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Quantum numbers and quark-parton fragmentation models*

Robert N. Cahn† and E. William Colglazier‡
Stanford Linear Accelerator Center, Stanford, California, 94305
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Cascade models for quark and hadron fragmentation are displayed. These models illustrate many of the predictions of the parton model for inclusive reactions. Multiplicities are constructed to go as $C_h \ln(s/m^2)$ in hadron-hadron collisions and as $C_{e^+e^-}\ln(Q^2/m^2)$ in e^+e^- annihilation, implying a multiplicity in leptoproduction of $C_{e^+e^-}\ln(Q^2/m^2) + C_h\ln(\omega-1)$. However, Feynman's conjecture that quark quantum numbers are retained, on the average, in the parton fragmentation region is not necessarily true. This was first noted by Farrar and Rosner in a model with meson emission only. The conjecture (as a general principle) is shown to fail as well with baryon emission included if multiplicities grow no faster than logarithmically. In cascade models a weaker version of Feynman's conjecture is found to be true in general and this version is accessible experimentally. Also, triality is found to play a significant role, suggesting that $C_{e^+e^-}$ need not equal C_h. Other implications of cascade models are also explored for hadron-hadron collisions, e^+e^- annihilation, and leptoproduction, for both small and large transverse momentum of the produced particles.

I. INTRODUCTION

The hypothesis of limiting fragmentation[1] or Feynman scaling[2] has been eminently successful in describing high-energy collisions when only one final-state particle is observed.[3] The hypothesis suggests that at very high energy a struck hadron will fragment in a fashion independent of the energy and type of the particle striking it. More precisely, for a final-state particle with longitudinal momentum a finite fraction z of the beam momentum, the Lorentz-invariant inclusive cross section $(E/\sigma)\, d\sigma/d^3p$ is expected to become a function only of z and the transverse momentum $p_\perp$. In parton models, similar behavior is predicted for the parton struck by the current in leptoproduction or produced in e^+e^- annihilation.[4-6] In these instances, an isolated (and unobservable) parton is converted into observed final-state hadrons, which are anticipated to have a distribution independent of the initial state and determined only by the parton type and by kinematic variables analogous to $p_\perp$ and z.[7]

Two characteristic features of hadron fragmentation are the existence of a flat plateau in rapidity (dz/z distribution for small z) and the retention of quantum numbers, on the average, in the fragmentation region. Berman, Bjorken, and Kogut[4] and Feynman[5] have speculated that parton fragmentation should also develop a plateau. In addition, Feynman[5] has suggested that the quantum numbers of the (quark) parton are retained, on the average, in the fragmentation region. The existence of a flat, nonzero plateau would imply a $\ln Q^2$ contribution to the multiplicity in e^+e^- annihilation and in leptoproduction from the current-fragmentation region.[8] The retention of fractional quark quantum numbers, on the average, in the parton-fragmentation region would be a striking indication of a quark substructure for the hadrons even if quarks are not seen. A dynamical mechanism which could produce a plateau in the current-fragmentation region is not at all understood; in fact most traditional calculable models lead to a finite multiplicity—that is a zero plateau—for this region.[9] However, we shall assume that the multiplicities from current fragmentation are logarithmic in Q^2 (which makes possible Feynman scaling while avoiding the problem of observing particles with quarklike quantum numbers). With this constraint, we construct cascade models to describe parton and hadron fragmentation which are useful for studying Feynman's quantum-number-retention hypothesis as well as other properties of inclusive lepton-hadron reactions.

We find that Feynman's quantum-number conjecture for parton fragmentation is not true in general in our models, although it could happen "accidentally." This was first noted by Farrar and Rosner[10] in a model in which fragmenting partons produce only mesons. Although a small amount of baryon emission can save the conjecture, the multiplicity is then forced to increase too quickly with Q^2 (see Appendix). With logarithmic multiplicities, only a weaker version of the conjecture is valid (see below), but it does provide an experimental test, although not as striking as that of the original proposal.

The major conclusion is that triality plays a central role in cascade processes. All triality +1 cascades evolve into a particular asymptotic form which is the charge conjugate of the asymptotic form of triality -1 cascades, but not necessarily related to the asymptotic form of triality zero cascades. Thus the coefficients of $\ln Q^2$ and $\ln s$ in the multiplicities in e^+e^- annihilation and pp collisions need not be the same. In general, the quantum-number-retention hypothesis need hold only for cascades which become eigenstates of charge conjugation asymptotically, as in the triality 0 cascade from a fragmenting hadron. In the case of triality +1 cascades, the difference between the quantum numbers of the quark and those left in its fragmentation region is a constant, independent of the quark type. That constant is not necessarily zero and is not known *a priori*. Consequently, the baryon number or electric charge left in the parton-fragmentation region cannot be predicted.

II. FRAGMENTATION AS A CASCADE

The models we use to describe fragmentation may heuristically be described as cascades. The fast-moving hadron or quark which fragments is pictured as throwing off particles in a cascade which proceeds towards lower rapidities. Feynman, Bjorken, and others introduced the concept to avoid the problem of observing particles with quarklike quantum numbers. Consider, for example, e^+e^- annihilation into hadrons, which proceeds via a $q\bar{q}$ intermediate in the quark-parton model. If asymptotically each quark fragments into a finite number of hadrons separated by a gap in longitudinal momentum, the fractional quantum numbers must appear in the final state. Consequently, it was proposed that the quark and the antiquark initiate cascades which terminate when they meet by annihilating the quarklike quantum numbers. As Feynman speculated, the quantum numbers of the quark could be retained in the fragmentation region on the average.

These cascades may be thought of as a stepwise process that deposits final-state hadrons (or partons which are then converted into final-state hadrons) at each step in rapidity. For example, if the cascade occurs stepwise though emissions (like $q \to qM$ and $q \to \bar{q}\bar{q}B$, where M is a meson and B is a baryon) in which the products (including the quarks which continue cascading) share the initial momentum, then each step corresponds to a finite step in rapidity. The density of emitted particles per unit rapidity is presumed to become constant away from the initiating end of the cascade (the assumption of a plateau).

We are not prepared to say whether such a stepwise process ought to be imagined to occur in physical space-time or whether it is really a mnemonic for some transformation between two representations of physical states—one as hadrons and one as quark partons. A literal interpretation in space-time may lead to problems if the q and $\bar{q}$ systems get so far apart at high energies that annihilation and removal of the quark quantum numbers are impossible. However, the use of the cascade to represent the transformation of the fragmenting particle into final-state hadrons is more general than a specific space-time evolution.

An example of a cascade and its mathematical description can be seen by reformulating a model given by Feynman[5] in his book. Feynman considers a simple model for e^+e^- annihilation in which the rapidity gap between the initial quark and antiquark is filled with N isosinglet $\overline{q}q$ pairs ($N \propto$ rapidity). Adjacent quarks and antiquarks (beginning at either end) are then assumed to convert into pions [see Fig. 1(a)]. Feynman uses a density-matrix formalism to show that, on the average, the z component of isospin of all the pions to the left (insensitive to where in the plateau the average is stopped) is the I_z of the left-moving fragmenting quark. This model can easily be cast into a cascade formalism to derive the same results. The first step of the cascade is the initial quark throwing off the first pion and producing a quark, which then initiates the second step, etc. [see Fig. 1(b)]. Various alternatives are offered at each step depending on the type of pion (π^+, π^0, or π^-) produced. Since a particular state after a certain number of steps is uniquely labeled by the initial quark (produced incoherently in the parton model) and the position and type of each pion in rapidity, there is no interference between states. Consequently, probabilities can be used in the place of amplitudes to describe the cascade. Moreover, the quantum numbers deposited in the hadronic final state after n steps is calculable solely from knowledge of the initial quark and the quark present at the nth step. Thus a probability vector representing the type of quark present at a particular step and a matrix describing the transition to the next step are sufficient to describe the cascade process. The quantum numbers deposited in the fragmentation region are equal to those of the initial particle if the probability vector at the end of the cascade ($N \to \infty$) is neutral in those quantum numbers. With only $\mathcal{P}$ and $\mathcal{N}$ quarks, as in Feynman's example, the probabilities for emission at each step are

$$\begin{aligned} &P(\mathcal{P} \to \mathcal{P}\pi^0) = \tfrac{1}{3} , \\ &P(\mathcal{P} \to \mathcal{N}\pi^+) = \tfrac{2}{3} , \\ &P(\mathcal{N} \to \mathcal{N}\pi^0) = \tfrac{1}{3} , \\ &P(\mathcal{N} \to \mathcal{P}\pi^-) = \tfrac{2}{3} . \end{aligned} \tag{1}$$

Then if the probability of having a $\mathcal{P}$ or $\mathcal{N}$ quark present at the Nth step is represented by a two-component vector

$$P_N = \begin{pmatrix} P(\mathcal{P}) \\ P(\mathcal{N}) \end{pmatrix} \tag{2}$$

the probability vector at the $N+1$st step is

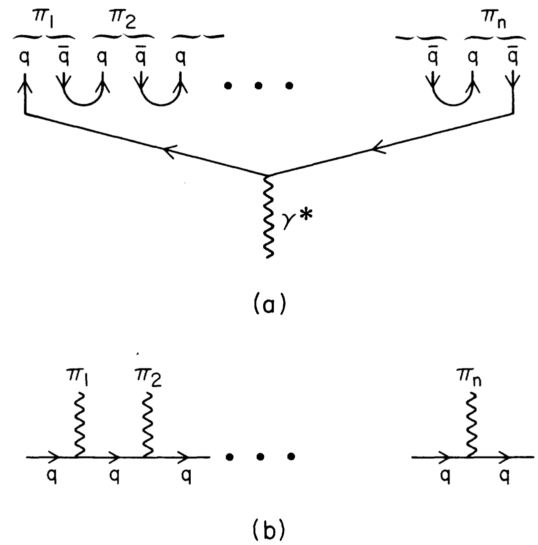

FIG. 1. (a) A simple model for hadronic final states in e^+e^- annihilation. The rapidity gap between the initial quark and antiquark is filled with N isosinglet $q\overline{q}$ ($N \propto$ rapidity gap). Adjacent quarks and antiquarks (beginning at either end) are assumed to convert into pions. (b) The above model pictured as a cascade. The first step is the initial quark throwing off the first pion and producing a quark which initiates the second step.

$$P_{N+1} = \begin{pmatrix} \frac{1}{3} & \frac{2}{3} \\ \frac{2}{3} & \frac{1}{3} \end{pmatrix} P_N \equiv TP_N . \tag{3}$$

The eigenvectors of T are $u_1 = \binom{1}{1}$ and $u_2 = \binom{1}{-1}$ with eigenvalues $\lambda_1 = 1$ and $\lambda_2 = -\frac{1}{3}$, respectively. Thus if

$$P_0 = \binom{1}{0} = \tfrac{1}{2}(u_1 + u_2) , \tag{4}$$

then

$$P_N = \tfrac{1}{2}u_1 + (-\tfrac{1}{3})^N \tfrac{1}{2} u_2 \xrightarrow{N \to \infty} \tfrac{1}{2} u_1 . \tag{5}$$

Therefore, the z component of isospin carried by P_∞ (limit of P_N as $N \to \infty$) is zero, which implies that the isospin of the initial quark is retained, on the average, in the fragmentation region. Feynman's hypothesis works, in this model, for the z component of isospin.[11]

However, Farrar and Rosner[10] showed that in certain cases Feynman's hypothesis is not true for electric charge. Their devastatingly simple argument paraphrased in terms of the above model[12] is that P_∞ carries electric charge, i.e.,

$$Q(P_\infty) = \tfrac{1}{6} ,$$

so that the average charge left in the fragmentation region (ΔQ) is

$$\Delta Q = Q(P_0) - Q(P_\infty) = \tfrac{1}{2} . \tag{6}$$

This counterexample destroys the hypothesis as a general principle.

When λ quarks are included, $Q(P_\infty)$ can be zero if SU(3) is exact; but this is an unlikely assumption since many more pions than kaons are expected in the plateau (in analogy with hadronic reactions). The general model with cascade steps of the form $q \to Mq$ is described by the following probability vector and cascade matrix:

$$P_N = \begin{pmatrix} P(\mathcal{P}) \\ P(\mathfrak{N}) \\ P(\lambda) \end{pmatrix} ,$$
$$T = \begin{pmatrix} a & b & c \\ b & a & c \\ 1-a-b & 1-a-b & 1-2c \end{pmatrix} . \tag{7}$$

The largest eigenvalue and its eigenvector are

$$\lambda_1 = 1, \quad u_1 = \begin{pmatrix} c \\ c \\ 1-a-b \end{pmatrix} , \tag{8}$$

so that $Q(P_\infty) \neq 0$ unless $a+b+c=1$ and $c \neq 0$ (the second condition ensures that the leading eigenvalue is nondegenerate).

That Feynman's hypothesis fails for models with only mesons emitted is obvious from consideration of baryon number. Since baryons are not produced, baryon number ($+\frac{1}{3}$ for q, $-\frac{1}{3}$ for $\overline{q}$) cannot be retained in the fragmentation region, contrary to the hypothesis. Moreover, the failure for electric charge then follows from the Gell-Mann–Nishijima relation since the hypothesis holds for I_z and fails for $Y = B + S$ [unless there is a compensating failure for S, as in the exact SU(3) version of the meson-emission model].

This suggests that the hypothesis might be valid if baryon emission is included. In fact, as we show in the Appendix, any amount of baryon emission resurrects Feynman's hypothesis in models where the cascade is a discrete branching Markov process (quarks cascading independently). Unfortunately, the independence assumption causes an avalanche of quarks, resulting in a multiplicity which grows as a power of Q^2. The reason for the success of the hypothesis in these models is that the number of quarks in the cascade increases so rapidly that the densities of quarks and antiquarks become equal. However, we must examine the consequences of baryon emission in more realistic models where the cascade saturates (where there is a finite number of quarks in the asymptotic cascade) to produce a logarithmic multiplicity.

We display next an example of the cascade with baryon emission and logarithmic multiplicity which can be solved in a fashion similar to the meson-emission case. Once that is done we shall be able to conclude what will result in more complex models with more degrees of freedom. Consider a model in which the "quarks" are SU(3) singlets, carrying only baryon number, $+\frac{1}{3}$ for quarks and $-\frac{1}{3}$ for antiquarks. Again suppose that a single quark q has been isolated, as in e^+e^- annihilation or electroproduction. Now we shall suppose that in a cascade step the q can emit a meson M and continue as a q, or emit a baryon B and continue as a $\overline{q}\overline{q}$ state. The $\overline{q}\overline{q}$ state we shall assume can emit a meson (or mesons) and continue as $\overline{q}\overline{q}$, or emit an antibaryon and continue as a q.[13] Thus we have a closed system (setting aside the emitted hadrons), which asymptotically produces a constant density of final-state hadrons in rapidity. Also, we may use probabilities rather than amplitudes since there is no interference. Let the probabilities for emission at each step be[14]

$$\begin{aligned} &P(q \to Mq) = \alpha , \\ &P(q \to B\overline{q}\overline{q}) = 1-\alpha , \\ &P(\overline{q}\overline{q} \to M\overline{q}\overline{q}) = \beta , \\ &P(\overline{q}\overline{q} \to \overline{B}q) = 1-\beta . \end{aligned} \tag{9}$$

Then if the probabilities of having a q or $\overline{q}\overline{q}$ present at the Nth step are represented as a vector,

$$P_N = \begin{pmatrix} P(q) \\ P(\overline{q}\overline{q}) \end{pmatrix}_N , \tag{10}$$

we have

$$P_{N+1} = \begin{pmatrix} \alpha & 1-\beta \\ 1-\alpha & \beta \end{pmatrix} P_N \tag{11}$$
$$= TP_N .$$

The eigenvectors of T are

$$u_1 = \begin{pmatrix} 1 \\ \dfrac{1-\alpha}{1-\beta} \end{pmatrix}$$

and

$$u_2 = \begin{pmatrix} 1 \\ -1 \end{pmatrix} ,$$

with eigenvalues $\lambda_1 = 1$ and $\lambda_2 = \alpha + \beta - 1$, respectively. Thus if

$$P_0 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$
$$= \frac{1-\beta}{2-\alpha-\beta} \left[u_1 + \left(\frac{1-\alpha}{1-\beta} \right) u_2 \right] , \tag{12}$$

then

$$P_N = \frac{1-\beta}{2-\alpha-\beta}\left[u_1 + \lambda_2{}^N\left(\frac{1-\alpha}{1-\beta}\right)u_2\right]. \tag{13}$$

If either α or β is less than 1, λ_2 is also less than 1 and the eigenvalues are nondegenerate. Thus

$$\lim_{N\to\infty} P_N = P_\infty = \frac{1-\beta}{2-\alpha-\beta}u_1 = \begin{pmatrix} \dfrac{1-\beta}{2-\alpha-\beta} \\ \dfrac{1-\alpha}{2-\alpha-\beta} \end{pmatrix}. \tag{14}$$

The baryon number carried by P_∞ is

$$\frac{1}{3}\left(\frac{2\alpha-\beta-1}{2-\alpha-\beta}\right), \tag{15}$$

so that the baryon number emitted is

$$P_0 - P_\infty = \frac{1-\alpha}{2-\alpha-\beta}. \tag{16}$$

Feynman's conjecture that the baryon number of the original quark ($+\frac{1}{3}$) is left in the emitted hadrons fails unless $2\alpha = 1+\beta$. Since there is no *a priori* reason for this constraint we see that the conjecture is not a general property of cascade models with logarithmic multiplicities.

Consider on the other hand a system with triality 0. For simplicity, we examine only baryon number and suppose that a hadron fragments by cascades among the following states: baryon (qqq), meson ($q\bar{q}$), and antibaryon ($\bar{q}\bar{q}\bar{q}$). Each of these states can be connected to the other two at each cascade step by depositing hadronic quantum numbers at that position in rapidity. Starting with a particular state, the cascade is again described by a probability vector and a transition matrix, i.e.,

$$\begin{pmatrix} P(qqq) \\ P(\bar{q}\bar{q}\bar{q}) \\ P(q\bar{q}) \end{pmatrix}_{N+1} = \begin{pmatrix} \alpha & \beta & \gamma \\ \beta & \alpha & \gamma \\ 1-\alpha-\beta & 1-\alpha-\beta & 1-2\gamma \end{pmatrix} \times \begin{pmatrix} P(qqq) \\ P(\bar{q}\bar{q}\bar{q}) \\ P(q\bar{q}) \end{pmatrix}_N. \tag{17}$$

The eigenvalues and eigenvectors of the cascade matrix are

$$u_1 = \begin{pmatrix} \gamma \\ \gamma \\ 1-\alpha-\beta \end{pmatrix}, \quad \lambda_1 = 1,$$
$$u_2 = \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix}, \quad \lambda_2 = \alpha+\beta-2\gamma, \tag{18}$$
$$u_3 = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}, \quad \lambda_3 = \alpha-\beta.$$

Excluding special cases which produce degenerate eigenvalues, P_∞ is proportional to u_1. Since u_1 carries no baryon number, the initial baryon number must have been emitted into hadrons, on the average, in contrast with the case of a fragmenting quark. Of course, the retention of quantum numbers in a hadronic fragmentation region is well known from the ideas of limiting fragmentation and Regge-Mueller theory.

In terms of a cascade, the reason for the quantum-number retention for triality 0 and not for triality ± 1 is charge-conjugation invariance. The matrices describing the triality $+1$ and 0 cascades transform under charge conjugation as follows:

$$CT_0C^{-1} = T_0,$$
$$CT_{+1}C^{-1} = T_{-1}, \tag{19}$$
$$CT_{-1}C^{-1} = T_{+1}.$$

For triality 0, nondegenerate eigenvectors of T_0 must be eigenvectors of C with eigenvalue $+1$. As a result, P_∞ must be neutral in additive quantum numbers like B, which then implies the quantum-number-retention hypothesis. For nonzero triality, P_∞ is not an eigenvector of C, and consequently is not necessarily neutral with respect to B or Q. Another way of stating the condition sufficient for satisfying the hypothesis is as follows: If a particle and its antiparticle are connected through the cascade, then the quantum numbers of the fragmenting particle are retained in its fragmentation region. For example, a cascade step cannot connect a q with a $\bar{q}$ ($q\to\bar{q}$ + final-state particles) or else fractionally charged particles would appear in the final state. In the case of hadronic fragmentation, particle and antiparticle can be connected through the cascade. (Note that in a special model where baryons and antibaryons are not connected through the cascade the quantum-number hypothesis fails for hadronic fragmentation; this case corresponds to a cascade matrix with degenerate leading eigenvalues.)

The above results can be generalized as follows: We assume that cascades develop in a stepwise process of the form $P_{N+1}=T_t P_N$, where t is the triality (±1,0). The probability vectors P_N give the probabilities for various states, which we assume to be finite in number, to be occupied after N steps. We assume that the cascade matrix T_t has a unique leading eigenvector, which must have an eigenvalue unity. Physically, this means we assume that there is a unique cascade for each triality which developes asymptotically. For example, a cascade begun with a $\mathcal{P}$ quark will develop asymptotically into the same cascade as an $\mathfrak{N}$ or λ quark or any state with triality +1. The asymptotic state, P^t_∞, is independent of the fragmenting particle except for its triality. Because $CT_{+1}C^{-1}=T_{-1}$, the asymptotic cascades for triality +1 and −1 are simply charge conjugates. Let the asymptotic probability vectors be P^+_∞, P^-_∞, and P^0_∞ for triality +1, −1, and 0, respectively. A cascade begun, say, by a $\mathcal{P}$ quark could be represented by

$$P_{N+1}=T_{+1}P_N\ ,$$
$$P_0=(\text{Prob}(\mathcal{P})=1,\ \text{Prob}(\mathfrak{N},\lambda,\bar{\mathcal{P}}\bar{\mathfrak{N}},\text{etc.})=0)\ . \tag{20}$$

The amount of any additive quantum number Q left in the fragmentation region would be $\Delta Q=Q(P_0)-Q(P^+_\infty)$. Feynman's hypothesis was that $Q(P^+_\infty)=0$ for all additive quantum numbers Q. If this were the case, then ΔQ would be the quantum number of the quark initiating the cascade. As we have seen, this is not necessarily true. What we can expect is that if we compare cascades initiated by different states with the same triality, then $\Delta Q-Q(P_0)$ is universal. This result follows from the uniqueness of the asymptotic state; the quantum-number hypothesis fails when this state is not neutral. In other words,

$$\Delta Q(\mathcal{P})-Q(\mathcal{P})=\Delta Q(\mathfrak{N})-Q(\mathfrak{N})$$
$$=\Delta Q(\lambda)-Q(\lambda),\quad \text{etc.} \tag{21}$$

For example, the electric charge left in the fragmentation region of a $\mathcal{P}$ quark should be one greater than that left in the fragmentation region of an $\mathfrak{N}$ quark or a λ quark. Similar results follow for the other additive quantum numbers. Also note that charge conjugation implies $Q(P^+_\infty)=-Q(P^+_\infty)$ so that $\Delta Q(q)=-\Delta Q(\bar{q})$. Of course a similar argument in the case of triality zero cascades implies the asymptotic state P^0_∞ is neutral, i.e., $\Delta Q(\text{hadron})=Q(\text{hadron})$.

For the case $Q=I_z$ the situation is somewhat different from the cases $Q=B$ and $Q=Y$. Since T_t must be invariant under a reflection in isospin space (charge symmetry), P^+_∞, P^-_∞, and P^0_∞ must have $I_z=0$, i.e., the z component of isospin is retained in the fragmentation region. Thus $\Delta I_z(\mathcal{P})=\frac{1}{2}$, $\Delta I_z(\mathfrak{N})=-\frac{1}{2}$, etc., as Feynman found. On the other hand, T_t need not be SU(3)-symmetric, so we cannot draw any conclusions about Y.

Summarizing the results for cascades initiated by a single quark, we have for the quantum numbers left in the fragmentation region

$$\Delta I_z(\mathcal{P})=\tfrac{1}{2},\quad \Delta I_z(\mathfrak{N})=-\tfrac{1}{2},\quad \Delta I_z(\lambda)=0,$$
$$\Delta B(\mathcal{P})=\Delta B(\mathfrak{N})=\Delta B(\lambda), \tag{22}$$
$$\Delta Y(\mathcal{P})=\Delta Y(\mathfrak{N})=\Delta Y(\lambda)+1,$$

where ΔI_z, ΔB, and ΔY are the average amounts of the quantum numbers observed in the fragmentation region. The electric charge and strangeness are related to the above via the Gell-Mann–Nishijima relations: $\Delta Q_{\text{el}}=\Delta I_z+\frac{1}{2}\Delta Y$, where $\Delta Y=\Delta B+\Delta S$. The fragmentation region need not be precisely defined since the adjoining plateau is neutral at very high energies.

The experimental quantities ΔQ can be represented in the notation of Gronau, Ravndal, and Zarmi[6] as follows:

$$\Delta I_z(q)=\sum_h\int dz\, D^h_q(z)\, I_z(h)\ ,$$
$$\Delta B(q)=\sum_h\int dz\, D^h_q(z)\, B(h)\ , \tag{23}$$
$$\Delta Y(q)=\sum_h\int dz\, D^h_q(z)\, Y(h)\ ,$$

where the sum is over hadrons h, and $D^h_q(z)$ describes the probability of a $\mathcal{P}$ quark producing a hadron h with a fraction z of the momentum of the quark. Our assumption that there is a unique cascade for the system with triality +1 gives $D^h_{\mathcal{P}}\sim D^h_{\mathfrak{N}}\sim D^h_\lambda\sim a/z$, for small z, where a is the same constant in all three cases. In other words, the plateau height, which determines the dominant contribution to the logarithmic multiplicity, is the same for all fragmenting states of the same triality. (The plateau heights are the same for triality +1 and −1 by C invariance.) In particular, since there is no relation between triality 0 and triality ±1 cascades, the constants multiplying $\ln s$ in the multiplicities in e^+e^- annihilation and pp collisions need not be equal, i.e., $C_{e^+e^-}\neq C_h$. However, the height of the current plateau in electroproduction or neutrino-induced production is equal to that in e^+e^- annihilation (see Fig. 2), so that the multiplicity in leptoproduction at high Q^2 and/or high ω is

$$\langle n\rangle\simeq C_{e^+e^-}\ln(Q^2/m^2)+C_h\ln(\omega-1)\ , \tag{24}$$

where m^2 is on the order of a hadron mass squared.

The length of the parton-fragmentation region is determined by the nonleading eigenvalues in the cascade picture. If each cascade step corresponds to a fixed rapidity interval L, then nonasymptotic contributions should vanish as $\lambda^N \sim \lambda^{Y/L}$, where λ is the second-largest eigenvalue. The correlation length is $l \sim L/(-\ln\lambda)$. An order-of-magnitude estimate is $l \approx 2$ (as in hadronic fragmentation). $L \approx \ln 2$ (if two particles share the momentum of the initial one), so that $\lambda \approx 0.4$.

The experimental consequences of the weaker version of the quantum-number conjecture [Eq. (22)] can be tested in inclusive neutrino reactions ($\nu N \to \mu h X$). These processes permit a determination of the type of quark ejected. For example, at high energy, the left-handed W^+ boson (σ_L) takes only a $\mathfrak{N}$ or λ quark into a $\mathcal{P}$, while a right-handed W^+ boson (σ_R) takes only a $\bar{\mathcal{P}}$ into a $\bar{\mathfrak{N}}$ or $\bar{\lambda}$. The strangeness-changing processes, which are suppressed by the Cabibbo angle, can be separated in principle (although it may be extremely difficult in practice) by observing the strangeness of the final state.

III. HIGH-ENERGY PROCESSES WITHIN CASCADE MODELS

To clarify Sec. II and to extend the dynamical framework, we outline how a number of high-energy processes would be described in the context of our cascade models. The essential features of the models are that the cascades exist among states of the same triality and proceed to a unique asymptotic state (for that triality), and that the hadrons are emitted by the cascade in a stepwise fashion leading to a constant density in rapidity. Although many of the results obtained are not new,[4-8] it is interesting to view them from this different perspective.

A. e^+e^- annihilation

In e^+e^- annihilation, the timelike virtual photon decays through a $q\bar{q}$ intermediate with each quark having an initial rapidity $\ln(Q/M)$ in the relative center-of-mass system (rapidities being measured with respect to the $q\bar{q}$ axis). The q and $\bar{q}$ cascade independently for about N steps, where $N \sim O(\ln(Q/M))$. If N is large, the cascades will be given approximately by P_∞^+ and $P_\infty^- = CP_\infty^+$. Thus the heights of the cascades are the same and the quarklike quantum numbers disappear when the two cascades "meet."

B. Deep-inelastic leptoproduction

For $\nu P \to \mu^- hX$ and $e^-P \to e^-hX$ we follow Feynman[5] and work in a frame in which q, the virtual W or γ momentum, is purely spacelike and defines

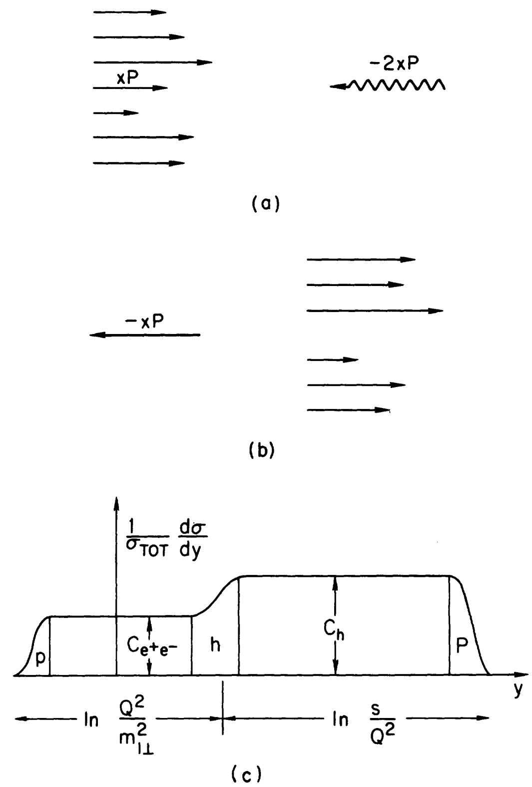

FIG. 2. Parton distributions (a) before and (b) after interaction with a virtual photon in the Breit frame of the virtual photon and struck parton. (c) The inclusive distribution, $\sigma^{-1}d\sigma/dy$, versus the rapidity y for deep-inelastic leptoproduction at large ω ($m_{1\perp}$ is the average transverse mass).

the negative z axis. If the target's momentum is P and the observed hadron's momentum is p, the standard invariants are

$$M\nu = P\cdot q\,,$$
$$M\kappa = p\cdot P\,,$$
$$\mu\nu_1 = p\cdot q\,,$$
$$Q^2 = -q\cdot q\,,$$
$$\omega = 2M\nu/Q^2 \equiv 1/x\,,$$
$$\omega_1 = 2\mu\nu_1/Q^2\,.$$

Thus in this frame $Q^2 = q_z^2$ and $-2xP_z = q_z$.

The target hadron cascades according to the prescription of Sec. II for triality 0 until the cascade reaches the point at which it contains a parton of momentum xP. The virtual photon strikes this parton (let us assume it is a $\mathcal{P}$ quark) and precisely reverses its motion. The hadron cascade is thus transformed into a system of triality -1 which proceeds via T_{-1}, while the struck parton decays via T_{+1}.

In this same frame, the initial hadron rapidity is $y \simeq \ln(\omega Q/M)$. When the virtual photon strikes the cascade, the cascade rapidity has decreased to $\sim\ln(Q/M)$. The struck parton begins its cascade with approximately the negative of this rapidity, while the hadron minus the quark continues from $y \simeq \ln(Q/M)$. For large Q/M, the cascades meet as before in e^+e^- annihilation, with C invariance guaranteeing that they have the same height [Fig. 2(c)].[15] Note that the hadron minus the quark has the quantum numbers of a $\bar{q}$ and asymptotically develops the same plateau (P_∞^-).

C. Hadron-hadron scattering

As described in Sec. III B, we imagine that hadrons evolve into final states through a cascade similar to that by which quark-partons turn into hadrons. The cascade prescription guarantees that a neutral plateau is present in the center of mass of the colliding high-energy hadrons, and that this plateau is universal (independent of the colliding hadrons). The quantum numbers are retained in the respective fragmentation regions.

According to Feynman's parton model, the dominant scattering mechanism producing the above picture is the exchange of "wee" partons—partons with finite c.m. momenta—resulting in a final hadrons distribution with limited transverse momenta. In addition there may be "hard" parton-parton scattering, resulting in partons knocked out with large transverse momenta. A similar description of this process in the parton model has been given by Savit,[16] but we shall review the analysis in terms of the cascade. Suppose two hadrons, each having a c.m. energy $E \simeq \frac{1}{2}\sqrt{s}$, collide such that the partons with momenta p_1 and p_2 suffer a hard collision and exit as p_1' and p_2'. Let us focus our attention on p_1 and p_1'. We shall consider the cases of large fixed $p_{1\perp}'$ ($p'_{1\perp} = E_1' \sin\theta_1 \gg M$, but $E_1' \gg p'_{1\perp}$) and fixed angle ($E_1' \simeq p'_{1\perp}$), and the relation to limited transverse momenta ($p'_{1\perp} \simeq \langle p_\perp \rangle$).

First we boost to a frame in which p_1 and p_1' are collinear and oppositely directed. Partons moving initially in the same direction as p_1 and with $x > 0$ are also collinear with p_1 and p_1' in this frame. Setting aside the partons associated with p_2 and p_2', the partons in this collinear frame have the same distribution as they would if they were the result of leptoproduction, with p_1' being the struck-parton momentum and p_1 being the hole momentum. Accordingly we expect them to evolve into hadrons in the same fashion as they do in this previously considered situation. Thus, typical hadron momenta will have limited transverse components in the collinear frame. What does this look like in the c.m. system? Boosting back, we find that a "fragment of the hole" will also have limited transverse momentum with respect to the original beam direction. If we consider a fragment of the struck parton, the hadron momentum lies near the direction of the struck parton and also with a spread $\sim\langle p_\perp \rangle$ away from this axis. The two cylinders centered on the hole and struck-parton directions will overlap for final-state hadrons with a c.m. energy E_0 such that $E_0 \sin\theta_1 \sim \langle p_\perp \rangle$ (θ_1 is the parton scattering angle as before). The hadrons with $E < E_0$ are not simply associated with just the hole or the parton. It is natural to assume that the dynamics in this region are those of triality 0, i.e., governed by T_0. In this heuristic picture, we see a triality -1 and a triality $+1$ system merging and continuing as a triality 0 system. The extent of this triality 0 system depends on θ_1. For finite θ_1, $E_0 = \langle p_\perp \rangle / \sin\theta_1$ is finite. On the other hand, if $p'_{1\perp}$ of the parton is large and fixed while $E \to \infty$, $\theta_1 \sim p'_{1\perp}/E$ and there is an increasing domain, $E < E_0$, in which the hadrons are controlled by triality zero dynamics.

A virtue of this description is that if we let $p'_{1\perp}$ decrease towards $\langle p_\perp \rangle$, the triality zero system engulfs the triality nonzero systems and we move continuously to the case in which all transverse momenta are limited (see Fig. 3). We find directly that the multiplicity is given by[16,17]

$$\langle n \rangle \simeq C_h \ln(s/4E_1^2) + 2C_{e^+e^-} \ln(4E_1^2/M^2),$$

where E_1 is the energy of the parton with large

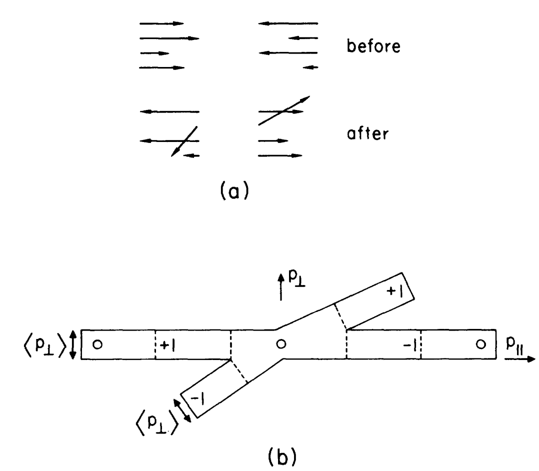

FIG. 3. (a) Parton distributions before and after a "hard" parton-parton scattering which produces large transverse momenta events in hadron-hadron scattering. (b) Schematic representation of the final-state hadron distribution in a large-transverse-momentum hadron-hadron scattering event. The triality of each cascade is indicated for an event in which a quark and antiquark suffer the hard collision.

transverse momentum (seen as a jet of hadrons) and M is an average hadron mass. (The first term contributes only for $s \gg 4E_1^2$ and the second for $E_1^2 \gg M^2$.) This formula makes manifest the smooth transition to the limited transverse momentum domain.

IV. CONCLUSIONS

We have presented a framework for parton cascades which reproduces many of Feynman's conjectures. In particular, hadron plateaus are universal (independent of the initiating hadron). Similarly, plateaus initiated by quarks are universal (independent of quark type), and dependent only on triality. However, there is no required connection between the triality zero and triality nonzero plateaus, suggesting that the coefficients of the logarithmic multiplicities in pp collisions and e^+e^- annihilation may well be different. The two distinct cascade types—triality zero and triality nonzero—play a fundamental role in the description of a variety of high-energy processes.

Within the context of our models, all of which have logarithmic multiplicities, Feynman's quantum-number-retention hypothesis for parton fragmentation need not necessarily hold. A weaker form [see Eq. (22)] is obtained, which would require, for example, that the electric charge in the $\mathcal{P}$-quark fragmentation region be one greater than that in the $\mathcal{N}$-quark or λ-quark fragmentation regions. In all our models, I_z is retained in the fragmentation region, unlike Y and B. Again, triality seems to play a central role in determining that quantum numbers must be retained in the fragmentation region of a hadron, but not necessarily in the fragmentation region of a quark.

While it is encouraging that a framework consistent with many postulates of the parton model can be produced, the far more difficult problem of understanding the actual dynamics remains.

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APPENDIX

We display here a class of models different from those in the main text. Here we assume all quarks and antiquarks act independently. This scheme is a specific type of Markov process called a discrete branching process. It suffices to consider the number of the various kinds of quarks in the cascade at each step. We find that Feynman's conjecture is satisfied as long as there is baryon production (unlike the situation in the Farrar-Rosner model), but that multiplicities grow geometrically rather than logarithmically.

We can express the population of the cascade by a column vector:

$$P = \begin{bmatrix} P_{\mathcal{P}} \\ P_{\mathcal{N}} \\ P_{\lambda} \\ P_{\bar{\mathcal{P}}} \\ P_{\bar{\mathcal{N}}} \\ P_{\bar{\lambda}} \end{bmatrix} . \tag{A1}$$

The average value of some additive quantum number carried by the cascade is

$$\langle Q \rangle = \sum_i P_i Q_i , \tag{A2}$$

where the sum is over quarks and antiquarks. Under what conditions does $\langle Q \rangle$ vanish so that Feynman's conjecture is satisfied? Obviously it suffices to have $P_{\mathcal{P}} = P_{\bar{\mathcal{P}}}$, $P_{\mathcal{N}} = P_{\bar{\mathcal{N}}}$, and $P_{\lambda} = P_{\bar{\lambda}}$, i.e., $CP = P$. If we consider only I_z and Y, it suffices to have $P_{\mathcal{P}} = P_{\mathcal{N}} = P_{\lambda}$, etc., i.e., SU(3) symmetry.

We recapitulate the Farrar-Rosner counterexample of Feynman's conjecture in this formalism as follows. The gap between a q and $\bar{q}$ arising in e^+e^- annihilation is filled in with isosinglet $q\bar{q}$ pairs. Neighboring pairs recombine to form mesons which break up the isosinglet pairs. Thus, for the cascade initiated by a $\bar{q}$ we have $\bar{q}, (q\bar{q}), (q\bar{q}), \ldots, (q|\bar{q}), (q\bar{q}), \ldots$. The quarks to the left of the break form the residue of hadrons, and the first antiquark to the right of the break is the cascade. Thus the probability vector is

$$P = \begin{bmatrix} 0 \\ 0 \\ 0 \\ a \\ a \\ 1-2a \end{bmatrix} .$$

Clearly Feynman's hypothesis fails for B and is satisfied for Y only in the SU(3) limit (though, of course, always working for I_z in any event).

Suppose, on the other hand, that there is some baryon emission. Thus, in addition to processes in which a quark is transformed into another quark with meson emission ($q \to Mq$), there are processes in which a quark turns into two antiquarks with the emission of a baryon ($q \to B\bar{q}\bar{q}$), and possibly more complex processes (e.g., $q \to Mqq\bar{q}$, $q \to B\bar{q}\bar{q}\bar{q}q$, etc.). If we suppose that each quark cascades independently, the development of the cascade can be described by

$$P_{N+1} = TP_N, \tag{A3}$$

where T is a 6×6 matrix. By C invariance of the strong interactions, T is necessarily of the form

$$T = \begin{pmatrix} T_1 & T_2 \\ T_2 & T_1 \end{pmatrix}, \tag{A4}$$

where the rows and columns are labeled by $\mathcal{P}$, $\mathcal{N}$, λ, $\bar{\mathcal{P}}$, $\bar{\mathcal{N}}$, and $\bar{\lambda}$. By isospin invariance, T_i $(i=1,2)$ is of the form

$$T_i = \begin{pmatrix} a_i & b_i & c_i \\ b_i & a_i & c_i \\ d_i & d_i & e_i \end{pmatrix}. \tag{A5}$$

Using the orthogonal 6×6 matrix

$$U = \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} \end{pmatrix}, \tag{A6}$$

we have

$$P' = UP = \frac{1}{\sqrt{2}} \begin{bmatrix} \mathcal{P} + \bar{\mathcal{P}} \\ \mathcal{N} + \bar{\mathcal{N}} \\ \lambda + \bar{\lambda} \\ \mathcal{P} - \bar{\mathcal{P}} \\ \mathcal{N} - \bar{\mathcal{N}} \\ \lambda - \bar{\lambda} \end{bmatrix} \tag{A7}$$

and

$$T' = UTU^{-1} = \begin{pmatrix} T_1 + T_2 & 0 \\ 0 & T_1 - T_2 \end{pmatrix}. \tag{A8}$$

In this representation, Feynman's hypothesis is satisfied if as $n \to \infty$

$$P_N' \to \begin{bmatrix} \alpha \\ \alpha \\ \beta \\ 0 \\ 0 \\ 0 \end{bmatrix}, \tag{A9}$$

while the Farrar-Rosner model yields

$$P_N' = \frac{1}{\sqrt{2}} \begin{bmatrix} a \\ a \\ 1-2a \\ \pm a \\ \pm a \\ \pm(1-2a) \end{bmatrix} \tag{A10}$$

for all N except the initial state.

It is straightforward to find the eigenvectors v_j (which are not orthogonal in general) and eigenvalues λ_j of a matrix of the form (A5). They are

$$\begin{aligned} v_1 &= (\mathcal{P} - \mathcal{N}), \quad \lambda_1 = a - b, \\ v_2 &= \mathcal{P} + \mathcal{N} + \frac{\lambda}{2c}\{e - a - b + [(e-a-b)^2 + 8cd]^{1/2}\}, \\ \lambda_2 &= \tfrac{1}{2}\{a + b + e + [(e-a-b)^2 + 8cd]^{1/2}\}, \\ v_3 &= \mathcal{P} + \mathcal{N} + \frac{\lambda}{2c}\{e - a - b - [(e-a-b)^2 + 8cd]^{1/2}\}, \\ \lambda_3 &= \tfrac{1}{2}\{a + b + e - [(e-a-b)^2 + 8cd]^{1/2}\}. \end{aligned} \tag{A11}$$

Thus the eigenvectors of T' are

$$\begin{aligned} u_1 &= \begin{pmatrix} v_1(+) \\ 0 \end{pmatrix}, \quad u_2 = \begin{pmatrix} v_2(+) \\ 0 \end{pmatrix}, \\ u_3 &= \begin{pmatrix} v_3(+) \\ 0 \end{pmatrix}, \quad u_4 = \begin{pmatrix} 0 \\ v_1(-) \end{pmatrix}, \\ u_5 &= \begin{pmatrix} 0 \\ v_2(-) \end{pmatrix}, \quad u_6 = \begin{pmatrix} 0 \\ v_3(-) \end{pmatrix}, \end{aligned} \tag{A12}$$

where $v_1(\pm)$ is given by (A10), with $\mathcal{P}$ replaced by $(\mathcal{P} \pm \bar{\mathcal{P}})/\sqrt{2}$, etc., a replaced by $a_1 \pm a_2$, etc. Now if the initial probability vector is

$$UP_0 = P_0' = \sum \alpha_i u_i, \tag{A13}$$

then

$$P_N' = (T')^N P_0' = \sum \alpha_i (\eta_i)^N u_i, \tag{A14}$$

where η_i are the eigenvalues $\eta_1 = \lambda_1(+)$, $\eta_2 = \lambda_2(+)$, $\eta_3 = \lambda_3(+)$, $\eta_4 = \lambda_1(-)$, $\eta_5 = \lambda_2(-)$, and $\eta_6 = \lambda_3(-)$.

The Feynman hypothesis is satisfied if the differences $\mathcal{P} - \bar{\mathcal{P}}$, $\mathcal{N} - \bar{\mathcal{N}}$, and $\lambda - \bar{\lambda}$ tend to zero asymptotically, i.e., if η_4, η_5 are less than unity. Since $\eta_6 < \eta_5$, it suffices that

$$\eta_4 = a_1 - a_2 - b_1 + b_2 < 1 \tag{A15}$$

and

$$\eta_5 = \tfrac{1}{2}\{a_1 - a_2 + b_1 - b_2 + e_1 - e_2 + [(e_1 - e_2 - a_1 + a_2 - b_1 + b_2)^2 + 8(c_1 - c_2)(d_1 - d_2)]^{1/2}\} < 1. \quad \text{(A16)}$$

The significance of the elements of $T_1 - T_2$ can be determined by considering what happens to a $\mathcal{P}$ or λ quark after a single cascade step. We have for these one-step processes

$$T\begin{bmatrix}1\\0\\0\\0\\0\\0\end{bmatrix} = \begin{bmatrix}a_1\\b_1\\d_1\\a_2\\b_2\\d_2\end{bmatrix}, \quad T\begin{bmatrix}0\\0\\1\\0\\0\\0\end{bmatrix} = \begin{bmatrix}c_1\\c_1\\e_1\\c_2\\c_2\\e_2\end{bmatrix}. \quad \text{(A17)}$$

From these vectors we calculate the average baryon number emitted by a $\mathcal{P}$ quark per cascade step:

$$\Delta B(p) = \tfrac{1}{3} - \tfrac{1}{3}(a_1 + b_1 + d_1 - a_2 - b_2 - d_2). \quad \text{(A18)}$$

Similarly, the hypercharge of the hadrons emitted by a $\mathcal{P}$ quark per cascade step is

$$\Delta Y(p) = \tfrac{1}{3} - \tfrac{1}{3}(a_1 + b_1 - 2d_1 - a_2 - b_2 + 2d_2). \quad \text{(A19)}$$

In the same fashion we find

$$\Delta B(\lambda) = \tfrac{1}{3} - \tfrac{1}{3}(2c_1 + e_1 - 2c_2 - e_2),$$
$$\Delta Y(\lambda) = -\tfrac{2}{3} - \tfrac{2}{3}(c_1 - e_1 - c_2 + e_2). \quad \text{(A20)}$$

In terms of these quantities,

$$\eta_5 = 1 - \tfrac{1}{2}[\Delta Y(\mathcal{P}) + 2\Delta B(\mathcal{P}) - \Delta Y(\lambda) + B(\lambda)] + \tfrac{1}{2}(\{[\Delta Y(\mathcal{P}) + 2\Delta B(\mathcal{P}) - \Delta Y(\lambda) + \Delta B(\lambda)]^2 - 12[\Delta B(\lambda)\Delta Y(\mathcal{P}) - \Delta B(\mathcal{P})\Delta Y(\lambda)]\}^{1/2}). \quad \text{(A21)}$$

We can expect quite generally that $\Delta Y(\lambda) < 0$ and $\Delta Y(\mathcal{P}) > 0$, i.e., quarks produce more negative hypercharge hadrons than positive, and vice versa for $\mathcal{P}$ quarks. Additionally, we expect that $\Delta B(\mathcal{P}) > 0$ and $\Delta B(\lambda) > 0$, i.e., λ and $\mathcal{P}$ quarks produce more baryons than antibaryons. From the expression for η_5 we see that the introduction of a small amount of baryon production reduces η_5 from unity to a value less than unity, thus guaranteeing the success of Feynman's conjecture in these models.

The eigenvalues η_4 and η_6 can be expressed similarly:

$$\eta_4 = 1 - 2\Delta I_z(\mathcal{P}),$$
$$\eta_6 = 1 - \tfrac{1}{2}[\Delta Y(\mathcal{P}) + 2\Delta B(\mathcal{P}) - \Delta Y(\lambda) + \Delta B(\lambda)] - \tfrac{1}{2}\{[\Delta Y(\mathcal{P}) + 2\Delta B(\mathcal{P}) - \Delta Y(\lambda) + \Delta B(\lambda)]^2 - 12[\Delta B(\lambda)\Delta Y(\mathcal{P}) - \Delta B(\mathcal{P})\Delta Y(\lambda)]\}^{1/2}, \quad \text{(A22)}$$

where $\Delta I_z(\mathcal{P})$ is the average z component of isospin of the hadrons emitted by a $\mathcal{P}$ quark per cascade step. Of course we expect $\Delta I_z(\mathcal{P}) > 0$. We see now that the requirements $\eta_4 < 1$, $\eta_5 < 1$, and $\eta_6 < 1$ are met quite generally. Of the six eigenvectors, only u_4 carries $I_z \neq 0$. Thus the equilibration of I_z is governed by η_4. Since u_5 carries both B and Y and since $\eta_5 > \eta_6$, the equilibration of these quantum numbers is governed by η_5. Since η_5 is reduced below unity only by the strange-particle and baryon production, we anticipate that $\eta_5 > \eta_4$, and thus I_z should equilibrate more quickly than Y or B.

In the Farrar-Rosner model $\Delta B(\mathcal{P}) = 0$ and $\Delta B(\lambda) = 0$, so that $\eta_5 = \eta_2 = 1$. Here we have two degenerate systems which are completely independent: the system initiated by quarks and the one initiated by antiquarks.

When $\eta_5 < 1$ we have also $\eta_2 > 1$, so that the number of quarks in the cascade grows as $(\eta_2)^N$. Consequently the number of hadrons emitted per step grows as $(\eta_2)^N$. This geometric particle growth is incompatible with a flat plateau and is the primary motivation for constraining our cascades discussed in the main text to have a bounded number of quarks at each step.

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†Present address: Department of Physics, University of Washington, Seattle, Washington 98195.

‡Present address: Institute for Advanced Study, Princeton, New Jersey 08540.

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[12]Although we use their basic argument, we do not use their formalism, which is not correct in general. Knowledge of particle ratios does not necessarily determine the cascade probabilities.

[13]One way of achieving this would be to require first independent emission of $q \to qM$, $\bar{q} \to \bar{q}M$, $q \to \bar{q}\bar{q}B$, $\bar{q} \to qq\bar{B}$, and then condensation of all $q\bar{q}$ pairs into mesons and all qqq triplets into baryons, etc., before the next step begins.

[14]Since crossing is required to relate $q \to \bar{q}\bar{q}B$ and $\bar{q}\bar{q} \to q\bar{B}$, $1-\alpha$ is not necessarily equal to $1-\beta$.

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