Role of the Pomeron in tensor-meson dominance*

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(Received 15 October 1973)

We include the Pomeron contribution in the tensor-meson dominance relations and find that (i) the previous discrepancies are resolved, (ii) the mass radii satisfy the Gell-Mann–Okubo type relations, and (iii) some bounds on the Pomeron slope and the tensor F/D ratio, consistent with experiments, are obtained.

The success of the vector-meson dominance of the electromagnetic current has led to efforts towards a study of tensor-meson dominance of the energy-momentum tensor.[1] Several authors have investigated saturation schemes with known tensor mesons.[2,3] If only the f and f' meson contributions are retained, certain universality-type relations for their couplings are obtained which are in conflict with the available experimental information.[2] This has led to the inclusion of subtraction constants,[3,4,5] the existence of exotic SU(3) singlet tensor mesons,[6] etc. Although it has been suggested that the Pomeron also contributes to the matrix elements of the energy-momentum tensor,[3,6] an explicit calculation taking its contribution into account has not been carried out. In the present work we investigate the role of the Pomeron in tensor dominance at a phenomenological level without *a priori* restrictions based on the notions of duality, strong exchange degeneracy, universality, etc. Also we consider the matrix elements of the energy-momentum tensor for the pseudoscalar meson and the baryon octets.

We begin by defining the couplings and the form factors. The matrix elements of the energy-momentum tensor $\theta_{\mu\nu}$ for the mesons M_i are given by

$$\langle M^i(p')|\theta_{\mu\nu}|M^i(p)\rangle = \tfrac{1}{2}P_\mu P_\nu F_1^M(q^2) + (q^2 g_{\mu\nu} - q_\mu q_\nu)F_2^M(q^2), \tag{1}$$

where $P=(p+p')$ and $q=p-p'$, and the states are normalized such that

$$\langle M^i(p')|M^i(p)\rangle = 2p_0(2\pi)^3\delta^3(\vec{p}-\vec{p}\,'). \tag{2}$$

For the baryons we have

$$\langle B^i(p')|\theta_{\mu\nu}|B^i(p)\rangle = \bar{u}(p')\left[(\gamma_\mu P_\nu + \gamma_\nu P_\mu)\tfrac{1}{4}F_1^B(q^2) + P_\mu P_\nu \frac{1}{4M_B}F_2^B(q^2) + (q^2 g_{\mu\nu} - q_\mu q_\nu)\frac{1}{M_B}F_3^B(q^2)\right]u(p), \tag{3}$$

with the same normalization as for mesons. At zero momentum transfer the form factors are normalized to

$$F_1^M(0)=1, \quad F_1^B(0)=1, \quad F_2^B(0)=0. \tag{4}$$

The form factors allow us to define the tensor-mass radii to be

$$r_M{}^2 = 6\frac{F_1^{M\prime}(0)}{F_1^M(0)}, \tag{5a}$$

$$r_B{}^2 = 6\left(\frac{F_1^{B\prime}(0)+F_2^{B\prime}(0)}{F_1^B(0)+F_2^B(0)}\right), \tag{5b}$$

where $F'(0)$ denotes $dF(q^2)/dq^2|_{q^2=0}$. The tensor-meson couplings to the stress tensor are given by

$$\langle T|\theta_{\mu\nu}|0\rangle = m_T{}^3 g_T \epsilon_{\mu\nu}, \tag{6}$$

and the couplings of the tensor mesons to the pseudoscalar mesons and the baryons are

$$\langle M^i(p')|T|M^i(p)\rangle = \frac{G_{TMM}}{2M_T}P_\mu P_\nu \epsilon_{\mu\nu}, \tag{7}$$

$$\langle B^i(p')|T|B^i(p)\rangle = \epsilon_{\mu\nu}\bar{u}_B(p')\left[(\gamma_\mu P_\nu + \gamma_\nu P_\mu)\frac{G_{TBB}^{(1)}}{4m_T} + \frac{P_\mu P_\nu}{4m_B m_T}G_{TBB}^{(2)}\right]u_B(p). \tag{8}$$

Here $i=1\cdots 8$ is the SU(3) index.

Now the nature of the Pomeron singularity has not been resolved as yet. For our purpose we shall treat it as a factorizable linear Regge trajectory with intercept unity and slope α_P'. The Pomeron contribution will be taken into account by introducing a spin 2^+ particle of mass $M_P{}^2 = 1/\alpha_P'$, with the Pomeron couplings given by (6), (7), and (8) with the replacement of T by the Pomeron (P). We consider this as simply a convenient way of parametrizing the Pomeron contribution and the actual existence of such a particle is not crucial. Moreover, the Pomeron will be regarded as an SU(3) singlet. This seems to be in agreement with high energy experiments to within 10–20%.

On saturating the form factors F_1^M and F_1^B, F_2^B with the f and f' mesons and the Pomeron poles we obtain from the equations (4) and (5) the relations

$$1 = g_f G_{fM_iM_i} + g_{f'} G_{f'M_iM_i} + g_P G_{PM_iM_i}, \tag{9}$$

$$1 = g_f G^{(1)}_{fB_iB_i} + g_{f'} G^{(1)}_{f'B_iB_i} + g_P G^{(1)}_{PB_iB_i}, \tag{10}$$

$$0 = g_f G^{(2)}_{fB_iB_i} + g_{f'} G^{(2)}_{f'B_iB_i} + g_P G^{(2)}_{PB_iB_i}, \tag{11}$$

and

$$r_M^2 = 6\left(\frac{g_f G_{fM_iM_i}}{m_f^2} + \frac{g_{f'} G_{f'M_iM_i}}{m_{f'}^2} + \frac{g_P G_{PM_iM_i}}{m_P^2}\right), \tag{12}$$

$$r_B^2 = 6\left[\left(\frac{g_f G^{(1)}_{fB_iB_i}}{m_f^2} + \frac{g_{f'} G^{(1)}_{f'B_iB_i}}{m_{f'}^2} + \frac{g_P G^{(1)}_{PB_iB_i}}{m_P^2}\right) + (1 \to 2)\right]. \tag{13}$$

The octet and singlet components of f and f' are given by

$$f_0 = f\cos\theta - f'\sin\theta, \quad f_8 = f\sin\theta + f'\cos\theta, \tag{14}$$

and the SU(3)-symmetric tensor couplings are

$$G_{TM_iM_i} M_i M_i T = G_{f\pi\pi}\{f[(\vec{\pi}\cdot\vec{\pi}) + (\cos^2\theta - \tfrac{1}{2}\sin^2\theta)(2\bar{K}K) + (\cos^2\theta - \sin^2\theta)\eta_0\eta_0] + f'[(-2\sin\theta\cos\theta\,\eta_0\eta_0 + (-\tfrac{3}{2})\sin\theta\cos\theta\,(2\bar{K}K)]\} \tag{15}$$

and

$$G_{TB_iB_i}\bar{B}_i B_i T = \frac{G_{fNN}}{(3F-D)}\{f[(3F-D)\bar{N}N + (3F(\cos^2\theta - \sin^2\theta) - D)\bar{\Xi}\Xi + (D(2\sin^2\theta - \cos^2\theta) + 3\cos^2\theta F)\bar{\Sigma}\Sigma + (D(-2\sin^2\theta - \cos^2\theta) + 3F\cos^2\theta)\bar{\Lambda}\Lambda] + (-3\sin\theta\cos\theta)f'[(2F)\bar{\Xi}\Xi + (F-D)\bar{\Sigma}\Sigma + (F + \tfrac{1}{3}D)\bar{\Lambda}\Lambda]\}, \tag{16}$$

where $F+D=1$. In agreement with experiment f' has been decoupled from $\pi\pi$ and $\bar{N}N$ in the above.

In order that the normalization conditions (9), (10), and (11) be satisfied for the entire multiplet it is clearly necessary that the octet parts of f and f' cancel in these equations. From (15) and (16) it is seen that this can be achieved by taking

$$\frac{g_{f'}}{g_f} = -\tan\theta. \tag{17}$$

From mass formulas for the tensor nonet $\theta \cong 30.5°$. Then all of the equations (9), (10), and (11) simplify to the following three relations:

$$1 = g_f G_{f\pi\pi} + g_P G_{P\pi\pi}, \tag{18}$$

$$1 = g_f G^{(1)}_{fNN} + g_P G^{(1)}_{PNN}, \tag{19}$$

$$0 = g_f G^{(2)}_{fNN} + g_P G^{(2)}_{PNN}. \tag{20}$$

Since the Pomeron exchange in elastic scattering is known to be consistent with s-channel helicity conservation[7] we expect $G^{(2)}_{PBB} \approx 0$, and this leads to $G^{(2)}_{fBB} = 0$. This is consistent with one of the determinations of the f couplings.[8] If in future, experiments indicate that $G^{(2)}_{fBB} \neq 0$ but $G^{(2)}_{PBB}$ is still zero then (20) will have to be modified either by a subtraction constant or extra contributions. For the present we assume that

$$G^{(2)}_{fBB} = G^{(2)}_{PBB} \approx 0. \tag{21}$$

Now we use Eqs. (18)–(20) to eliminate g_f and g_P in the expressions for tensor mass radii. Defining

$$X = G_{PNN}/G_{P\pi\pi}, \quad Y = G_{fNN}/G_{f\pi\pi}, \tag{22}$$

and using $\alpha'_P m_P^2 = 1$, $m_f^2/m_{f'}^2 = 0.7$, we have

$$r_\pi^2 = 6\left[\frac{(Y-1)}{(Y-X)}\alpha'_P - \frac{(X-1)}{(Y-X)m_f^2}\right], \tag{23}$$

$$r_K^2 = r_\pi^2 + 6\left[\frac{(X-1)}{(Y-X)m_f^2}(0.45\sin^2\theta)\right], \tag{24}$$

$$r_\eta^2 = r_\pi^2 + 6\left[\frac{(X-1)}{(Y-X)m_f^2}(0.6\sin^2\theta\right], \tag{25}$$

and

$$r_N^2 = 6\left[\frac{X(Y-1)}{(Y-X)}\alpha'_P - \frac{Y(X-1)}{(Y-X)m_f^2}\right], \tag{26}$$

$$r_\Lambda^2 = r_N^2 + 6\,\frac{Y(X-1)}{(Y-X)m_f^2}\left(\frac{0.2F+0.1}{4F-1}\right)(3\sin^2\theta), \tag{27}$$

$$r_\Sigma^2 = r_N^2 + 6\,\frac{Y(X-1)}{(Y-X)m_f^2}\left(\frac{0.6F-0.3}{4F-1}\right)(3\sin^2\theta), \tag{28}$$

$$r_\Xi^2 = r_N^2 + 6\,\frac{Y(X-1)}{(Y-X)m_f^2}\left(\frac{0.6F}{4F-1}\right)(3\sin^2\theta). \tag{29}$$

Tensor-mass radii represent the distribution of the hadronic matter in particles. Here we have expressed these radii in terms of experimentally determinable quantities by using tensor-meson dominance. While a direct observation of these radii requires probing particles by the graviton, information about hadronic distribution of matter is beginning to be available from various other theoretical approaches also. For example a simple connection between pp scattering and the proton electromagnetic form factor suggests some equivalence of matter density and charge density.[9] Also Y. S. Kim and one of us (K. V.) have considered inverse scattering formalism to obtain the nucleon and the ρ-meson strong-interaction radii from the S-matrix approach using experimental information on phase shifts and bound-state parameters.[10]

Quite independent of any numerical values of the parameters in Eqs. (23)–(29) we see that the (radius)^2 satisfy Gell-Mann–Okubo type relations. Thus

$$4r_K{}^2 = 3r_\eta{}^2 + r_\pi{}^2 \tag{30}$$

and

$$2(r_N{}^2 + r_\Xi{}^2) = r_\Sigma{}^2 + 3r_\Lambda{}^2 . \tag{31}$$

Note that the saturation of baryon and meson matrix elements of the commutators of the conformal generator K_0 with the vector or axial-vector currents j_μ and their divergences $\partial_\mu j_\mu$, in the infinite-momentum limit, leads to the result that the radii within the meson or baryon multiplets are equal.[4,5] However, here we shall not invoke properties of the currents under the conformal transformations.

If we assume that the mass radii *within the same multiplets* increase as the masses increase, then we at once obtain restrictions on the coupling constants. Thus $r_\eta{}^2 > r_K{}^2 > r_\pi{}^2$ requires that

$$\frac{X-1}{Y-X} > 0 . \tag{32}$$

In the following we will show that experiments indicate $X > 1$; hence we have $Y > X > 1$. If we requre $r_\Xi{}^2 > r_\Sigma{}^2 > r_\Lambda{}^2 > r_N{}^2$ we obtain a constraint on the F/D ratio in $G^{(1)}_{fB_iB_i}$ couplings. In particular, it is found that $F > 1$, or F/D is negative. This is in excellent agreement with the Regge pole fits of Barger *et al.*[11] and Berger and Fox.[12] A typical value of $F = 1.25$ to 1.4 is obtained in Refs. 11 and 12.

Further constraints can be obtained by demanding that all the r^2 are positive since the matter density is expected to be positive. The only additional restrictions arise from $r_\pi{}^2 > 0$ and $r_N{}^2 > 0$ and we have

$$\alpha'_P > (X-1)/m_f{}^2(Y-1) \tag{33}$$

and

$$\alpha'_P > Y(X-1)/m_f{}^2 X(Y-1), \tag{34}$$

respectively. Since $Y > X$ as shown above the condition (34) is the more restrictive one.

Now we consider some typical numerical estimates. The Pomeron couplings can be obtained by comparing the contribution of tensor meson (2^+) pole on the P trajectory with the factorizable Regge pole contribution at $t=0$ which is simply related to the total cross sections. An elementary calculation then gives

$$\sigma_{\pi\pi}(\infty) = CG_{P\pi\pi}{}^2 , \tag{35}$$

$$\sigma_{\pi N}(\infty) = CG_{P\pi\pi}G^{(1)}_{PNN} , \tag{36}$$

$$\sigma_{NN}(\infty) = C(G^{(1)}_{PNN})^2 , \tag{37}$$

where $C = \frac{1}{2}\pi(\alpha'_P)^2 S_0$, S_0 being the usual factor for Regge contributions. Thus if α'_P is known, $G_{P\pi\pi}$, $G^{(1)}_{PNN}$ can be determined from the asymptotic cross sections. Since we regard the Pomeron as a factorizable Regge pole with $\alpha_P(0)=1$, possible logarithmic rise of asymptotic cross sections is outside the scope of our work. At any rate this determination should be approximately valid. For the present, however, we will just consider the ratios of the coupling constants. From (35), (36), and (37) we have

$$X = G^{(1)}_{PNN}/G_{P\pi\pi} = \sigma_{\pi N}(\infty)/\sigma_{\pi\pi}(\infty) = \sigma_{NN}(\infty)/\sigma_{\pi N}(\infty) . \tag{38}$$

The quark model predicts the last ratio to be $\frac{3}{2}$ and this is consistent with experiment to within 15%. (Thus $X > 1$.) The ratio Y is more difficult to extract. If we make a similar comparison with the Regge pole $(P' = f)$ exchange we find

$$Y = G^{(1)}_{fNN}/G_{f\pi\pi} = \begin{cases} 2.05\text{–}2.43 \text{ (Ref. 11)} \\ 1.9 \text{ (Ref. 12)}. \end{cases} \tag{39}$$

In our normalization $G_{f\pi\pi}$ is related to the $f \to 2\pi$ decay width by

$$G_{f\pi\pi}{}^2/4\pi = \tfrac{5}{2}\Gamma m_f{}^4/|\vec{k}|^5 \tag{40}$$

leading to $G_{f\pi\pi} \simeq 10.8$. Dispersion relation estimates for Y vary considerably. Engels[8] finds $Y = 3.44 \pm 0.33$ and $G^{(2)}_{fNN} \approx 0$, whereas Schlaile and Strauss[13] find nonzero values for $G^{(2)}_{fNN}$ and $Y = 2.31$ and $Y = 3.7 \pm 1.1$, respectively. Goldberg[14] obtains $Y = 1.95$ using fixed-u dispersion relations and Liu and McGee[15] have $Y = 2.3$–2.75 from continuous moment sum rules with nonzero $G^{(2)}_{fNN}$ having large uncertainties. Note that in the absence of Pomeron contributions Eqs. (18) and (19) lead to a unique value[2,3] $Y = 1$. On the other hand Freund[6] has suggested that strong exchange degeneracy and universality of the vector-meson couplings induces a universality into the tensor couplings leading to $Y = \frac{3}{2}$. In spite of the variations in experimental numbers it appears that $Y = 1$ or $\frac{3}{2}$ are not consistent with experiment.

With $X = \frac{3}{2}$ the following lower bounds are obtained for α'_P, i.e.,

$$\alpha_P' > \begin{Bmatrix} 0.43\ \text{GeV}^{-2} \\ 0.32\ \text{GeV}^{-2} \\ 0.29\ \text{GeV}^{-2} \end{Bmatrix} \text{ for } Y = \begin{cases} 2 \\ 3 \\ 4 \end{cases} . \tag{41}$$

Alternately, for a given value of α_P' we can obtain bounds on Y (or on $G_{fNN}^{(1)}$). Recent fits[16] based on Serpukhov and CERN ISR data indicate that the Pomeron slope lies in the range $0 < \alpha_P' < 0.45$.

Now in Ref. 10 it was found that the strong interaction radius r_N of the nucleon is in the range of 0.15 to 0.50 fermi. The bootstrap hypothesis ascribes essentially the whole mass of the particles to self-consistent strong interactions. In accordance with this if we assume that this strong interaction radius is the same as the tensor mass radius found here we obtain $0.3 < \alpha_P' < 0.9\ \text{GeV}^{-2}$ for values of Y between 2 and 4. Also it is clear that r_N cannot be arbitrarily large unless Y is close to $\frac{3}{2}$ since there is an upper limit on α_P' ($\approx 1\ \text{GeV}^{-2}$). An interesting point is that in Regge-pole potential scattering theory the radius R of a particle is given by

$$R^2 = 4(2\alpha + 1)\alpha' .$$

This gives a typical value of $R \simeq 0.6$ fermi. The proportionality of R^2 to α' is consistent with our formulation. Thus in spite of the current experimental inaccessibility of the mass radii, a consistent interesting picture emerges.

As we have already mentioned, Freund[6] has examined the question of tensor dominance in a broad framework covering several ideas. In that work, however, strong exchange degeneracy, universality, and strict duality were imposed and the mass radii were not considered. One result was that $X = Y = \frac{3}{2}$ and hence the Pomeron contribution could not resolve the discrepancies of f couplings. Large contributions of some exotic multi-quark SU(3) singlet tensor mesons were required. It remains for the future experiments to discover such exotic mesons. In the meantime it appears that many of the above-mentioned theoretically attractive concepts are found to be broken in various contexts. In addition the nature of Pomeron itself is not clear. It has not yet been possible to generate the Pomeron through dual quark models. This has motivated our reexamination of this question on a phenomenological basis, although we can not rule out contributions from exotic mesons. We can, however, remove the discrepancies by using the experimental information without imposing the duality, exchange degeneracy, and universality constraints. Apart from resolving the inconsistency arising from saturation of tensor dominance relations by f and f' mesons only, we are able to obtain various bounds on the Pomeron slope and the F/D ratio for tensor-meson couplings. In addition the mass radii have been shown to obey the Gell-Mann–Okubo type relations.

We wish to thank the Purdue Particle Physics Group for discussions and Professor Rosen, Professor Sugawara, and Professor Loeffler for hospitality during the summer of 1973. One of us (K. V.) wishes to aknowledge the award of an I. U. Summer Research Fellowship.

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*Work supported in part by the U. S. Atomic Energy Commission.

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