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$e^+e^- \to \gamma \to$ four mesons and three mesons in a fermion-loop model*

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The low-energy limits of the single-fermion-loop contribution to the reactions $e^+e^- \to \gamma \to \pi^+\pi^-\pi^0$, $K^+K^-\pi^0$, $\pi^+\pi^-\pi^0\pi^0$, $\pi^+\pi^-\pi^+\pi^-$, $K^+K^-\pi^0\pi^0$, and $K^+K^-K^+K^-$ were calculated without any adjustable parameters. Results were found to be comparable to recent experiments. This model's significance and its possible modification were discussed.

It is well known that the π^0 decay into 2γ is suppressed in a standard current-algebra calculation.[1] This discrepancy is resolved by the introduction of the so-called Adler-Schwinger anomaly,[2] which arises in the triangular fermion-loop diagram as shown in Fig. 1. This diagram gives a finite though arbitrary result that is dependent on the average charge of the fermion quarks circulating the loop. In practice, this picture is not much different from Steinberger's[3] phenomenological calculation of π^0 decay using the same diagram with the nucleon only. Subsequently, a series of papers[4,5] treated the possibility of testing the anomaly directly in the processes $\gamma\gamma \to \pi^+\pi^-\pi^0$ and $3\pi^0$. Based on the anomaly, the low-energy relation of these processes to the π^0 decay is derived, which in principle can be tested experimentally. In terms of the perturbative field theory, e.g., the linear σ model, the above considerations can again be realized explicitly[5] in the one-loop diagrams, in which the fermion loop is a major contribution. In a more phenomenological approach,[6] it is found that when the fermion-loop model is suitably modified for weak interaction it provides a qualitative explanation for $K_2^0 \to \gamma\gamma$, $\pi^+\pi^-\gamma$ and $K^\pm \to \pi^\pm\pi^0\gamma$ decays with no adjustable parameters.

Recently, data[7,8] on

$$e^+e^- \to \pi^+\pi^-\pi^+\pi^- \tag{1}$$

and

$$e^+e^- \to \pi^+\pi^-\pi^0 \; (\pi^+\pi^-\pi^0\pi^0) \tag{2}$$

became available. It is assumed that e^+e^- first annihilate into an off-shell photon, which decays into the appropriate hadronic final states. What will be the contribution of the fermion-loop model to these exclusive production processes? We can consider the following: (a) In any simple field theory, the lowest-order approximation to the

TABLE I. Low-energy-limit prediction of the photon-meson vertices in a fermion loop model. The virtual photon carries a momentum p. q's are the mesonic momenta.

	$A_\mu (q_i \cdot q_j \ll m^2)$
$\gamma \to \pi^+\pi^-\pi^0$	$-\frac{ieg^3}{2\pi^2 m^3} d(3f^2+d^2)\epsilon_{\mu\nu\lambda\delta}p^\nu q^\lambda_{\pi^+} q^\delta_{\pi^-}$
$\gamma \to K^+K^-\pi^0$	$-\frac{ieg^3}{2\pi^2 m^2} d(3f^2+d^2)\epsilon_{\mu\nu\lambda\delta}p^\nu q^\lambda_{K^+} q^\delta_{K^-}$
$\gamma \to \pi^+\pi^-\pi^0\pi^0$	$-\frac{ieg^4}{3\pi^2 m^2}(\frac{43}{9}d^4+\frac{70}{3}d^2f^2+19f^4)(q_{\pi^+}-q_{\pi^-})_\mu$
$\gamma \to \pi^+\pi^-\pi^+\pi^-$	$-\frac{i2eg^4}{3\pi^2 m^2}(\frac{43}{9}d^4+\frac{70}{3}d^2f^2+19f^4)(q_{+1}+q_{+2}-q_{-1}-q_{-2})_\mu$
$\gamma \to K^+K^-\pi^0\pi^0$	$-\frac{i8eg^4}{3\pi^2 m^2}(\frac{5}{9}d^4+\frac{5}{3}f^2d^2+2f^4)(q_{K^+}-q_{K^-})_\mu$
$\gamma \to K^+K^-\pi^+\pi^-$	$-\frac{ieg^4}{3\pi^2 m^2}(\frac{43}{9}d^4+\frac{70}{3}d^2f^2+19f^4)(q_{K^+}+q_{\pi^+}-q_{K^-}-q_{\pi^-})_\mu$
$\gamma \to K^+K^-K^+K^-$	$-\frac{i2eg^4}{3\pi^2 m^2}(\frac{43}{9}d^4+\frac{70}{3}d^2f^2+19f^4)(q_{+1}+q_{+2}-q_{-1}-q_{-2})_\mu$

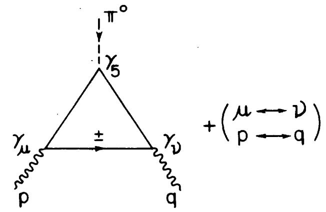

FIG. 1. Single-fermion-loop contribution to $\pi^0 \to \gamma\gamma$.

direct emission of mesons from a single photon comes from one-loop diagrams, in which the fermion loop is a major contribution. Thus processes (1) and (2) offer a chance to test whether the fermion-loop model is able to give a global, qualitative result for different classes of problems. (b) When the total e^+e^- c.m. energy is above the production threshold of kaon pairs, no compelling separation of pions from kaons has been made in experiments. In the model we consider below, it is simple and straightforward to calculate the kaon pair production as well, which offers an estimation of the kaon contamination.

The results of the model calculation are summarized in Tables I and II. They are found to be comparable to the experiments.

The effective interaction Lagrangian is

$$\mathcal{L}_I = -\sqrt{2}\, gf\, \mathrm{Tr}([\bar{B}i\gamma_5, B]M) + \sqrt{2}\, gd\, \mathrm{Tr}(\{\bar{B}i\gamma_5, B\}M) - \tfrac{1}{2} eA^\mu\, \mathrm{Tr}([\bar{B}\gamma_\mu, B]Q), \qquad (3)$$

where $B = \lambda_i \psi_i/\sqrt{2}$ (Ref. 9) is the traceless baryon matrix, $M = \lambda_i \phi_i/\sqrt{2}$ is the traceless Hermitian meson matrix, and $Q = \lambda_3 + \lambda_8/\sqrt{3}$. As in Ref. 6, we use $g^2/4\pi = 14.6$ and $d/f = 1.8$, where d and f are the symmetric and antisymmetric MBB couplings, so that there are no adjustable parameters in our calculation.

We will discuss the construction of the amplitude $e^+e^- \to \gamma \to \pi^+\pi^-\pi^+\pi^-$ in detail. To the lowest order, the (direct emission) vertex $\gamma \to \pi^+\pi^-\pi^+\pi^-$ is given by the sixteen sets of Feynman diagrams in Fig. 2; each has a definite charge state running along the fermion loop, and each gives rise to a definite SU(3) factor. For example, Figs. 2(a) and 2(b) have a factor of $16g^4(\frac{1}{3}d^2 - f^2)^2$, Fig. 2(c) has a factor of $4g^4[(f+d)^4 + 4(\frac{1}{3}d^2 + f^2)^2]$, and Fig. 2(d) has a factor of $4g^4[(f-d)^4 + 4(\frac{1}{3}d^2 + f^2)^2]$ coming from a product of the four MBB vertices. In the low-energy limit, the basic tensor structure of the loop is fairly simple. We define a quantity I_μ in accordance with Fig. 3(a):

$$I_\mu = \int \frac{d^4t}{(2\pi)^4} \mathrm{Tr}\left[\gamma_\mu \frac{1}{\gamma\cdot(t-q_1-q_2-q_3-q_4)-m} \gamma_5 \frac{1}{\gamma\cdot(t-q_1-q_2-q_3)-m} \gamma_5 \frac{1}{\gamma\cdot(t-q_1-q_2)-m} \gamma_5 \frac{1}{\gamma\cdot(t-q_1)-m} \gamma_5 \frac{1}{\gamma\cdot t-m} \right]. \qquad (4)$$

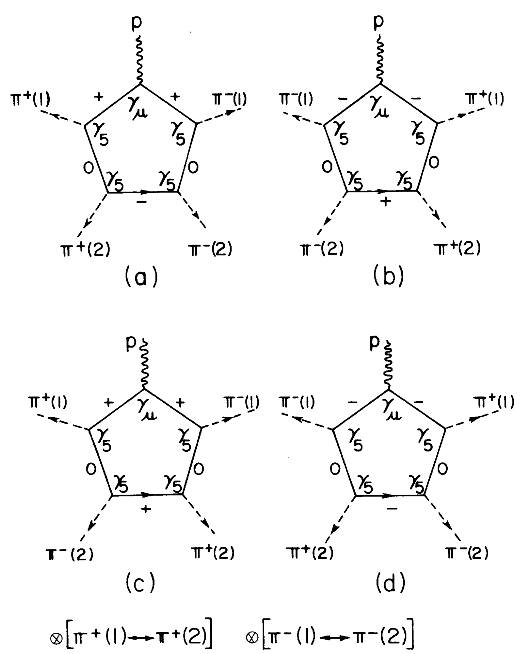

FIG. 2. Single-fermion-loop contribution to $\gamma \to \pi^+\pi^-\pi^+\pi^-$.

For simplicity, we neglect the mass difference among the fermion octet and assume an average fermion mass $m = 1$ GeV. When the energy of the virtual photon is not much above production threshold, we shall neglect $q_i \cdot q_j$ as compared with m^2. Thus in this low-energy limit we have

$$I_\mu \to \frac{1}{12\pi^2 i m^2}[(q_1 - q_4) - \tfrac{1}{2}(q_2 - q_3)]_\mu . \qquad (5)$$

Supplying the other factors and summing over all

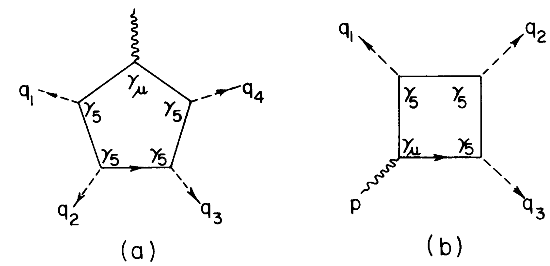

FIG. 3. (a) Basic fermion-loop diagram for $\gamma \to 4$ mesons. (b) Basic fermion-loop diagram for $\gamma \to 3$ mesons.

TABLE II. Cross sections for $e^+e^- \to \gamma \to$ mesons predicted in a fermion loop model. 1 nb $=10^{-33}$ cm^2. The second column, σ, is the absolute prediction, and the third column, σ_n, is normalized to π^0 decay into 2γ. For the 4-meson final states, cross sections are evaluated with a Monte Carlo program with 5000 points and the errors represent the estimated statistical error. In the experimental values, the number in the parentheses represents the systematic error and the second number represents the statistical error.

	$2E$ (GeV)	σ (nb)	σ_n (nb)	σ_{exp} (nb)
$\pi^+\pi^-\pi^0$	0.8	2.05	0.73	
	1.0	11.07	3.96	
	1.2	36.12	12.94	
	1.35	72.93	26.12	
	1.50	132.54	47.48	
$K^+K^-\pi^0$	1.2	0.00	0.00	
	1.4	0.29	0.10	
	1.6	2.60	0.93	
	1.8	10.67	3.82	≤ 6 (Ref. 7)
	2.0	30.23	10.83	
$\pi^+\pi^-\pi^0\pi^0$	0.8	0.70 ± 0.01	0.25 ± 0.00	
	1.0	4.85 ± 0.06	1.74 ± 0.02	
	1.2	14.46 ± 0.17	5.18 ± 0.06	
	1.35	24.76 ± 0.30	8.87 ± 0.11	
	1.5	38.65 ± 0.48	13.65 ± 0.17	
$\pi^+\pi^-\pi^0$ $\pi^+\pi^-\pi^0\pi^0$	0.8	2.75 ± 0.01	0.98 ± 0.00	
	1.0	15.92 ± 0.06	5.70 ± 0.02	
	1.2	50.58 ± 0.17	18.12 ± 0.06	$(30 \pm 2) \pm 15$
	1.35	97.69 ± 0.30	34.99 ± 0.11	Ref. 8
	1.5	171.19 ± 0.48	61.13 ± 0.17	$(30 \pm 4) \pm 11$
$\pi^+\pi^-\pi^+\pi^-$	0.8	2.80 ± 0.03	1.00 ± 0.01	
	1.0	19.44 ± 0.21	6.96 ± 0.08	
	1.2	56.65 ± 0.62	20.29 ± 0.22	$(3 \pm 3) \pm 3$
	1.35	98.11 ± 1.07	35.15 ± 0.39	Ref. 8
	1.5	152.97 ± 1.67	54.79 ± 0.60	$(18 \pm 3) \pm 3$
$K^+K^-\pi^0\pi^0$	1.4	0.01 ± 0.00	0.00	
	1.6	0.36 ± 0.00	0.13 ± 0.00	
	1.8	2.07 ± 0.02	0.74 ± 0.01	
	2.0	6.02 ± 0.07	2.16 ± 0.03	
	2.2	13.04 ± 0.16	4.67 ± 0.06	
$K^+K^-\pi^+\pi^-$	1.4	0.04 ± 0.00	0.00	
	1.6	1.80 ± 0.04	0.64 ± 0.00	
	1.8	10.88 ± 0.28	3.92 ± 0.08	
	2.0	31.76 ± 0.76	11.36 ± 0.28	
	2.2	65.16 ± 1.64	23.36 ± 0.60	
$K^+K^-K^+K^-$	2.0	0.00	0.00	
	2.2	0.20 ± 0.00	0.07 ± 0.00	
	2.4	2.67 ± 0.03	0.95 ± 0.01	
	2.6	11.33 ± 0.13	4.06 ± 0.05	
	2.8	30.29 ± 0.34	10.85 ± 0.12	

permutations, we find that the vertex is given by

$$A_\mu[\gamma \to \pi^+(q_{+,1})\pi^-(q_{-,1})\pi^+(q_{+,2})\pi^-(q_{-,2})]$$

$$= -i\,\frac{2}{3}\,\frac{eg^4}{\pi^2 m^2}\left(\tfrac{43}{9}d^4 + \tfrac{70}{3}d^2f^2 + 19f^4\right) \times (q_{+,1} + q_{+,2} - q_{-,1} - q_{-,2})_\mu \,. \tag{6}$$

The invariant amplitude for $e^+(p', s') + e^-(p, s) \to \gamma \to \pi^+\pi^-\pi^+\pi^-$ is then[10]

$$M = -\frac{e}{(2E)^2}\,\bar{v}(p', s')\gamma_\mu u(p, s) A^\mu \,, \tag{7}$$

and the cross section σ is given by

$$\sigma = \frac{1}{|\vec{v}-\vec{v}'|}\frac{m_e m_e}{E_p E_{p'}}\left(\frac{1}{2!2!}\right)\frac{1}{4}\sum_{ss'}\int |M|^2 \prod_i^4 \frac{d^4 q_i}{(2\omega_i)(2\pi)^3}(2\pi)^4 \delta(p+p'-q_{+,1}-q_{+,2}-q_{-,1}-q_{-,2}), \tag{8}$$

with

$$\frac{m_e^2}{E^2}\frac{1}{4}\sum_{ss'}|\bar{v}\gamma_\mu u A^\mu|^2 = \tfrac{1}{2}(A_1^2 + A_2^2). \tag{9}$$

The phase-space integral is evaluated by a Monte Carlo[11] program with 5000 points. The result is given in Table II. At $2E = 1.5$ GeV, the cross section is about nine times larger than the experimental value. Notice that when we generalize Steinberger's calculation of $\pi^0 \to \gamma(\mu p)\gamma(\nu q)$ to SU(3) we get an amplitude

$$-\frac{2i\alpha g}{\pi m} d\epsilon_{\mu\nu\lambda\delta} p^\lambda q^\delta \tag{10}$$

which gives a decay rate Γ_{th} of 20.1 eV, almost three times larger than the experimental value $\Gamma_{\text{exp}} = 7.2$ eV. We would like to argue that the cross section normalized to π^0 decay may be more meaningful. We then define the normalized cross section

$$\sigma_n = \sigma\left(\frac{\Gamma_{\text{exp}}}{\Gamma_{\text{th}}}\right). \tag{11}$$

This is also listed in Table II.

The other amplitudes for different four-meson final states are similarly constructed. They are summarized in Tables I and II. For completeness, we have also calculated $\gamma \to \pi^+\pi^-\pi^0$ and $\gamma \to K^+K^-\pi^0$ in this model. Here the basic box diagram is given in Fig. 3(b). We define

$$J_\mu = \int \frac{d^4 t}{(2\pi)^4} \mathrm{Tr}\left[\gamma_\mu \frac{1}{\gamma\cdot(t-p)-m}\gamma_5 \frac{1}{\gamma\cdot(t-p+q_1)-m}\gamma_5 \frac{1}{\gamma\cdot(t-p+q_1+q_2)-m}\gamma_5 \frac{1}{\gamma\cdot t - m}\right]$$
$$\to -\frac{1}{24\pi^2 m^3}\epsilon_{\mu\nu\lambda\delta} p^\nu q_1^\lambda q_2^\delta . \tag{12}$$

With this low-energy result, the γ-3-meson vertices are constructed; these are given in Table I. The phase-space integral near the threshold in this case has been worked out by Aviv and Zee.[12] Using their result, Eq. (A20) of Ref. 12, we have calculated the corresponding cross sections as listed in Table II.

As can be seen in Table I, the vertex for four-charged-meson production has a quite different momentum dependence than the two-charged–two-neutral-meson vertex. Also, the latter does not vanish when $q_0 \to 0$, while the opposite might be expected from current algebra. Our model also predicts that the kaon production vertex is of the same order of magnitude as the pion vertex. However, they are not related to each other in a simple way. From Table II, the absolute prediction of our model is typically 10 times larger than the experimental value, while the normalized cross sections are typically 3 times larger, so that the amplitudes based on a fermion loop alone do have the right order of magnitude. We emphasize that this model is the most economical one that gives both an anomalylike contribution and a simple generalization to kaon pair production. If we were to work out all the one-loop contributions systematically, e.g., in a linear σ model, we would have other classes of diagrams like Fig. 4(a), whose contribution is of the same order of magnitude as the fermion loop, and perhaps with opposite phase. This possible destructive interference would easily bring our result closer to the experiment. Also, both functions I_μ and J_μ defined in Eqs. (4) and (12) could be evaluated exactly with numerical methods, and this should give a more reliable result. Moreover, a complete one-loop calculation will definitely include diagrams like Fig. 4(b), where the $\gamma\pi\pi$ vertex should be renormalized properly. In this respect, a renormalizable, unified gauge theory will be a perfect model to work with. This is currently being investigated.

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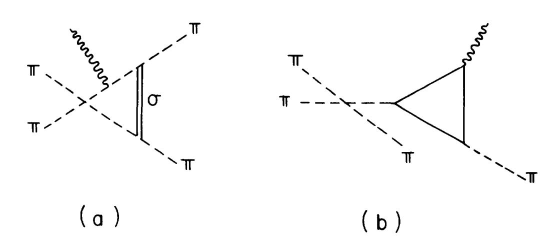

FIG. 4. (a) A possible one-loop diagram for $\gamma \to 4\pi$ in a linear σ model. (b) Another possible one-loop diagram for $\gamma \to 4\pi$. The $\gamma\pi\pi$ vertex is divergent.

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[10]We use the same convention and notation as J. D. Bjorken and S. D. Drell, *Relativistic Quantum Fields* (McGraw-Hill, New York, 1965).

[11]We use the relation

$$\prod_{i=1}^{n}\int \frac{d^3p_i}{(2\omega_i)}\,|M|^2=\left[\prod_{i=1}^{n}\int \frac{d^3p_i}{(2\omega_i)}\right]\frac{1}{m}\sum_{i=1}^{m}|M(p_1,\ldots,p_n)|^2,$$

where

$$\{p_1,\ldots,p_n\}\in\prod_{i=1}^{n}\frac{d^3p_i}{2\omega_i},$$

i.e., the matrix element squared is averaged over all Monte Carlo points, which are selected to give a uniform density in phase space. The averaged value is then multiplied by the n-body total phase space.

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Are second-class currents present in hyperon beta decay?*

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Within the framework of Cabibbo $V-A$ theory bounds are obtained on the pseudotensor form factor which is induced by SU(3) violations. Such an estimate is necessary to distinguish second-class effects from effects of SU(3) violations. It is argued that calculations based on an analogy with the dispersion-theory evaluation of the nucleon anomalous magnetic moment are suspect since the vector and axial-vector currents have very different structures. A different dispersion-theory calculation involving only the matrix elements of the axial-vector currents is presented, which predicts the F/D ratio of the pion-nucleon octet coupling to be $\pm(\frac{5}{9})^{1/2}$. It is concluded that, if the controversial large experimental value of this term is confirmed, this would then require the presence of a second-class current.

I. INTRODUCTION AND SUMMARY

The concepts of first- and second-class currents were introduced by Weinberg,[1] following the invention of G parity by Michel[2] and Lee and Yang.[3] Recently there has been renewed interest in the possible existence of second-class currents, following experiments of the semileptonic decays of the Λ hyperon[4] and measurements of the ft values of the decays of mirror nuclei to a common daughter nucleus.[5] The relevant matrix element of the weak current may be written in terms of spinor functions and form factors. In the limit of exact SU(3) symmetry, the symmetries of the currents represented by the various form factors are known precisely. However, this relationship between the symmetries of the currents and the form factors is less precise the more SU(3) is broken. In order to establish the origin of a form factor which in an SU(3)-symmetric world would represent a second-class current, and which is nonzero in fact, one must estimate the size of the form factor obtained by assuming only first-class currents in the SU(3)-symmetric world and appropriately modifying the matrix element for SU(3) violations.[6] Recently, attempts have been made to estimate such induced effects.[7] In this work it is argued that these previous calculations are suspect. Implementation of the indicated modifications yields effects which are somewhat smaller than those obtained previously.

The semileptonic Hamiltonian density is