Relations between the Ball amplitudes and the zeros of ρ^0-decay density matrix elements*

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(Received 17 December 1973)

We present experimental evidence that in $\pi^- p \to \rho^0 n$, ${}^t\rho^{11}_{1-1} \approx 0$ in an extended range of s and t and $\mathrm{Re}\,{}^s\rho^{11}_{10} = 0$ at $t \approx -\mu^2$ independently of s. These two facts are shown to be correlated and imply fundamental relations between the Ball invariant amplitudes.

Recent experimental results and their phenomenological analyses have revived interest in the study of the ρ^0-meson production process.[1] Comparisons of recent high-statistics polarized photoproduction[2] $(\gamma + N \to \pi + N)$ and ρ^0 production[3] $(\pi^- + p \to \rho^0 + n)$ experiments have given us a clue to find fundamental dynamical properties of the invariant amplitudes associated with these processes.

We denote t- and s-channel density matrix elements of the ρ^0 meson by ${}^t\rho^{11}_{mm'}$ and ${}^s\rho^{11}_{mm'}$ respectively. In brief, we observe the following experimental results. In Fig. 1 we show the density matrix elements ${}^t\rho^{11}_{1-1}$ for $\pi^- p \to \rho^0 n$ at pion lab momenta[4] of 2.7, 4.1, 6, 7, 8, 15, and 17 GeV/c for the momentum transfer range $0 < |t| < 8\mu^2$. The truly striking fact that ${}^t\rho^{11}_{1-1} \approx 0$ is observed. Note that the zero is nontrivial in the sense that it is obtained from nonvanishing amplitudes which contribute significantly to ${}^t\rho^{11}_{11}$ (${}^t\rho^{11}_{11} \approx 0.1$). In Fig. 2 we demonstrate another remarkable fact, that $\mathrm{Re}\,{}^s\rho^{11}_{10}$ passes through zero at $t \approx -\mu^2$ in an energy-independent manner.[3,5] In addition we note that the zero of the D_- amplitude[6] (defined below), the maximum of the asymmetry ratio in pion photoproduction,[7] and the ratio ${}^s\rho^{11}_{1-1}/{}^s\rho^{11}_{11}$ in ρ^0 production[3,8] all occur at the same value of t. The observed similarities between the two reactions and their energy independence suggest that we are observing a manifestation of a general result.

Now we discuss the implications of these results. We introduce the usual s-channel helicity amplitudes $H^{\lambda}_{\lambda_p,\lambda_n}$ where λ, λ_n, and λ_p denote the ρ-meson, neutron, and proton helicities, respectively. It is also useful to introduce the amplitudes $G^{m}_{\lambda_n,\lambda_p}$ by the relation

$$G^{m}_{\lambda_n,\lambda_p}(s,t) = \sum_{\lambda} d^1_{m\lambda}(\chi) H^{\lambda}_{\lambda_n,\lambda_p}(s,t) \,. \qquad (1)$$

Here $d^1_{m\lambda}(\chi)$ is the usual d function of the ρ-meson crossing angle χ. For large s and small t

$$\cos\chi \approx \frac{m_\rho^2 - \mu^2 + t}{m_\rho^2 - \mu^2 - t}\,, \quad \sin\chi \approx \frac{2(-t)^{1/2} m_\rho}{m_\rho^2 - \mu^2 - t}\,. \qquad (2)$$

It is well known that although the G's are not t-channel helicity amplitudes, their products give the t-channel density matrix elements after summing over nucleon helicities. The condition ${}^t\rho^{11}_{1-1} = 0$ then implies that[9] $\mathrm{Re}(G^1_{++}G^{-1*}_{++} + G^1_{+-}G^{-1*}_{+-}) = 0$.

For small t, one-pion exchange (OPE) and associated absorptive corrections have been known to be the dominant contributions. It is also known that the $++$ amplitudes are down by a factor of $1/s$ compared with the $+-$ amplitudes in such models. Hence the above relation cannot be satisfied by cancellation between the two terms. It might be thought that the relation could be satisfied by demanding the relative phase between $G^1_{+\pm}$ and $G^{-1}_{+\pm}$ to be $\pi/2$. However, note that the relation has to hold for a large range of s and t and that Eq. (1) would then require extremely complicated and pathological phase conditions between different helicity amplitudes. Hence we regard this possibility as unlikely. Also, taking $G^1_{+\pm} = 0$ leads to ${}^t\rho^{11}_{11} = 0$, in disagreement with the data. So we are led to the conclusion that $G^{-1}_{+\pm} = 0$. (The condition $G^{-1}_{++} = 0$ leads to $G^1_{++} = 0$ when $H^1_{++} = -H^{-1}_{++}$ is used.) In terms of $H^{\lambda}_{\lambda_n,\lambda_p}$ these conditions give the following model-independent constraints:

$$(1-\cos\chi)H^1_{+\pm} + (1+\cos\chi)H^{-1}_{+\pm} + \sqrt{2}\,\sin\chi H^0_{+\pm} = 0 \,. \qquad (3)$$

Thus the helicity amplitudes with different helicity flips have to be correlated in a special way. Note that simple angular momentum arguments show that for one elementary pion exchange all $G^{\pm 1}_{\lambda_n,\lambda_p}$ are zero. Hence in such models the extra contributions (ΔH) will satisfy the same constraints. These contributions could be the absorptive corrections in OPE-plus-absorption models, nucleon Born terms in gauge-invariant Born models, or Regge-pole–Regge-cut terms in cut models. Equation (3) can also be derived directly from the t-channel helicity amplitudes by using the crossing matrix. The constraints (3) should be valid for an

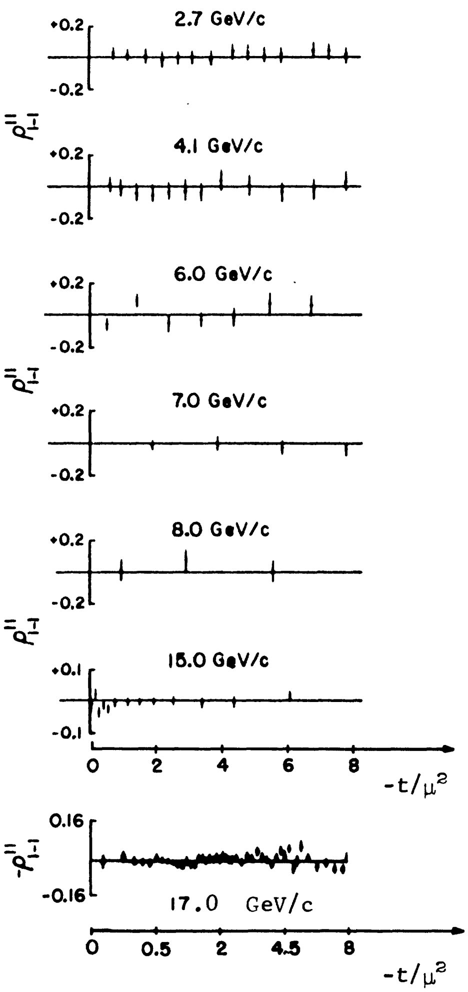

FIG. 1. The experimentally determined density matrix elements ${}^t\rho^{11}_{1-1}$. As indicated in the graph, the data were taken at 2.7, 4.1, 6, 7, 8, 15, and 17 GeV/c incident momenta. The z axis is along the direction of the incident pion in the dipion rest frame (t-channel helicity frame).

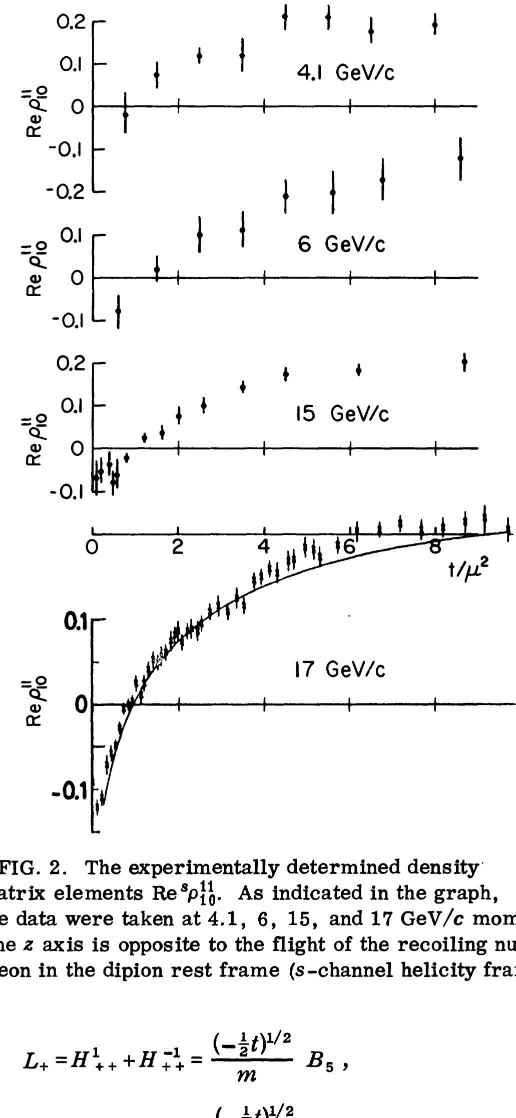

FIG. 2. The experimentally determined density matrix elements $\mathrm{Re}\,{}^s\rho^{11}_{10}$. As indicated in the graph, the data were taken at 4.1, 6, 15, and 17 GeV/c momenta. The z axis is opposite to the flight of the recoiling nucleon in the dipion rest frame (s-channel helicity frame).

extended range of s and at least up to $|t| \approx 8\mu^2$. A model by Froggatt and Morgan[10] satisfies these constraints. In the following, however, we attempt to understand them in a more general way.

First we express the helicity amplitudes in terms of the well-known Ball invariant amplitudes $B_1, B_2, \ldots, B_8$. Since the general equations are rather complicated, it is more transparent to neglect terms of the order of $1/s$ and obtain[11]

$$T_+ = H^0_{++} = \frac{m_\rho s}{2m}\left(\frac{t+m_\rho^2-\mu^2}{2m_\rho^2}B_8 + \frac{B_5}{m_\rho^2} - \frac{sB_6}{2m_\rho^2}\right), \tag{4}$$

$$T_- = H^0_{+-} = -\frac{m_\rho(-t)^{1/2}}{m}\left(\frac{t+m_\rho^2-\mu^2}{2m_\rho^2}B_3 - \frac{sB_2}{2m_\rho^2}\right), \tag{5}$$

$$L_+ = H^1_{++} + H^{-1}_{++} = \frac{(-\frac{1}{2}t)^{1/2}}{m}B_5, \tag{6}$$

$$D_+ = H^1_{++} - H^{-1}_{++} = \frac{(-\frac{1}{2}t)^{1/2}}{m}sB_8, \tag{7}$$

$$L_- = H^1_{+-} + H^{-1}_{+-} = -\frac{1}{\sqrt{2}\,m}(-sB_1 + 2mB_5), \tag{8}$$

$$D_- = H^1_{+-} - H^{-1}_{+-} = -\frac{1}{\sqrt{2}\,m}(-sB_1 - 2tB_3 + 2mB_5). \tag{9}$$

In what follows we make use of the fact that $H^1_{++} = -H^{-1}_{++}$. One-pion exchange satisfies this relation exactly. Since both the amplitudes have the same net helicity flip it is known that the absorptive corrections are also the same.[12,13] In fitting data on ρ production this condition was also found correct empirically.[14] This gives $B_5 = 0$. We also note that the s- and u-channel nucleon poles do not contribute to B_5.

Now assuming that all the Ball amplitudes are independent of k^2 [(off-shell vector-meson mass)^2], Cho and Sakurai[15] and Achasov and Shestakov[16] have derived the following relations between the Ball amplitudes:

$$sB_2-(t-\mu^2)B_3=0\ , \qquad (10)$$

$$-2B_5+sB_6-(t-\mu^2)B_8=0\ . \qquad (11)$$

These relations constitute the smoothness assumption. Note that our constraint equations (3) are different from the equations connecting longitudinal and transverse amplitudes given in Ref. 15.

Now it can be easily seen that the relation (3) for ++ amplitudes is automatically satisfied by imposing $H^1_{++}=-H^{-1}_{++}$ and Eq. (11), i.e.,

$$-\sqrt{2}\cos\chi\,\frac{(-t)^{1/2}}{2m}\,sB_8+\sqrt{2}\,\frac{\sin\chi}{4m}\,m_\rho sB_8\approx 0 \qquad (12)$$

irrespective of the value of B_8. Trivial satisfaction of the constraint, however, will require B_5, B_6, $B_8=0$. Our numerical experience shows that although the ++ amplitudes are small, they are not negligible, especially at low energies and at momentum transfers where other amplitudes vanish. Later on, however, we will use the fact that D_+ (or B_8) is small compared with D_-, especially at higher energies. In leading order of s, this has the quantum numbers of A_1 exchange. Analyses of the photoproduction experiments are consistent with the smallness of this term. On the other hand, even granting $B_8=0$, there will be still a question of sB_6 unless the smoothness equations are used. In spite of this, because of the smallness of the ++ amplitudes, the ++ constraints cannot be said to imply smoothness although they are mutually consistent.

For the +− amplitudes we find the constraint to be

$$(1-\cos\chi)(-sB_1)+\sin\chi\,\frac{(-t)^{1/2}}{m_\rho}\,(-sB_2) + \left[2t\cos\chi+\frac{\sin\chi(-t)^{1/2}}{m_\rho}(t+m_\rho{}^2-\mu^2)\right]B_3=0\ . \qquad (13)$$

The last term is readily seen to vanish. Cancellation of the first two terms requires the dynamical condition $B_1=-B_2$ for the range of s and t under consideration. Note that the smoothness conditions have not been used yet. If we do use Eq. (10), however, the condition $B_1=-B_2$ has a very interesting consequence for the amplitude D_-. Since $B_5=0$ and $B_1=-B_2$, Eq. (10) gives $D_-=(1/\sqrt{2}\,m)\times(\mu^2+t)B_3$. *This shows that, irrespective of the value of* B_3, D_- *will have a zero at* $t=-\mu^2$. Now the density matrix element $\mathrm{Re}\,{}^s\rho^{11}_{10}$ is given by

$$6\,\mathrm{Re}\,{}^s\rho^{11}_{10}=\mathrm{Re}(D_+T^*_+ + D_-T^*_-)\ . \qquad (14)$$

If D_+ is small, Eq. (14) implies that $\mathrm{Re}\,{}^s\rho^{11}_{10}$ passes through zero at $t\approx-\mu^2$. This is in remarkable agreement with the experimental data plotted in Fig. 2 at several energies. In an earlier amplitude analysis of ρ^0 production by Estabrooks and Martin,[17] this zero arose from the phase incoherence between H^1_{+-} and H^0_{+-} amplitudes. Subsequently a second solution with phase coherence was found by them.[18] *Our discussion here relates the zero in* $\mathrm{Re}\,{}^s\rho^{11}_{10}$ *to a zero in the amplitude* D_- *in a much more fundamental manner without requiring any phase incoherence whatsoever.* It is clear that the same zero arises in $\mathrm{Re}\,{}^s\rho^{10}_{10}$ and in photoproduction.

For the ratio of density matrix elements we have

$$\frac{{}^s\rho^{11}_{1-1}}{{}^s\rho^{11}_{11}}=\frac{|L_+|^2+|L_-|^2-|D_+|^2-|D_-|^2}{|L_+|^2+|L_-|^2+|D_+|^2+|D_-|^2}\ . \qquad (15)$$

Clearly, if $|D_+|\approx 0$ this ratio becomes ≈ 1 at $t\approx-\mu^2$. Again this is confirmed by the data.[3,8,9] This ratio in the case of photoproduction is the asymmetry parameter for the isovector photons and is also found[7] to be ≈ 1 at $t\approx-\mu^2$. These facts have been discussed for the case of photoproduction and ρ production by Ross *et al.*,[12] Cho and Sakurai,[19] Contogouris *et al.*,[19] and Williams,[19] using various models. Their implications for the phase coherence have been discussed recently by the present authors.[20] Our results are much more general than any specific model.

In conclusion we wish to emphasize that the condition $B_1\approx-B_2$ is demanded by the experimentally observed fact that ${}^t\rho^{11}_{1-1}\approx 0$ for a wide range of incident beam momenta (2.7–17 GeV/c) and for at least $0<|t|<8\mu^2$. Primary use of the smoothness assumption has been made for the region $|t|\sim\mu^2$ in producing a zero in $\mathrm{Re}\,{}^s\rho^{11}_{10}$. Thus up to $|t|\approx\mu^2$ the smoothness assumption is experimentally correct. Possible breakdown of smoothness (vector dominance) for larger values of $|t|$ when ${}^t\rho^{11}_{1-1}$ is still zero does raise interesting questions. However, these are beyond the scope of the present work. Our model-independent constraints [Eq. (3)] must be satisfied by any theoretical model (e.g., absorptive model, gauge-invariant Born model, Regge-cut model, Veneziano model, etc.) in the energy and momentum-transfer range under consideration. It is truly amazing that a large number of features of data extending over a vast range of energy and momentum transfer can be explained by such simple relationships without assigning any numerical or functional model-dependent form to the amplitudes.

We wish to thank A. Contogouris for many illuminating discussions and F. J. Loeffler for his generous support. L. G. would like to thank F.

Turkot and Brookhaven National Laboratory for a summer appointment. K. V. wishes to acknowledge award of an I. U. Summer Research Fellowship. Our thanks are also due to our colleagues at the Universities of Colorado, Pennsylvania, Toronto, Wisconsin, Notre Dame, and at the Stanford Linear Accelerator Center for participating in the data compilation.

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*Work supported in part by the U. S. Atomic Energy Commission.

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