PHYSICAL REVIEW D VOLUME 9, NUMBER 9 1 MAY 1974

Prediction of the SU(3) ⊗ SU(3) configuration mixing as a relativistic effect in the naive quark model

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(Received 29 June 1973)

The harmonic-oscillator quark model of hadrons implies relativistic internal volocities for the quarks. If, consequently, we use a relativistic treatment of the quark spin (by replacing Pauli spinors with Dirac spinors), we obtain not only corrections to the "static SU(6)" results for the axial-vector matrix elements G_A/G_V or G^* (as shown by Bogoliubov), but the whole SU(3)⊗SU(3) configuration mixing at $p_z=\infty$, in a semiquantitative manner. The mixing operator is just the product of the Wigner rotations of quark spins. For the low-lying $\frac{1}{2}^+$ octet, the dominant representations are (6,3), $(3,\overline{3})$, and (8,1), in agreement with the phenomenological analysis of Buccella, De Maria, and Lusignoli. The SU(6) classification of hadrons is preserved; at rest, the hadron wave function corresponds to an SU(6) group whose spin generators are the sum of the mean spin operators (of Foldy and Wouthuysen) of the quarks.

I. INTRODUCTION

As is well known, the "static SU(6)" (or nonrelativistic quark model) predictions concerning the axial-vector current matrix elements, e.g., $G_A/G_V=\frac{5}{3}$, $G^*=\frac{4}{3}$ are in serious disagreement with experiment. The same results are reproduced by the saturation of chiral algebra at $p_z=\infty$ with the $\frac{1}{2}^+$ and $\frac{3}{2}^+$ low-lying states. In this scheme, the discrepancy can be explained by the so-called "configuration mixing."[1] In fact, the presence of a considerable mixing is clearly indicated by the important contribution of higher isobars to the Adler-Weisberger relation.[2] Various phenomenological mixing schemes have been proposed since 1966.[3] The discrepancy can also be explained by the "realistic" quark model (i.e., with real quarks, endowed with a real spatial motion inside the hadron). Bogoliubov[4] has shown that with the Dirac equation in a central potential G_A/G_V is reduced to $\frac{5}{3}(1-2\delta)$, where δ is a positive quantity related to the norm of the small components in the quark Dirac spinors. Thus, the discrepancy with the SU(6) results could be a relativistic effect due to the quark internal motion. In order to have the right order of magnitude for the correction, the internal velocities must be large, in contradiction with the traditional picture initiated by Morpurgo[5] (heavy quarks with nonrelativistic velocities). Precisely, the work made by various authors on the harmonic-oscillator quark model[6] indeed shows that the internal quark velocities are highly relativistic. From the mass spectrum interval ΔE and the wave-function radius squared R^2, one deduces a mean internal velocity of the order $v\simeq R\Delta E\simeq 1$.

The aim of the present paper is to show that, if in the quark-model wave functions we take into account the small components of Dirac spinors implied by the momentum distributions of the harmonic oscillator, we are able to explain not only G_A/G_V (as shown by Bogoliubov), but the whole mixing scheme of the chiral algebra at $p_z=\infty$ in a quantitative manner.

In Sec. II we present the introduction of a relativistic treatment of spin, in the spin part of the quark-model wave function. In Sec. III we make the SU(3)⊗SU(3) configuration mixing appear by boosting the hadron wave function to $p_z=\infty$. In Sec. IV we make an explicit calculation of the $\frac{1}{2}^+$ and $\frac{3}{2}^+$ baryon mixing. Section V is devoted to the numerical predictions and to the comparison with the phenomenological analysis of Buccella, De Maria, and Lusignoli[7] and with previous configuration-mixing hypotheses. In Sec. VI we discuss related theoretical ideas of Gell-Mann[1,8] and we establish some relations of our work with the mixing operators of Buccella, Kleinert, Savoy, Celeghini, and Sorace[9] and of Melosh.[10] Section VII is a conclusion.

II. THE BASIC HYPOTHESIS FOR THE RELATIVISTIC TREATMENT OF THE SPIN IN THE NAIVE QUARK MODEL

Usually, the quark spin states in a SU(6) hadron wave function are understood as Pauli spinors in the frame of nonrelativistic quantum mechanics. In such a frame, the famous prediction for the nucleon magnetic moments $\mu_p/\mu_n=-\frac{3}{2}$ starts from a purely phenomenological magnetic moment operator

$$\sum_i \vec{\mu}_i=\sum_i \frac{e_i}{2m_q}\vec{\sigma}_i=\mu_q\sum_i e_i\vec{\sigma}_i\,.$$

However, it is known that the Dirac equation leads to a theoretical prediction of the phenomenological magnetic moment at least for a structureless fermion; it endows the quark with a "normal" magnetic moment $\mu_q^i=(1/2m_q)e_i$, where m_q is the quark mass.[11]

Now, if we replace Pauli spinors by Dirac spinors in the SU(6) wave function and if we take the relativistic quark current operator $\bar{q}\gamma_\mu q$, we are led to the nucleon moments

$$\mu_p=\frac{e}{2m_q}, \quad \mu_n=-\frac{2}{3}\frac{e}{2m_q}. \tag{1}$$

If the quark mass m_q is taken to be the effective mass $\frac{1}{3}M_B$ for baryons, $\frac{1}{2}M_M$ for mesons, we obtain a very encouraging result for the absolute magnitude of μ_p and μ_n, 3 and −2 nuclear magnetons, respectively; moreover, the transition rates $\omega\to\pi\gamma$, $\Delta\to N\gamma$ are roughly predicted.

Note that for this calculation of the magnetic moments, the small components come merely from the external (center-of-mass) motion, namely, the momentum transfer, since, for an active quark we set

$$u_i\left(-\frac{\vec{q}}{2}\right)=\begin{bmatrix}\chi_i\\ -\dfrac{\vec{\sigma}_i\cdot\frac{1}{2}\vec{q}}{2m_q}\chi_i\end{bmatrix},$$
$$\bar{u}_i'(+\tfrac{1}{2}\vec{q})=\left[\chi_i'^{+},\ -\chi_i'^{+}\frac{\vec{\sigma}_i\cdot\frac{1}{2}\vec{q}}{2m_q}\right] \tag{2}$$

(initial state and final state, respectively).

However, the quark motion cannot entirely be reduced to an "external" motion; even in the hadron center-of-mass system, the quarks have momenta, related by the uncertainty principle to the spatial extension of the hadron, and these momenta must also contribute to the small components; this phenomenon has been neglected by some previous authors.[12] But is it really negligible? The magnitude of these small components is of the order of $(\langle\vec{p}_i^{\,2}\rangle)^{1/2}/m_q$. Let us estimate this ratio in the harmonic-oscillator quark model,[6] which is now the most widely accepted model for the quark spatial motion. We have $(\langle\vec{p}_i^{\,2}\rangle)^{1/2}\sim 1/R$, R being the radius of the ground-state wave function. The analysis of various hadronic processes indicates that R^2 is about 6 to 12 GeV^{-2}. An effective mass $m_q\simeq 300$ MeV is necessary to explain the magnetic moments and to account for the mean level spacing $\Delta E=1/m_qR^2$ in the hadron spectrum. Thus, with $\Delta E\simeq 400$ MeV

$$(\langle\vec{p}_i^{\,2}\rangle)^{1/2}/m_q\simeq 1$$

and we conclude that the small components of the quark Dirac spinors are of the same order as the big ones.[13]

To treat such a highly relativistic situation, we are led to extend the hypothesis which allowed us to calculate the absolute magnitude of the magnetic moment: *In the SU(6) wave functions we should replace the Pauli spinors by Dirac spinors; these Dirac spinors should also take into account the internal quark motion.*

Let us comment on this hypothesis from a group-theoretical point of view. If we use Pauli spinors, the spin operators of SU(6) are simply the Pauli matrices $\sum_i\frac{1}{2}\vec{\sigma}_i$. We maintain the SU(6) form of the hadron wave functions introducing the Dirac spinors. Now, which are the spin generators of this new SU(6) corresponding to the new wave functions? Passing from Pauli spinors to Dirac spinors amounts to passing from the Foldy-Wouthuysen representation to the Dirac representation. Then the spin generators we are looking for are the components of the so-called "mean spin" operator $\sum_i\vec{S}_i^{(M)}$, where

$$\vec{S}_i^{(M)}=F_i^{-1}\tfrac{1}{2}\vec{\Sigma}_iF_i, \quad \vec{\Sigma}_i=\begin{pmatrix}\vec{\sigma}_i & 0\\ 0 & \vec{\sigma}_i\end{pmatrix}, \tag{3}$$

F_i being the Foldy-Wouthuysen transformation for the ith quark. These generators are clearly different from those of the current algebra SU(6) group,[14] which are just $\sum_i\frac{1}{2}\vec{\Sigma}_i$.

This fact bears the very important consequence that the matrix element of, e.g., the axial-vector current will not be equal to the static SU(6) results, where it is assumed that the current is exactly a generator of the symmetry group. Thus, the hypothesis of combining Dirac spinors according to SU(6) pure representations may explain, for example, the discrepancy between the SU(6) predictions and the observed values of G_A/G_V. This point has been emphasized by Bogoliubov, who calculated G_A/G_V using the Dirac equation in a square-well potential, as we mentioned in the Introduction.

Let us finally write the explicit wave function which will be used throughout the paper.

We take the SU(6) harmonic-oscillator wave functions[6] replacing the Pauli spinors χ_i (ith quark) by the following Dirac spinor:

$$u_i(s)=\begin{bmatrix}\chi_i(s)\\ \mu_q\vec{\sigma}_i\cdot\vec{p}_i\chi_i(s)\end{bmatrix}, \tag{4}$$

where we set μ_q to be roughly the normal quark magnetic moment, $\mu_q\simeq 1/2m_q$.[15] This amounts to adopting the *spinor* structure of *free* quarks, although there is an internal momentum distribution reflecting the binding. This is the usual hypothesis in the quark model. For a nucleon with spin $+\frac{1}{2}$, we write

$$\Psi_{+1/2} = N(\tfrac{1}{2})^{1/2}(\chi'_{+1/2}\phi' + \chi''_{+1/2}\phi'') \times \exp\left[-\tfrac{1}{6}R^2 \sum_{i<j} (\vec{p}_i - \vec{p}_j)^2\right], \tag{5}$$

where

$$\chi'_{+1/2} = (\tfrac{1}{2})^{1/2}[u_1(-\tfrac{1}{2})u_2(+\tfrac{1}{2}) - u_1(+\tfrac{1}{2})u_2(-\tfrac{1}{2})]u_3(+\tfrac{1}{2}) \tag{6}$$

and

$$\chi''_{+1/2} = -(\tfrac{2}{3})^{1/2}\{u_1(+\tfrac{1}{2})u_2(+\tfrac{1}{2})u_3(-\tfrac{1}{2}) - \tfrac{1}{2}[u_1(+\tfrac{1}{2})u_2(-\tfrac{1}{2}) + u_1(-\tfrac{1}{2})u_2(+\tfrac{1}{2})]u_3(+\tfrac{1}{2})\}. \tag{7}$$

The ϕ's are the usual quark SU(3) wave functions. Ψ is normalized to one with the Dirac scalar product. This wave function leads to[4]

$$\frac{G_A}{G_V} = \tfrac{5}{3}(1-2\delta), \tag{8}$$

with $\delta \simeq \frac{1}{6}$ for $R^2/\mu_q^2 \simeq 3$, which corresponds to the values of the harmonic-oscillator parameters emphasized in Ref. 16. This gives $G_A/G_V \simeq 1.1$, which is encouraging. We are aware of the arbitrariness of our choice of the quark spinors: μ_q could depend on internal momentum and there could be an over-all kinematic factor. Such effects do not seem important, however, for the calculation of integrated quantities once the global wave function is normalized.

III. BOOSTING TO THE INFINITE-MOMENTUM FRAME

As has been recalled in the Introduction, the discrepancy between the SU(6) predictions for the axial-vector current matrix elements and the observed values is interpreted by current algebra in terms of configuration mixing of chiral algebra at $p_z = \infty$.[1] When it is assumed that the nucleon is a $(6,3)_8$ ($h = +\frac{1}{2}$) or a $(3,6)_8$ ($h = -\frac{1}{2}$) of SU(3)⊗SU(3) [which corresponds to $\underline{56}$ of SU(6)], one gets

$$G_A/G_V = \tfrac{5}{3}$$

[to obtain this value it is not necessary to recourse to the whole SU(6) group]. To circumvent this difficulty one superposes a series of irreducible representations

$$|N, h=+\tfrac{1}{2}\rangle = \alpha\,|(6,3)_8, L_z=0\rangle + \alpha'\,|(3,6)_8, L_z=+1\rangle + \beta\,|(3,\bar{3})_8, L_z=+1\rangle + \beta'\,|(\bar{3},3)_8, L_z=0\rangle + \gamma\,|(8,1)_8, L_z=-1\rangle + \gamma'\,|(1,8)_8, L_z=+2\rangle. \tag{9}$$

Then one gets

$$\frac{G_A}{G_V} = \tfrac{5}{3}(\alpha^2 - \alpha'^2) + (\beta^2 - \beta'^2) + (\gamma^2 - \gamma'^2). \tag{10}$$

Stimulated by the previous calculation of G_A/G_V we suspect that this configuration mixing can be deduced from the hypothesis of Sec. II concerning the treatment of the internal relativistic motion of quarks. In fact, we shall demonstrate that the mixing is a logical consequence of this hypothesis and, moreover, that it can be predicted quantitatively.

Let us transform the Dirac spinors (4) to the infinite-momentum frame (we note the Lorentz transformation L). We get

$$Lu_i(s,\vec{p}) \simeq (\tfrac{1}{2})^{1/2}\cosh\tfrac{1}{2}\omega \begin{bmatrix}\psi_i\\ \sigma_{iz}\psi_i\end{bmatrix}, \tag{11}$$

where $\beta = \tanh\omega$ is the nucleon velocity (as $\beta \to 1$, we neglect terms of order $1/p_z$) and

$$\psi_i = [1 + \mu_q\sigma_{iz}(\vec{\sigma}_i\cdot\vec{p}_i)]\chi_i \tag{12}$$

or

$$\psi_i = [1 + \mu_q p_{iz} + \mu_q\sigma_i^{(+)}p_i^{(-)} + \mu_q\sigma_i^{(-)}p_i^{(+)}]\chi_i, \tag{13}$$

with

$$\sigma_i^{(\pm)} = \tfrac{1}{2}(\sigma_x \pm i\sigma_y), \quad p_i^{(\pm)} = \mp(p_x \pm ip_y).$$

As concerns the spatial part of the wave function, which we assume to be of the harmonic-oscillator-type, we suppose that, after extraction of the center-of-mass motion,

$$\psi_{p_z=\infty}\left(\left\{\frac{p_{iz}}{(1-\beta^2)^{1/2}}, \vec{p}_{i\perp}\right\}\right) = \psi_{\text{rest}}(\{p_{iz}, \vec{p}_{i\perp}\}). \tag{14}$$

This is the hypothesis of Licht and Pagnamenta,[17] which amounts to neglecting the relative time dependence.

Then the spinor (11) can be rewritten in the form (we just write the product of two pure Lorentz transformations as a product of a boost by a Wigner rotation of quark spins[18])

$$(\tfrac{1}{2})^{1/2}\cosh\tfrac{1}{2}\omega[1 + \mu_q p_{iz} + \mu_q\beta_i\Sigma_i^{(+)}p_i^{(-)} + \mu_q\beta_i\Sigma_i^{(-)}p_i^{(+)}]\begin{bmatrix}\chi_i\\ \sigma_{iz}\chi_i\end{bmatrix}, \tag{15}$$

where we recognize the W-spin operators $W_i^{(\pm)}$ of Lipkin and Meshkov.[19] If the internal momenta were really small, (15) would reduce to

$$\sim(\tfrac{1}{2})^{1/2}\cosh\tfrac{1}{2}\omega\begin{bmatrix}\chi_i\\ \sigma_{iz}\chi_i\end{bmatrix},$$

which behaves just like χ_i under the generators

$$\tfrac{1}{2}(1\pm\Sigma_{iz})$$

of SU(3)⊗ SU(3). Thus we would get at the baryon level the usual SU(3)⊗ SU(3) assignment, e.g., $(6,3)_8$ for the nucleon of helicity $h=+\frac{1}{2}$, resulting from the standard SU(6) classification with Pauli spinors. But since in fact $\mu_q(\langle\vec{p}_i^{\,2}\rangle)^{1/2}\simeq 1$, the terms $\beta_i\Sigma_i^{(\pm)}$ cannot be neglected. These terms reverse the quark helicity and consequently lead to a different SU(3)⊗ SU(3) representation. This is already clear at the quark level since a quark with $h=+\frac{1}{2}$ belonging to the representation (3, 0) will pass to the (0, 3) representation ($h=-\frac{1}{2}$) by application of $\beta_i\Sigma_i^{(-)}$.

Thus we propose an interpretation of the SU(3) ⊗ SU(3) representation mixing as a simple consequence of the presence of high quark velocities inside the hadrons.

In Sec. IV we will make the mixing more explicit and perform a quantitative calculation.

IV. CALCULATION OF SU(3) ⊗ SU(3) BARYON MIXING

We limit ourselves to the calculation of the mixing for the low-lying baryon states, $\frac{1}{2}^+$ and $\frac{3}{2}^+$. We begin with the $\frac{1}{2}^+$.

Before we apply the mixing operator, let us first rewrite in a suitable manner the starting SU(6) wave function, to make its SU(3)⊗ SU(3) content explicit. In the notation of Mitra and Ross,[20] the SU(6) part of the low-lying $\frac{1}{2}^+$ baryons is written as

$$(\tfrac{1}{2})^{1/2}(\chi'\phi'+\chi''\phi''), \tag{16}$$

where the prime or double prime expresses antisymmetry or symmetry with respect to the first and second quarks. To express the SU(3)⊗ SU(3) content of a spin term, e.g., we write it (31, 2), grouping up-spinors on the left, and down-spinors on the right. Note that the order of the indices inside each helicity group is irrelevant; we shall respect the cyclic order. For helicity $+\frac{1}{2}$, we have, with this notation,

$$\chi'_{+1/2}=[(23,1)-(31,2)],$$
$$\chi''_{+1/2}=-(\tfrac{2}{3})^{1/2}\{(12,3)-\tfrac{1}{2}[(31,2)+(23,1)]\}.$$

Then (16) is rewritten as

$$(\tfrac{1}{2})^{1/2}[-(\tfrac{2}{3})^{1/2}][\phi''(12,3)+(-\tfrac{1}{2}\phi'-\tfrac{1}{2}\sqrt{3}\phi'')(23,1) +(-\tfrac{1}{2}\phi'+\tfrac{1}{2}\sqrt{3}\phi'')(31,2)].$$

Obviously, the first term is $(6,3)_8$ under SU(3) ⊗ SU(3). By symmetry of the <u>56</u> SU(6) wave function, the coefficient of (23, 1) and (31, 2) is the same function of 2, 3, 1 and 3, 1, 2, respectively, as ϕ'' is of 1, 2, 3, as can be easily verified. Then we write in an obvious notation

$$(\tfrac{1}{2})^{1/2}(\chi'_{+1/2}\phi'+\chi''_{+1/2}\phi'')=-(\tfrac{1}{3})^{1/2}\sum_p\phi''_{123}(12,3). \tag{17}$$

This formulation has the advantage of showing both the (6, 3) behavior and the symmetry in the three quarks. The formulas

$$\phi''_{231}=-\tfrac{1}{2}\phi''_{123}-\tfrac{1}{2}\sqrt{3}\phi'_{123},\qquad \phi''_{312}=-\tfrac{1}{2}\phi''_{123}+\tfrac{1}{2}\sqrt{3}\phi'_{123} \tag{18}$$

will be very useful in the subsequent calculation.

We now pass to the effect of our mixing operator on the wave function (17). We have to replace the Pauli spinors χ_i ($i=1,2,3$) by

$$\chi_i\to(\tfrac{1}{2})^{1/2}\cosh\tfrac{1}{2}\omega[1+\mu_q p_{iz}+\mu_q W_i^{(+)}p_i^{(-)} +\mu_q W_i^{(-)}p_i^{(+)}]\begin{bmatrix}\chi_i\\ \sigma_{iz}\chi_i\end{bmatrix}; \tag{19}$$

with the notation

$$u_i(\pm\tfrac{1}{2})=\frac{1}{\sqrt{2}}\begin{bmatrix}\chi_i(\pm\frac{1}{2})\\ \sigma_{iz}\chi_i(\pm\frac{1}{2})\end{bmatrix};$$

it amounts to replacing χ_i by u_i in the SU(6) wave function (17) and to applying the over-all operator

$$U=\prod_i[1+\mu_q p_{iz}+\mu_q W_i^{(+)}p_i^{(-)}+\mu_q W_i^{(-)}p_i^{(+)}]. \tag{20}$$

(The over-all factor $\cosh(\omega/2)$ is unimportant since it disappears when the hadron wave functions are normalized.)

Since $W_i^{(\pm)}$ raises or lowers the quark helicity, its effect on a term such as (12, 3) is either to annihilate it or to exchange the position of the corresponding quark. Thus, also taking into account the symmetry of (17) and using (18), it is very easy to calculate the new wave function and to split it into irreducible representations of SU(3) ⊗ SU(3) (see Table I). The identification of the various SU(3)⊗ SU(3) representations is very easy:

(i) One quark alone with $h=\pm\frac{1}{2}$ corresponds to the representation 3 of SU(3)$^{\pm}$;

(ii) Two quarks with $h=\pm\frac{1}{2}$ correspond to $\bar{3}$ or 6 of SU(3)$^{\pm}$, depending on whether the SU(3) wave function is ϕ' (antisymmetric) or ϕ'' (symmetric);

(iii) Three quarks with $h=\pm\frac{1}{2}$ correspond to 8 of SU(3)$^{\pm}$.

In all the three cases, the representation of SU(3) (internal symmetry) remains an octet, since the mixing operator is an SU(3) scalar. We have noted in the table the value of L_z which ensures the conservation of helicity.

TABLE I. Decomposition of the $\frac{1}{2}^+$ low-lying octet wave function at $p_z=\infty$ into SU(3)⊗SU(3) irreducible representations.

$(6,3)_8$, $L_z=0$	$-(\frac{1}{3})^{1/2}\sum_P [(1+\mu_q p_{3z})-\frac{1}{2}\mu_q^2 p_3^+(p_1^- + p_2^-)]\phi''_{123}(12,3)$
$(3,6)_8$, $L_z=+1$	$-(\frac{1}{3})^{1/2}\sum_P [\mu_q^3 p_1^+ p_2^+ p_3^- - \frac{1}{2}\mu_q(p_1^+ + p_2^+)]\phi''_{123}(3,12)$
$(3,\bar{3})_8$, $L_z=+1$	$-(\frac{1}{3})^{1/2}\sum_P \frac{1}{2}\sqrt{3}\,\mu_q(p_1^+ - p_2^+)\phi'_{123}(3,12)$
$(\bar{3},3)_8$, $L_z=0$	$-(\frac{1}{3})^{1/2}\sum_P \frac{1}{2}\sqrt{3}\,\mu_q^2 p_3^+(p_2^- - p_1^-)\phi'_{123}(12,3)$
$(8,1)$, $L_3=-1$	$-(\frac{1}{3})^{1/2}\{\mu_q[p_3^- - \frac{1}{2}(p_1^- + p_2^-)]\phi''_{123}+\frac{1}{2}\sqrt{3}\,\mu_q(p_2^- - p_1^-)\phi'_{123}\}(123,0)$
$(1,8)$, $L_z=+2$	$-(\frac{1}{3})^{1/2}\{\mu_q^2[p_1^+ p_2^+ - \frac{1}{2}p_3^+(p_1^+ + p_2^+)]\phi''_{123}+\frac{1}{2}\sqrt{3}\,\mu_q^2 p_3^+(p_1^+ - p_2^+)\phi'_{123}\}(0,123)$

We now pass to the calculation of the mixing parameters defined by the formula (9); they are just the norms of the various components in Table I, once the total wave function has been normalized. Thus, for the representation (n,m),

$$\alpha^2(n,m)=\frac{(\psi_{(n,m)},\psi_{(n,m)})}{(\psi,\psi)} \qquad (21)$$

the norms being defined by

$$(\psi,\psi)=\int|\psi|^2\prod_i d^3p_i\,\delta^3\Big(\sum_j \vec{p}_j\Big).$$

(Lorentz contraction factors affecting the z axis are unimportant since they disappear in the ratio.)

In Table II we present the results. The parameter of the development x is just μ_q^2/R^2 and characterizes the intensity of the mixing. If $R^2\to\infty$ or $m_q\to\infty$ (nonrelativistic internal velocities), we would get no mixing.

Let us quote briefly the results for $\frac{3}{2}^+$ baryon mixing (the Δ and its strange partners). The starting SU(6) wave function is

$$\chi^s\phi^s \text{ (fully symmetric combinations)}. \qquad (22)$$

For helicity $\frac{1}{2}$ we can write

$$\chi^s_{+1/2}=(\tfrac{1}{3})^{1/2}\sum_P(12,3). \qquad (23)$$

We note that (23) is already in a symmetric form and the calculation is thus straightforward. Table III gives the splitting of the boosted wave function into irreducible SU(3)⊗SU(3) representations. The mixing parameters of the Δ are now defined by

$$|\Delta,h=+\tfrac{1}{2}\rangle =\delta|(6,3)_{10},L_z=0\rangle+\delta'|(3,6)_{10},L_z=+1\rangle +\epsilon|(10,1),L_z=-1\rangle+\epsilon'|(1,10),L_z=+2\rangle, \qquad (24)$$

$$\delta^2+\delta'^2+\epsilon^2+\epsilon'^2=1.$$

Table IV gives the predictions for these parameters as functions of $x=\mu_q^2/R^2$. We see that $\epsilon=0$. This follows from $\sum_i \vec{p}_i=0$, and the complete symmetry of χ^s.

TABLE II. Expression of the mixing parameters (for the low-lying octet) as functions of $x=\mu_q^2/R^2$. $\alpha^2+\alpha'^2+\beta^2+\beta'^2+\gamma^2+\gamma'^2=1$; $D_N=1+\frac{7}{3}x+\frac{16}{9}x^2+\frac{4}{9}x^3$.

$(6,3)_8$	$\alpha^2=(1-\frac{1}{3}x+\frac{2}{9}x^2)/D_N$
$(3,6)_8$	$\alpha'^2=(\frac{1}{6}x-\frac{1}{9}x^2+\frac{4}{9}x^3)/D_N$
$(3,\bar{3})_8$	$\beta^2=\frac{3}{2}x/D_N$
$(\bar{3},3)_8$	$\beta'^2=x^2/D_N$
$(8,1)$	$\gamma^2=x/D_N$
$(1,8)$	$\gamma'^2=\frac{2}{3}x^2/D_N$

V. QUANTITATIVE PREDICTIONS AND COMPARISON WITH PREVIOUS CONFIGURATION MIXING SCHEMES

To get quantitative predictions from Table II, we have now to estimate the parameter

$$x=\frac{\mu_q^2}{R^2}.$$

We emphasize the tentative character of the estimation. Concerning R^2, the radius of the ground-state wave function, we have various estimates:

TABLE III. Decomposition of the $\frac{3}{2}^+$ low-lying decuplet wave function at $p_z=\infty$ into SU(3) ⊗ SU(3) irreducible representations.

$(6,3)_{10}$, $L_z=0$	$(\frac{1}{3})^{1/2}\sum_P [(1+\mu_q p_{3z})+\mu_q^2 p_3^+(p_1^-+p_2^-)]\phi^s_{123}(12,3)$
$(3,6)_{10}$, $L_z=+1$	$(\frac{1}{3})^{1/2}\sum_P [\mu_q(p_1^++p_2^+)+\mu_q^3 p_1^+p_2^+p_3^-]\phi^s_{123}(3,12)$
$(10,1)$, $L_z=-1$	$(\frac{1}{3})^{1/2}\sum_P \mu_q p_3^-\phi^s_{123}(123,0)$
$(1,10)$, $L_z=+2$	$(\frac{1}{3})^{1/2}\sum_P \mu_q^2 p_1^+p_2^+\phi^s_{123}(0,123)$

first, from phenomena involving quark-model wave functions[6]; second, from the mass spectrum.[6,12,16] All the values found fall in the range $R^2=6$ to 12 GeV^{-2} (Ref. 16). Most fall between 6 and 10 GeV^{-2}.

Concerning μ_q, we may take, as is usually done, the "normal" expression $1/2m_q$. If m_q is the effective mass $\frac{1}{3}M_N$, it leads, in the nonrelativistic model, to

$$\mu_p=\mu_q=\frac{3}{2M_N}, \qquad (25)$$

which is roughly satisfactory. When one takes into account the small components due to the internal motion, one gets, as calculated by Bogoliubov,[4]

$$\mu_p=\mu_q(1-\delta), \qquad (26)$$

δ being the same quantity as in formula (8). As pointed out in Ref. 4, it is hard to get exactly the right correction to G_A/G_V and μ_p. With $\mu_q=3/2M_N$ and $x=0.255$, which means $R^2=11$ GeV^{-2}, one gets rough agreement[21]:

$$G_A/G_V=1.20, \quad \mu_p=2.60 \text{ nuclear magnetons}.$$

With this value of x, one predicts the mixing angles (Table V for $\frac{1}{2}^+$, Table VI for $\frac{3}{2}^+$).[22]

Let us show that, even if G_A/G_V is modified, we keep nevertheless the SU(6) prediction for the ratio $(F/D)_{\text{axial}}=\frac{2}{3}$, which agrees well with experiment. The axial-vector operator to be sandwiched between unmixed states will be

$$Q_5^\alpha=U^{-1}\Big(\sum_i \sigma_{iz}\lambda_i^\alpha\Big)U$$
$$=\sum_i U_i^{-1}\sigma_{iz}\lambda_i^\alpha U_i, \qquad (27)$$

where U_i is the corresponding mixing operator at the quark level. We have then

$$Q_5^\alpha=\sum_i[\sigma_{iz}(1-\mu_q^2\vec{p}_{i\perp}^{\,2}) - 2i\mu_q(\sigma_i^{(+)}p_i^{(-)}-\sigma_i^{(-)}p_i^{(+)})]\lambda_i^\alpha. \qquad (28)$$

Now, between states of the $\frac{1}{2}^+$ octet, the second term of (28) linear in $\vec{p}_i$ does not contribute. The first term will give the *same ratio* $(F/D)_{\text{axial}}$ that is given by the SU(6) operator $\sum_i\sigma_{iz}\lambda_i^\alpha$.

For the Δ-N axial-vector-coupling transition constant G^* we get

$$G^*=\frac{\frac{4}{3}(1+x-\frac{5}{9}x^2-\frac{4}{9}x^3)}{(D_N D_\Delta)^{1/2}}=0.95$$

to be compared with the PCAC (partial conservation of axial-vector current) result obtained from the $\Delta\Rightarrow N\pi$ decay,[23] $G^*_{\text{exp}}\simeq 1$.

Let us comment on the tables. In Table V, Buccella I stands for the phenomenological values of the mixing angles taken from sum rules, assuming approximate saturation by the resonances.[7] It is a model-independent determination of the mixing parameters. Buccella II corresponds to the assumption of a $\underline{56}\oplus\underline{70}$ configuration mixing[3] (also proposed by Lipkin, Rubinstein, and Meshkov[3]), plus a contribution from the Roper resonance. Harari and Gatto, Maiani, and Preparata[3] have a $\underline{56}\oplus\underline{20}$ mixing. Finally, Gershtein and Lee assume

$$|N,h=+\tfrac{1}{2}\rangle=(6,3)_8\oplus(3,\bar{3})_8\oplus(8,1).$$

As concerns the $\frac{3}{2}^+$, the last three references assume no mixing.

TABLE IV. Expression of the mixing parameters (for the $\frac{3}{2}^+$ low-lying decuplet) as a function of $x=\mu_q^2/R^2$. $\delta^2+\delta'^2+\epsilon^2+\epsilon'^2=1$; $D_\Delta=1+\frac{7}{3}x+\frac{20}{9}x^2+\frac{1}{9}x^3$.

$(6,3)_{10}$	$\delta^2=(1+\frac{5}{3}x+\frac{8}{9}x^2)/D_\Delta$
$(3,6)_{10}$	$\delta'^2=(\frac{2}{3}x+\frac{7}{9}x^2+\frac{1}{9}x^3)/D_\Delta$
$(10,1)$	$\epsilon^2=0$
$(10,1)$	$\epsilon'^2=\frac{5}{9}x^2/D_\Delta$

TABLE V. Comparison of our prediction for the mixing parameters ($\frac{1}{2}^+$ low-lying octet) with their phenomenological determination (Buccella I) and previous configuration-mixing hypotheses. $R^2=11\ \text{GeV}^{-2}$.

Mixing parameters	This work		Buccella I	Gershtein and Lee	Harari, Gatto, Maiani, and Preparata	Buccella II
$\alpha^2(6,3)_8$	0.57	$\alpha^2+\alpha'^2=0.59$	$\alpha^2+\alpha'^2=0.59$	0.375	0.64	0.580
$\alpha'^2(3,6)_8$	0.023			0	0	0.055
$\beta^2(3,\bar{3})_8$	0.21	$\beta^2+\beta'^2=0.24$	$0.26\le\beta^2+\beta'^2\le 0.41$	0.375	0.24	0.298
$\beta'^2(\bar{3},3)_8$	0.033			0	0.12	0.044
$\gamma^2(8,1)$	0.14	$\gamma^2+\gamma'^2=0.16$	$\gamma^2+\gamma'^2\le 0.15$	0.25	0	0.023
$\gamma'^2(1,8)$	0.022			0	0	0

One observes a rather striking agreement with the phenomenological values (Buccella I). We also have a qualitative agreement with the mixing hypothesis of Gershtein and Lee (the dominant representations are the same). Moreover, we disagree with the mixing schemes which assume a simple mixing between representations of SU(6): (i) We disagree with $\underline{56}\oplus\underline{70}$, because we do not have the $(\bar{3},3)_8$ representation at first order in $\vec{p}_i$ (however, the first order representations belong to $\underline{70}$), and (ii) with $\underline{56}\oplus\underline{20}$, because we have (8, 1) and we do not have $(\bar{3},3)_8$ at first order in $\vec{p}_i$. An interesting aspect is that we do have mixing of the ground state with the first radial excitation, often identified with the Roper resonance. To see this let us write the $(6,3)_8$ component of Table I in the form

$$\sum_P\left[(1+\mu_q p_{3z})-\tfrac{1}{2}\mu_q{}^2\vec{p}_{3\perp}{}^2\right]\phi''_{123}(12,3)\,.$$

The second term in the square brackets can be decomposed into a ground state and a first radial excitation polynomial.

As assumed by various authors, the $\frac{3}{2}^+$ is predicted to be almost unmixed (this is a consequence of the spin wave-function symmetry), as shown in Table VI.

TABLE VI. Comparison of our prediction for the mixing parameters ($\frac{3}{2}^+$ low-lying decuplet) with previous configuration-mixing hypotheses.

Mixing parameters	This work	Harari	Gershtein and Lee
$\delta^2(6,3)_{10}$	0.863	1	1
$\delta'^2(3,6)_{10}$	0.119	0	0
$\epsilon^2(10,1)$	0	0	0
$\epsilon'^2(1,10)$	0.018	0	0

On the whole, the success of the quark model is rather impressive. Once we have reasonable values for G_A/G_V and the nucleon magnetic moments which correspond to values of the quark effective mass and a value for R^2 roughly in agreement with what we can expect from the properties of the hadron spectrum (masses, as well as transition amplitudes), the whole mixing scheme of the baryons is predicted. This is a rather detailed test of the wave functions. One should not, however, overemphasize the purely quantitative aspect of the results, because there are serious uncertainties concerning the constants R^2 and $\mu_q{}^2$, and the results are rather sensitive to these constants (see Fig. 1). What is remarkable is that there exist reasonable values of R^2 and $\mu_q{}^2$ (included in the anticipated range) for which we predict the correct mixing. This success is not an accident; we have done the parallel calculations for mesons: The $\pi-A_1(0)$ and the $\rho(\pm1)-B(\pm1)$ mixing angles are, respectively, 45° and 30°, in agreement with the phenomenological calculations of Weinberg,[24] Gilman and Harari,[25] and Buccella, Kleinert, Savoy, Celeghini, and Sorace.[9] These results and other consequences of the mixing operator for mesons will be published elsewhere.

VI. RELATED THEORETICAL IDEAS

Related theoretical ideas can be found in some earlier works of Gell-Mann *et al.*[1,8] Namely, he is often led to use the naive quark model as a hint and he recognizes that the relativistic quark velocities may provide corrections to the SU(6) results. Thus, he finds an expression for G_A/G_V quite similar to that of Bogoliubov,

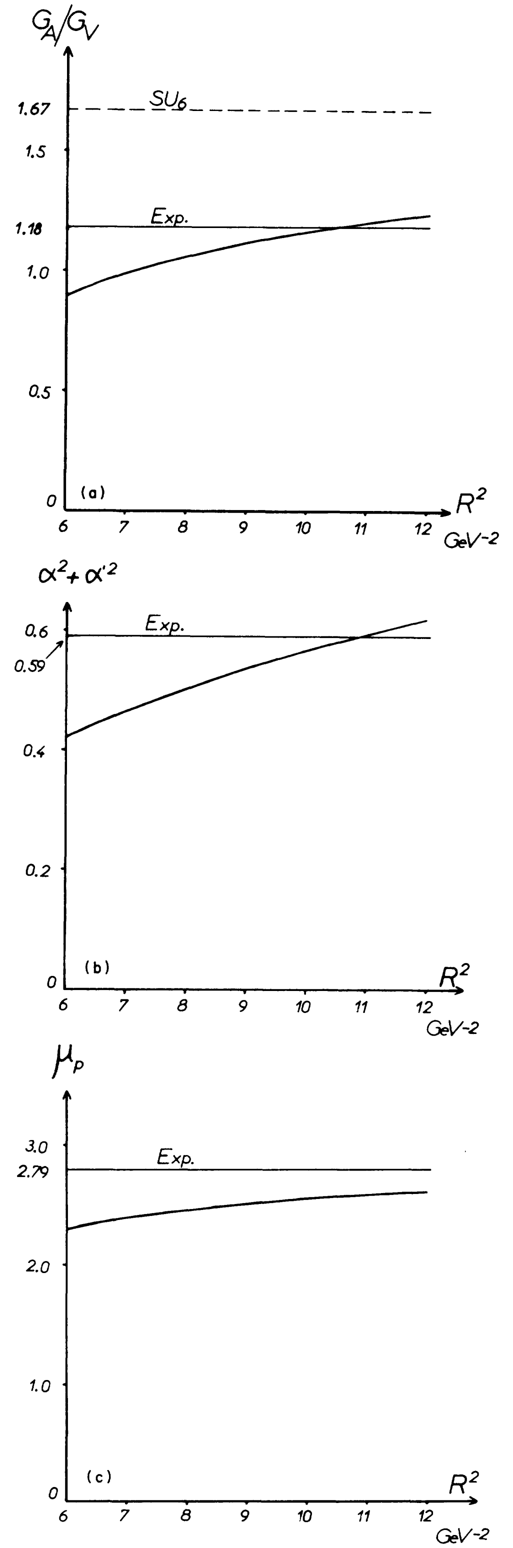

FIG. 1. The curves give G_A/G_V, $\alpha^2+\alpha'^2$, and μ_p as functions of R^2 for $\mu_q = 1/2m_q = 3/2m_n$. The horizontal solid lines indicate the experimental values. In Fig. 1(a) we indicate the SU_6 value by a dashed line.

$$\frac{G_A}{G_V} = \frac{5}{3}\left(1 - \frac{\langle \vec{p}^2 \rangle}{3m^2} + \cdots\right) .$$

He discusses the role of the Foldy-Wouthuysen transformation in the evaluation of such relativistic corrections. He represents the corrections by a phenomenological unitary transformation e^{iS} at the quark level, which is constrained to fulfill some "angular conditions." In fact, this e^{iS} operator is just our Wigner rotation of quark spins. Moreover, we have shown that the Foldy-Wouthuysen transformation and the Wigner rotation are two versions—in different frames—of the same relativistic phenomenon. We suspect that this may explain the similarity between our results and the structure of the Melosh operator,[10] as reported by Gilman and Kugler[26] and Hey and Weyers.[27] From (28), it is trivial to see that the corrective term to the SU(6) axial charge behaves like the corrective term in the Melosh operator,[28]

$$\{(3,\bar{3})_{\Delta h=+1}, L_z=-1\} - \{(\bar{3},3)_{\Delta h=-1}, L_z=+1\} .$$

Although there is undoubtedly a relation between our approach and that of Gell-Mann, we think that they are not equivalent. We try to keep real quarks endowed with a real spatial motion, described by a wave function. We believe that the main problem is to carefully analyze this motion; for instance the (harmonic) mass spectrum suggests that the motion is highly relativistic and moreover gives an estimate of the momentum distribution. The richness of a relativistic motion (revealed through the presence of small components, pair creation, etc.) provides an explanation for numerous involved phenomena such as the configuration mixing. We are also able to predict semiquantitatively the mixing parameters on the basis of the naive quark-model classification of hadrons.

Important efforts in order to formulate the SU(3) ⊗ SU(3) mixing have also been made by a number of physicists.[3,7,9,29,30] Besides their phenomenological analyses, Buccella *et al.* have proposed, while trying to sum up their conclusions, a mixing operator $e^{i\theta Z}$, where $Z = (\vec{W}\times\vec{M})_z$, $\vec{M}$ being some vector, and $\vec{W}$ the W-spin generators; our mixing operator has just this structure, the first-order term of U being

$$-i\mu_q \sum_i (\vec{W}_i \times \vec{p}_i)_z .$$

In a later work,[30] they try to satisfy the Weinberg conditions on the mass spectrum and the chiral charges.[24] They find that $\vec{M}$ "is the coordinate of the three-dimensional harmonic oscillator" and that the spectrum to lowest order in θ is the harmonic one plus a spin-orbit coupling. Thus they find some aspects of our results, but in their work, (i) the form of Z is introduced *a priori*, and (ii) the relation between the parameter θ and the hadron spectrum remains unexplained.

As shown in Refs. 9, 24, 25, and 26, the configuration mixing for mesons leads to a rough agreement with experiment for some axial-vector transition matrix elements ($A\to\rho\pi$, $B\to\omega\pi$) which were serious problems for the $SU(6)_W$ scheme. In the frame of the quark model, another way to describe these processes leading to similar results has been provided by the quark pair-creation model of strong-interaction vertices.[31] In our realistic formulation of the model[32] the decay process is described by a pair-creation operator which couples the spin and the relative momentum of the pair in a $^3P_0$ state. A sizable departure from the SU(6) results appears when one takes into account the internal motion (defined by the same wave functions as in the present paper). The interpretation of PCAC in the frame of the quark model would explain how the axial-vector coupling with mixing leads to the same results as the pure strong-interaction process of the quark pair-creation model.

VII. CONCLUSION

In the frame of the realistic quark model we have tried to explain, for axial-vector–current matrix elements, the systematic discrepancies between the experiment and the predictions of the nonrelativistic treatment. We follow the proposal by Bogoliubov that G_A/G_V can be reduced from $\frac{5}{3}$ to the experimental value by relativistic corrections due to the small components of the quark Dirac spinors. This proposal is strongly supported by the estimate of the quark velocity as suggested by the hadron spectrum and other phenomena. We make the hypothesis that under relativistic conditions, the SU(6) wave functions of the simple naive model are still valid, Pauli spinors being, however, replaced by Dirac spinors. We then predict at $p_z=\infty$, through the Wigner rotations of quark spins, the whole $SU(3)\otimes SU(3)$ configuration mixing used by current algebraists. Moreover, we predict semiquantitatively the whole set of mixing angles for baryons and mesons from the estimate of the quark internal velocity. We believe that this is a rather remarkable success for the "naive" or "realistic" quark model.

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[11]The presence of a magnetic moment operator $\vec{\sigma}_i$ (between Pauli spinors) comes from the small components of the Dirac spinors proportional to the momentum transfer.

[12]For instance, Feynman, Kislinger, and Ravndal apply the Bargmann-Wigner condition in their relativistic harmonic-oscillator quark model and this amounts to canceling the small components in the hadron center-of-mass system. See R. P. Feynman, M. Kislinger, and F. Ravndal, Phys. Rev. D 3, 2706 (1971).

[13]Note that many works on photoproduction and electroproduction in the quark model lead to the same conclusion. The convective term in the Foldy-Wouthuysen expansion of the quark current, which gives a measure of the internal quark velocity, needs to be important in order to correctly describe photoproduction amplitudes, as shown by Copley, Karl, and Obryk (Ref. 6). In fact, in the relativistically correct description of the electromagnetic interactions of composite systems,

the small components of the Dirac spinors in the bound-state center-of-mass system play a crucial role, as shown by S. J. Brodsky and J. R. Primack [Ann. Phys. (N.Y.) 52, 315 (1969)] and by F. E. Close and L. A. Copley [Nucl. Phys. B19, 477 (1970)], who emphasized the possible importance of such terms in the quark-model calculations.

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[21]Photoproduction amplitudes calculated in the harmonic-oscillator quark model by K. C. Bowler [Phys. Rev. D 1, 926 (1970)], including terms of the order m_q^{-2} in the Foldy-Wouthuysen expansion, seem to fit with a value $R^2 = 11$ GeV^{-2}. This may be another indication that a complete relativistic treatment of the quark spin leads to higher values of R^2 than the nonrelativistic one.

[22]In fact the spatial wave function in Ref. 5 plays the role of a cutoff in the calculation of the mean value $(\langle \vec{p}^2 \rangle)^{1/2}$, which determines the mixing parameters. Any wave function (of a Coulomb or a Yukawa potential, for instance) could play such a role, and by itself, this calculation is not a sensitive test of the harmonic oscillator. The Coulomb potential has been suggested because of the smooth q^2 behavior of the electromagnetic form factors. [H. Lipkin, in *Proceedings of the Fifth International Conference on Elementary Particles, Lund, 1969,* edited by G. von Dardel (Berlingska Boktryckeriet, Lund, Sweden, 1970).] However, the harmonic oscillator can explain the dipole shape of the nucleon form factors by the Lorentz contraction of the wave function (Ref. 17) and is well tested for the excited hadronic states (Ref. 12). So we think it better to stick to the harmonic-oscillator model. This allows us to compare our parameter R^2 to other determinations by other phenomena.

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[28]After the issue of the report version of this work, we have studied the Melosh thesis (Ref. 10). We think indeed that our works are equivalent on some main points, in particular on the result that the chiral SU(3) ⊗ SU(3) configuration mixing is produced by transverse-momentum components of the quarks, and on the relation between the Wigner rotation and the Foldy-Wouthuysen transformation. However, the two approaches are different. He uses the shortcut of the *W* spin and $SU(6)_W$ formalism to avoid the necessity of boosts, and obtains the mixing operator by constraining the generators of the $SU(6)_W$ group that classifies the hadron states to commute with the Hamiltonian (assumed to be the one of the quark free-field theory). In our approach we start from the highly relativistic motion of the quarks inside the hadrons (even in the hadron center-of-mass system) suggested by the mean level spacing of the hadronic spectrum or by the high value of the convective term in the quark electromagnetic current required by photoproduction (Ref. 13). Then we consequently modify the center-of-mass wave functions by substituting Pauli spinors by Dirac spinors. (We have adopted free Dirac spinors, as is implicit in the quark free-field theory approach of Melosh.) The calculation of the mixing angles at $p_z = \infty$ turns out to be a test of our hypothesis for the wave functions at rest. We relate in this way two apparently disconnected regions and phenomena: the successes of the harmonic-oscillator quark model at low energies and the mixing parameters needed by current algebraists at $p_z = \infty$.

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