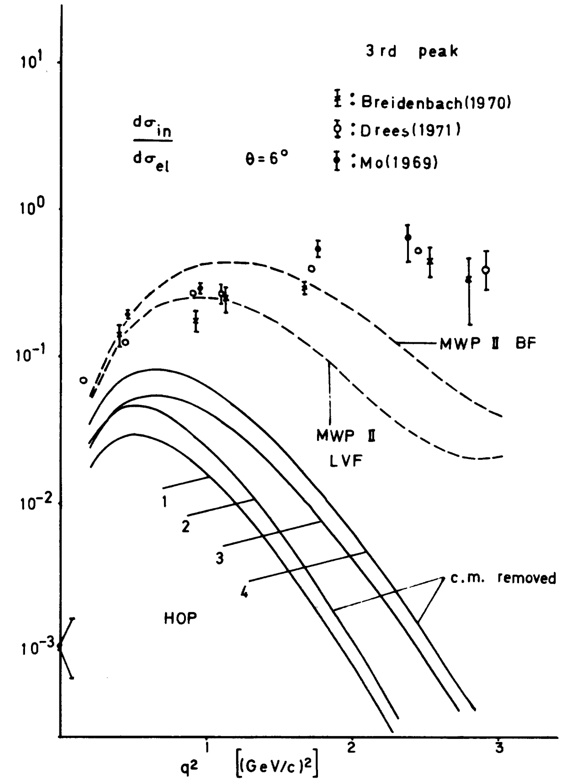

FIG. 6. The same as Fig. 5 but with a different peak.

proved by changing γ. In fact, the dipole formula is given only by a $1/r$ potential, and if we change the form of the bottom of MWP, the elastic form factor inevitably deviates from the dipole formula.

Thus we fail to obtain a good agreement by using two approximations presented in Sec. III A. Next we comment on them.

C. The separation of the c.m. motion

In the quark-diquark model the separation of the c.m. motion is easy and can be done analytically since the system consists of two particles. However, in the quark model the system consists of three particles and for general potentials the c.m. motion cannot be separated out analytically. For HOP the effect of separating out the c.m. motion is to multiply every form factor by a factor $\exp(\vec{q}^2/12\alpha^2)$ (α is the spring constant). Although this correction is not small, our predicted values are not greatly affected since in our model the spring constant α is determined by the elastic form factor in the limit of $q\to 0$.

The corrected α is smaller than the uncorrected α. Therefore, the cross sections after removing the c.m. motion are slightly larger than the previous ones (see Figs. 5 and 6). In this sense the correction is small at least for HOP and we believe that the corrections for the general potentials are small enough so that the conclusions in this paper are not altered.

IV. CONCLUDING REMARKS

The following conclusions are obtained: The results given by MWP are much better than those given by HOP or a $1/r$ potential.

If we use the same parameters for MWP as was used in the quark-diquark model,[8] MWP gives values correct for elastic cross section and the cross section corresponding to the production of the first peak. However, the predicted cross sections corresponding to the productions of the second and third peaks are too small.

It seems difficult to obtain a complete agreement with the experimental result by changing parameters of MWP.

Although we have neglected the c.m. motion, this correction may improve the theoretical prediction as is the case for HOP. Further, we have used the approximation $m_q=\infty$. If we assume, instead, that the mass of the quark is $m_p/2.793$, i.e., $g_q=1$, the predicted values further approach the experimental data. However, in this case the nonrelativistic approximation becomes poor.

Let us make one more remark. For $m_q=m_p/2.793$ the energy interval between the S state and P state in MWP is about 400 MeV, roughly in agreement with the interval between the S states [P(938) and Δ(1236)] and the P states [N(1520), N(1535), N(1670), and N(1700)]. However, if we assume that the quark is very heavy, say, 30 GeV, then the interval becomes only 4 MeV.

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