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Nucleon-nucleon scattering near 50 MeV. II. Sensitivity of various n-p observables to the phase parameters*

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In the first paper in this series, we reported on a phase-shift analysis of existing p-p and n-p data in the energy range of 47.5 to 60.9 MeV. Two results were emphasized. The first is that the available n-p data leave ϵ_1 undetermined within the range $-10°$ to $+3°$, resulting in a range of phase-parameter solutions, rather than a single solution. The second result is that while ϵ_1 is very poorly determined, $\delta(^1P_1)$ is rather well determined, but at a value which appears to conflict not only with values obtained at adjacent energies, but also with the value (or narrow range of values) predicted by meson-theoretical models. In that paper it is reported that the Harwell n-p $d\sigma/d\Omega$ data are responsible for this value of $\delta(^1P_1)$. The remaining data, consisting only of σ_{tot} data, polarization data, and other $d\sigma/d\Omega$ data, are consistent with the theoretical predictions. In this paper we look more closely at the sensitivity of experimental observables to variations in the partial-wave parameters. We extend the number of experimental observables under study to twenty, and consider the effect on these of varying seven different phase parameters: $\delta(^1S_0)_{np}$, $\delta(^3S_1)$, ϵ_1, $\delta(^1P_1)$, $\delta(^3D_1)$, $\delta(^3D_2)$, and $\delta(^3D_3)$. We discover that the best observable to fix $\delta(^1P_1)$ is still the differential cross section, and recommend, as in the first paper, that it be measured both at extreme forward and extreme backward angles. We also discover that the reason ϵ_1 is very poorly determined by the present data is that neither σ_{tot}, $d\sigma/d\Omega$, nor P is sensitive to changes in ϵ_1. We find that the experimental observables which are sensitive to ϵ_1 and can fix this parameter are, in order of decreasing sensitivity, A_{zz}, C_{pp}, A_t', C_{KK}, A_t, D_t, C_{nn}, and A_{xx}.

I. INTRODUCTION

In a paper by Arndt, Binstock, and Bryan,[1] hereafter referred to as paper I, a phase-shift analysis of n-p plus p-p elastic-scattering data in the laboratory energy range 47.5–60.9 MeV was carried out. Charge independence was assumed for all but $\delta(^1S_0)$, and F waves and higher partial waves were set to the OPEC (one-pion-exchange contribution) values. It was found that the available n-p data leave ϵ_1 undetermined within the range $-10°$ to $+3°$, resulting in a range of phase parameter solutions rather than a single solution. Furthermore, although ϵ_1 was poorly determined, $\delta(^1P_1)$ was found to be rather well determined by the data, but at an anomalous value. In particular, for ϵ_1 fixed at a reasonable 50-MeV value of $+2.78°$ (taken from Ref. 2), $\delta(^1P_1)$ searched to $-3.52 \pm 1.04°$ at 50 MeV, in conflict both with theoretical expectations of about $-9°$ and with any smooth interpolation of experimental values of $\delta(^1P_1)$ at neighboring energies.

A comparison of the experimentally determined observables (calculated from the phase shifts determined by the existing data) with the theoretically predicted observables [from a model[2] fit to the entire set of (0–450)-MeV nucleon-nucleon data] showed that (1) σ_{tot} and P determined from experiment agree with the theoretical predictions, and (2) $d\sigma/d\Omega$ determined from experiment disagrees with theoretical predictions, particularly in the extreme forward and extreme backward directions. These comparisons are shown in Fig. 3 of paper I. The data responsible for this discrepancy were found to be the Harwell[3] differential cross-section measurements at 47.5, 52.5, and 57.5 MeV.[4] Mechanisms were then discussed which might explain away the discrepancy in $\delta(^1P_1)$ if the Harwell data proved correct.

However, in the event that experimentalists might decide to redo the $d\sigma/d\Omega$ measurements, or perhaps carry out measurements of a new observable, we present in Sec. II of this paper a study of the sensitivity to $\delta(^1P_1)$ of several possible experimental observables, in order to determine which one can best pin down this phase shift. We discover that $\delta(^1P_1)$ is best determined by the n-p differential cross section, which has already been measured. We also study the problem of fixing ϵ_1 and discover that the reason ϵ_1 is undetermined in the range $-10°$ to $+3°$ is that none of the presently measured observables, σ_{tot}, $d\sigma/d\Omega$, and P, is at all sensitive to variations in this parameter. Therefore in Sec. III we make recommendations for experiments to be performed to properly determine ϵ_1 and to settle the $\delta(^1P_1)$ question.

II. SENSITIVITY OF VARIOUS n-p EXPERIMENTS TO THE PHASE PARAMETERS AT 50 MeV

To facilitate the choice of which n-p experiments are worth carrying out in order to pin down the 50-MeV $I=0$ S matrix, we have calculated the sensitivity of various n-p observables to changes in n-p phase parameters as mentioned in Sec. I. The results are displayed in Figs. 1 and 2.

A. Discussion of Fig. 1

Figure 1 shows observables for which there are existing n-p data in the (47.5–60.9)-MeV range[5]: σ_{tot}, $d\sigma/d\Omega$, and P.

1. Curve 0

Since we intend to show the changes in some twenty-odd experimental observables, brought about by variations in various of the relevant phase parameters, we must choose a set of phase shifts to be our standard, about which to make the variations. We could choose as our standard the current set of phase parameters determined by experiment, but the $I=0$ parameters of this set are not suitable: $\delta(^1P_1)$ of this set is very suspicious (and probably wrong) and ϵ_1 is undetermined over a range of some 13°. Therefore we have elected to choose as our standard the phase shifts predicted at 50 MeV by model C of Ref. 2 (Bryan-Gersten model C). This model, a one-boson-exchange potential, approximately fits the nucleon-nucleon data over the (0–450)-MeV range and probably constitutes a not unreasonable choice of standard. The 50-MeV phase shifts which comprise our standard set, set 0, are displayed in Table I. [Actually the $I=1$ phase parameters in this set differ slightly from the Bryan-Gersten model C set, but not enough to make more than very slight changes in the predicted observables shown in Figs. 1 and 2—less than a few line-widths at most and not enough to make a significant difference in the results of a sensitivity study. These slightly different phase parameters result from a search on the (excellent) p-p data.]

2. Curves 1 through 7

Curves 1–7 show the results of changing one phase parameter at a time, keeping the rest at the Table I values (set 0), represented by δ^{th} or ϵ^{th} in the legend.

3. Interpretation of Fig. 1

It can be seen that all the n-p observables measured so far, in the considered energy range, are insensitive to changes in the ϵ_1 phase parameter. This is the reason why ϵ_1 is poorly determined by present data. Greater accuracy in measuring σ_{tot}, $d\sigma/d\Omega$, and P will not help much in determining ϵ_1; some other observable must be measured.

The polarization [Fig. 1(c)] can be seen to be sensitive mainly to two of the triplet-D parameters, $\delta(^3D_1)$ and $\delta(^3D_3)$, so that the polarization data helps to fix these parameters. It is not sufficient, however, to fix all three D-wave phase parameters.

As for the determination of $\delta(^1P_1)$, we recall that in paper I we found for the differential cross section a great difference between the prediction from theory and the determination by the data. This discrepancy between theory and experiment occurs both in the forward and in the backward directions. We can see now in Fig. 1(b) of this paper that the differential cross section is sen-

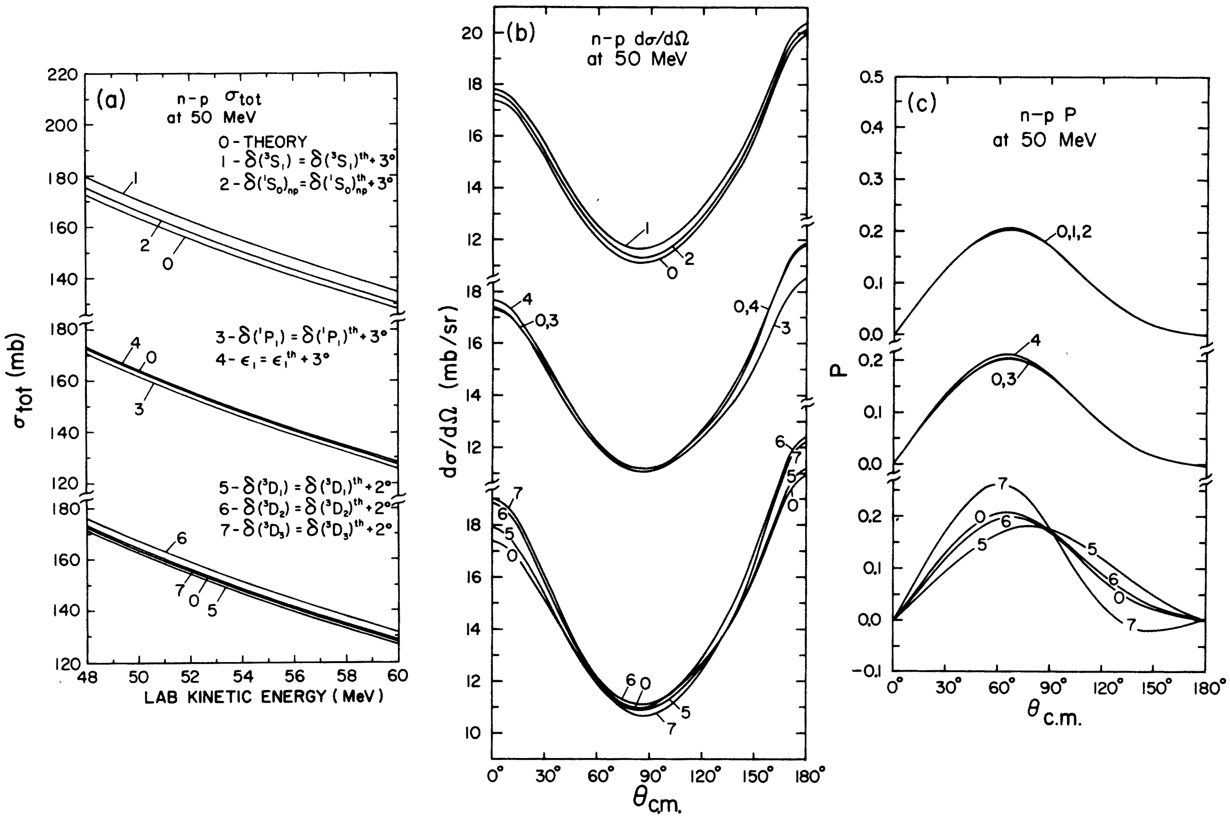

FIG. 1. Sensitivity of the n-p observables σ_{tot}, $d\sigma/d\Omega$, and P to variations in the lower partial-wave n-p phase parameters. Curve 0 is the standard curve predicted by phase-shift set 0 (presented in Table I), and represents the prediction of theory. Curves 1 through 7 correspond to a 2° or 3° increase over the set-0 value in a single phase parameter, chosen in turn to be $\delta(^3S_1)$, $\delta(^1S_0)_{np}$, $\delta(^1P_1)$, ϵ_1, $\delta(^3D_1)$, $\delta(^3D_2)$, and $\delta(^3D_3)$, respectively. The particular amount of increase for each phase parameter is given in the legend.

sitive to $\delta(^1P_1)$ at backward angles, and to the triplet-D parameters at both forward and backward angles. Inaccurate experimental *forward* $d\sigma/d\Omega$ data are therefore sufficient to throw off the value of $\delta(^1P_1)$ by fixing the triplet-D phases incorrectly from the forward data and then giving the wrong triplet-D contribution to the backward data. Therefore, good absolute determinations of n-p $d\sigma/d\Omega$ are needed both near 0° and near 180°.

B. Discussion of Fig. 2

Since σ_{tot}, $d\sigma/d\Omega$, and P are not sensitive to ϵ_1, we show in Fig. 2 the sensitivity of various other n-p observables to +3° changes in ϵ_1 [and also 3° changes in $\delta(^1P_1)$], in hopes of determining which observables are sensitive to ϵ_1. Schematic pictures of these various scattering experiments in the laboratory frame comprise Fig. 3. Circles with center dots represent polarizations normal to and out of the page. Arrows with double lines represent polarizations in the scattering plane. These observables are defined in the Appendix. The observables graphed in Fig. 2 turn out not to be particularly sensitive to the S- or D-wave parameters, which anyway will presumably be fixed from present σ_{tot}, P, and, hopefully more accurate, $d\sigma/d\Omega$ data. Therefore, we do not show the sensitivity to changes in these S- and D-wave parameters. Curves 0, 3, and 4 are the same as described in Sec. II A 2.

III. EXPERIMENTAL RECOMMENDATIONS

A. Experiments which will best determine ϵ_1

1. *Qualitative predictions*

One can study all of the graphs in Figs. 1 and 2 in order to determine which of the neutron-proton observables is most sensitive to ϵ_1 and whose measurement therefore will best pin down ϵ_1. However, to aid in selecting the most sensitive experiments, we plot in Fig. 4(a) the maximum variation within the 0° to 180° domain of

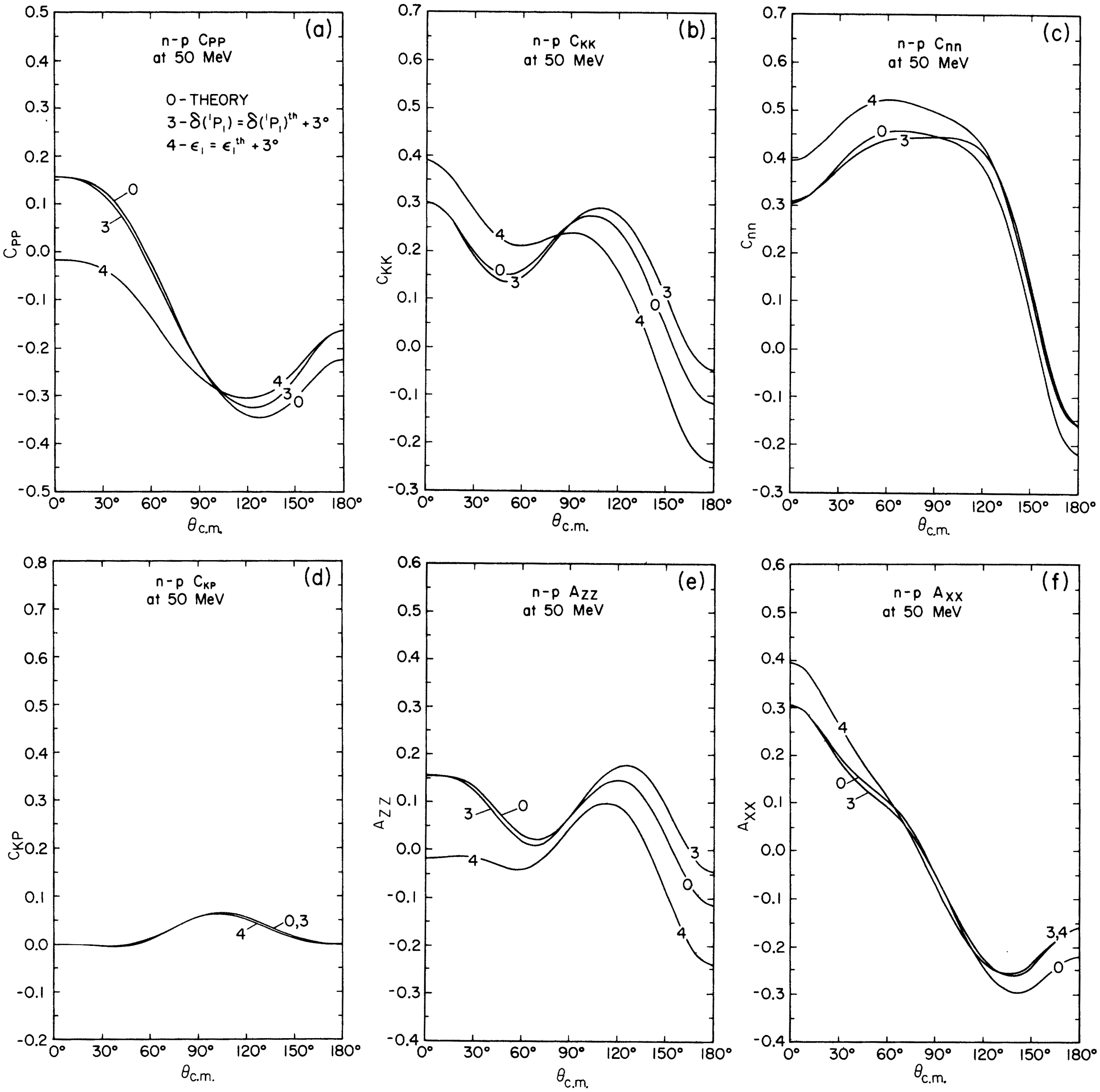

FIG. 2. *(Continued on next page)*

each observable when ϵ_1 is increased by 3°. One can see that A_{zz} and C_{PP} are the most sensitive, followed by A_t', C_{KK}, A_t, D_t, C_{nn}, and A_{xx}, in order of decreasing sensitivity. (Note that if either time-reversal or parity is good, $A_{yy} = C_{nn}$, as discussed in the Appendix.) A_{zz} is a difficult experiment to perform because it requires a target polarized along the beam direction, and therefore the poles of the polarizing magnet are in the way of the beam. A more feasible and equally sensitive experiment is C_{PP}. However, Fig. 2(a) reveals that this sensitivity is in the forward direction, and it may be difficult to measure the spin of the relatively slow recoil proton. A_t' is next most sensitive, for backward angles, according to Fig. 2(q); but its measurement requires the precession of the proton spin. C_{KK} is next in sensitivity, but again it is most sensitive at backward angles, according to Fig. 2(b), so the transverse spin of a slow neutron must be measured. Furthermore, it also requires the precession of the proton spin. A_t follows next

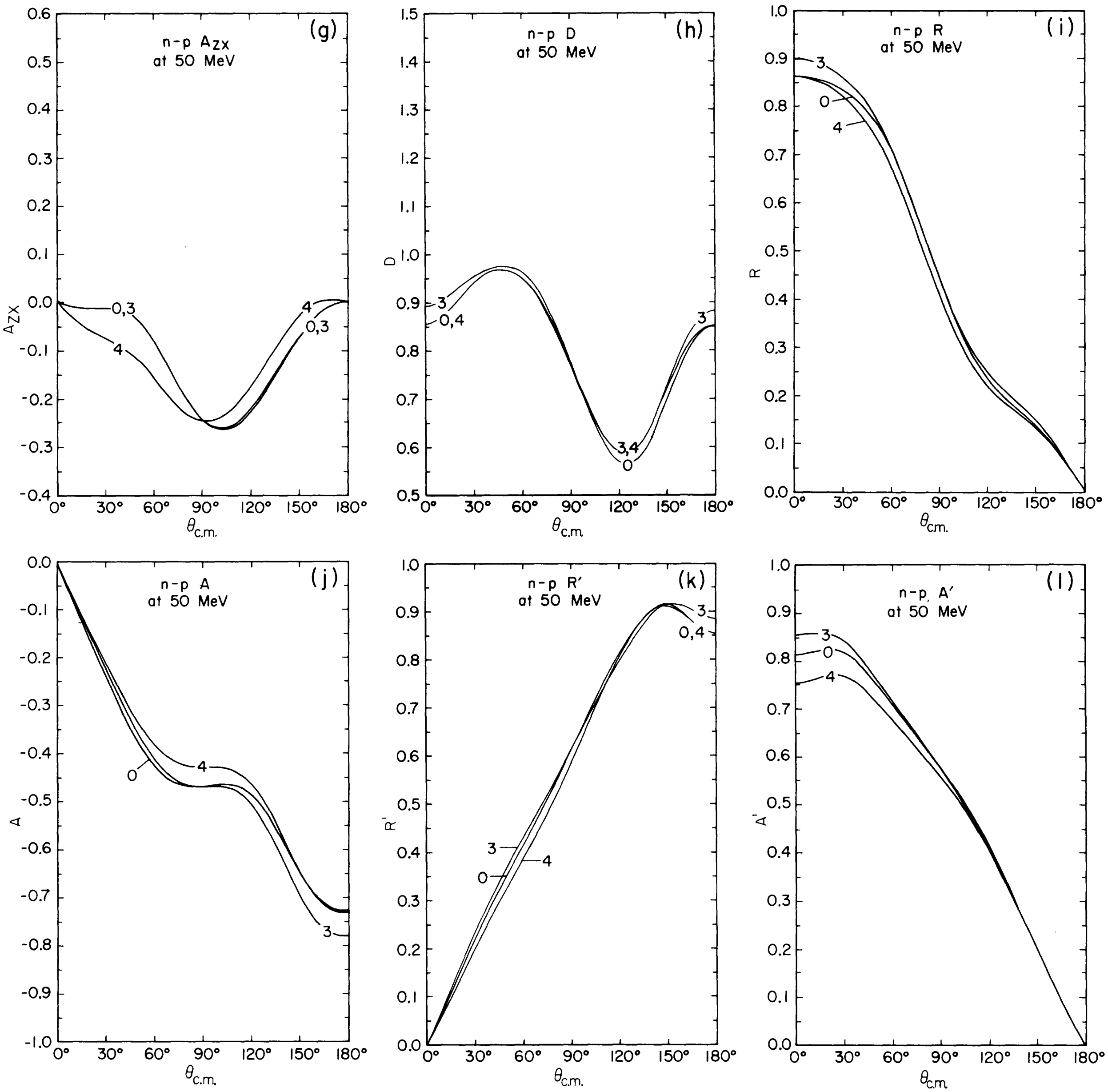

FIG. 2. *(Continued on next page)*

in sensitivity, at forward angles [Fig. 2(o)], so that the transverse spin of a slow proton must be measured. D_t may be the most feasible experiment. For $\theta_{c.m.} \geqslant 120°$ [Fig. 2(m)] it is reasonably sensitive to ϵ_1, and requires the measurement of the normal component of the spin of an energetic recoil proton. A_{yy} may also be a feasible experiment. It is sensitive to ϵ_1 over the entire angular range, but particularly so in the forward direction. Its measurement requires a polarized beam and polarized target, with both polarizations normal to the scattering plane.

2. *Quantitative predictions for D_t and A_{yy}*

To test our predictions that D_t and A_{yy} are good observables to measure to determine ϵ_1, we added one fake data point to the 50-MeV p-p and n-p data listed in Table I of paper I and computed the resulting χ^2 versus ϵ_1. Four of these so-called parameter studies were carried out, one

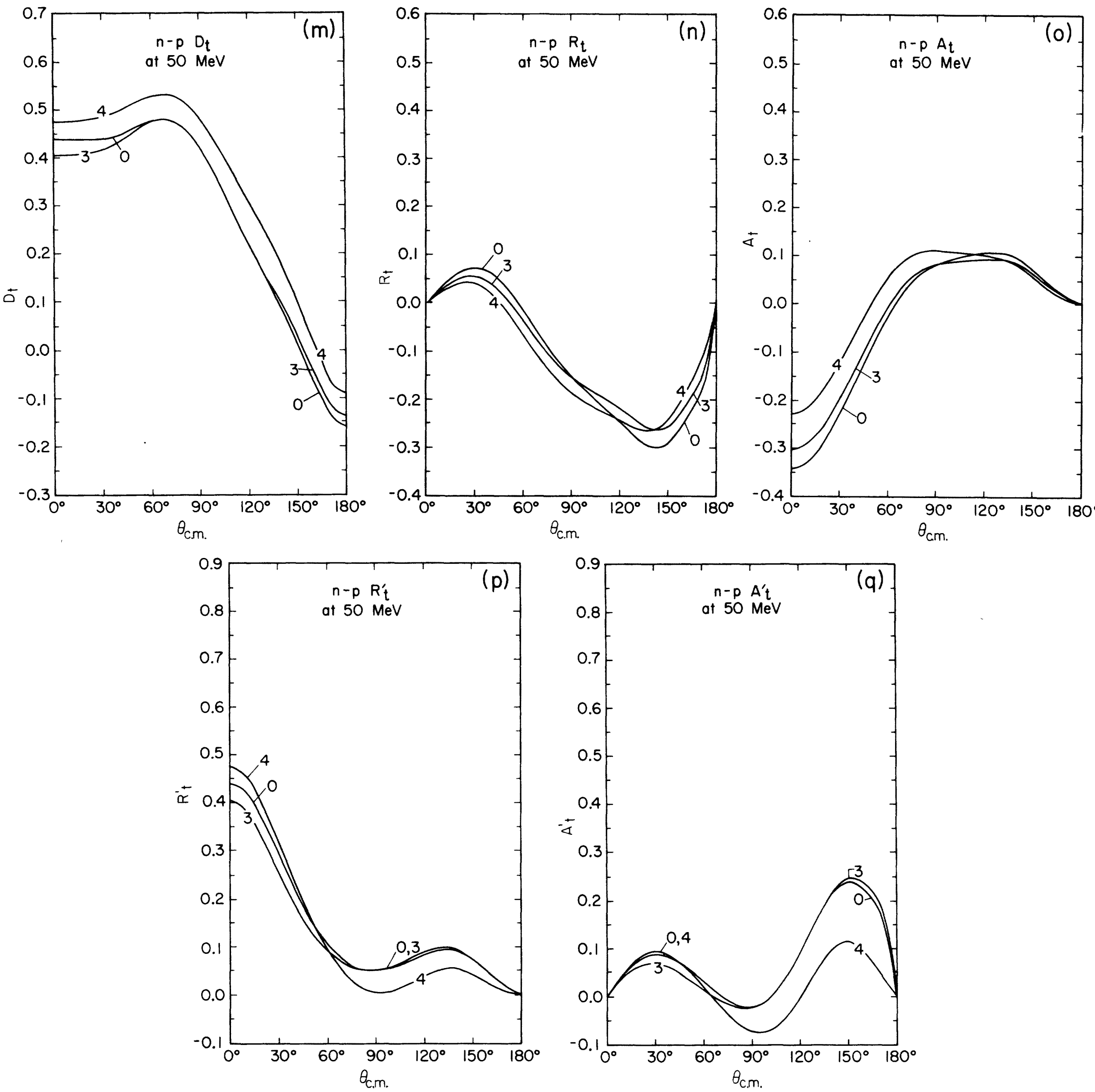

FIG. 2. Study of the sensitivity of several n-p observables to variations in the $\delta(^1P_1)$ and ϵ_1 phase parameters. Curve 0 is the standard curve and is defined in the caption of Fig. 1 and in Table I; curves 3 and 4 correspond to a 3° increase in $\delta(^1P_1)$ and ϵ_1, respectively, just as in Fig. 1.

study for each of four fake data points. The four data used were $D_t(80°)=0.467\pm0.01$, $D_t(151.5°)=0.00\pm0.01$, $A_{yy}(0°)=0.308\pm0.01$, and $A_{yy}(100°)=0.444\pm0.01$, all taken off the curve-0 predictions of Figs. 2(m) and 2(c). The results of these parameter studies are graphed in Fig. 5. As one can see, the addition of each datum results in a single well-defined minimum in χ^2 versus ϵ_1, rather than the broad, flat 13°-wide minimum determined by the current data (consisting for the n-p data of only σ_{tot}, $d\sigma/d\Omega$, and P). In each case, ϵ_1 is determined to about ±1°, corresponding to an increase in χ^2 of one above the minimum value. Thus the prediction that D_t and A_{yy} are good observables to fix ϵ_1 appears to be borne out.

It is interesting to note, however, that these predictions are not borne out quantitatively so much as qualitatively: $D_t(151.5°)$ and $A_{yy}(0°)$ are about 50% more sensitive to variations in ϵ_1 than

TABLE I. 50-MeV nuclear-bar phase parameters for set 0.

	Phase parameters [a]	Energy slopes [a] $d\delta/dE$ (deg/MeV)
$I=1$		
$\delta(^1S_0)_{pp}$	+38.92°	−0.390
$\delta(^3P_0)$	+11.66°	+0.000
$\delta(^3P_1)$	−8.30°	−0.117
$\delta(^3P_2)$	+5.93°	+0.125
$\delta(^1D_2)$	+1.72°	+0.041
ϵ_2	−1.74°	−0.032
$\delta(^1S_0)_{np}$	+40.32°	−0.390
$I=0$		
$\delta(^3S_1)$	+60.08°	−0.648
$\delta(^1P_1)$	−8.76°	−0.081
ϵ_1	+2.78°	+0.025
$\delta(^3D_1)$	−6.83°	−0.141
$\delta(^3D_2)$	+10.37°	+0.240
$\delta(^3D_3)$	+0.41°	+0.019

[a] Phase parameters described in text. The energy slopes are taken from model C of Bryan and Gersten (Ref. 2). The phase parameters not appearing in this table are assumed to be equal to the OPEC values, with $g_\pi{}^2=14.43$, $m_\pi=135.04$ MeV/c^2, and nucleon mass $=938.211$ MeV/c^2.

are $D_t(80°)$ and $A_{yy}(100°)$, according to Figs. 2(m) and 2(c), yet ϵ_1 is determined to about the same accuracy in all four parameter studies. This is because in the χ^2-versus-ϵ_1 parameter study, the other phase shifts below $L=3$ are allowed to vary as well as ϵ_1, and a variation in one or more of these other phase shifts may compensate for the variation in ϵ_1 so that χ^2 does not rise so steeply as ϵ_1 is stepped away from the minimum. This is what has apparently happened in the case of $A_{yy}(0°)$ and $D_t(151.5°)$. Well-defined minima are nevertheless obtained in all four cases. This is probably aided by the fact that of the $I=0$ phases searched in addition to ϵ_1, $\delta(^1P_1)$ is already well fixed by the n-p $d\sigma/d\Omega$ data (if somewhat incorrectly) and the S- and D-wave phase shifts are not phases to which D_t and A_{yy} are overly sensitive (as mentioned in Sec. II B).

One concern that we had is not borne out by the parameter studies. This was that one might find a double minimum in χ^2 versus ϵ_1 after adding the fake data point, rather than a single minimum. We were thus concerned because Wright, MacGregor, and Arndt show in Fig. 2 of their paper (Ref. 6) that 50-MeV measurements of $D_t(151.5°)$ near 0.05 or $A_{yy}(0°)$ near 0.2 will not differentiate between their solutions A_3 and B_3,

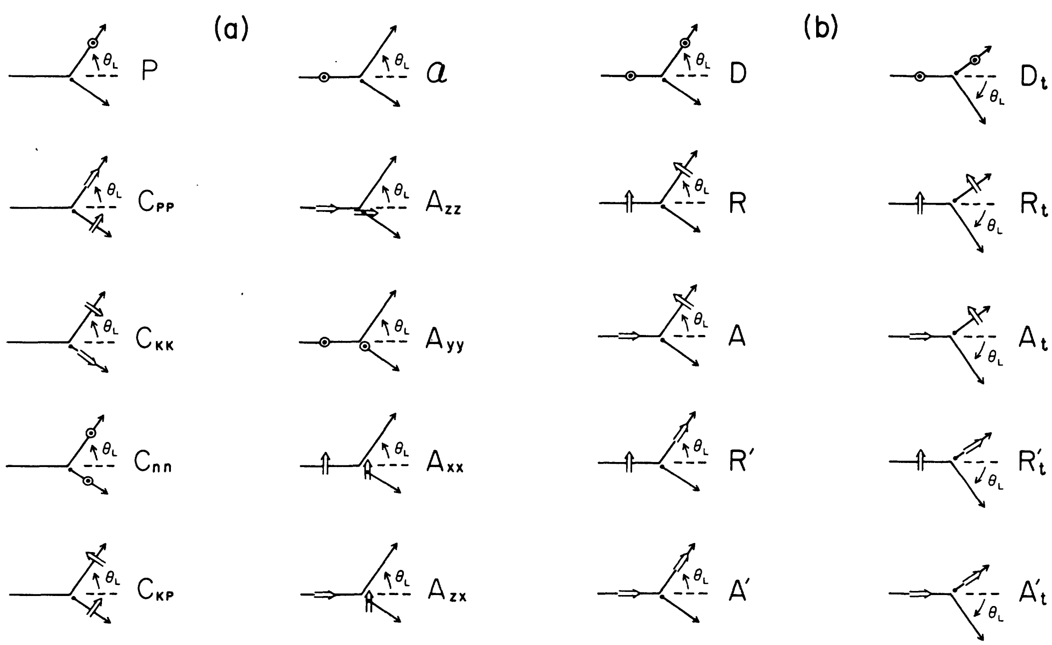

FIG. 3. Schematic pictures in the laboratory frame of several nucleon-nucleon observables; θ_L is the laboratory scattering angle; a circle with a center dot corresponds to spin normal to the scattering plane and up out of the plane of the page. An arrow with a wide shaft depicts spin lying in the scattering plane. Observables are defined in the appendix. Note that in the case of the transfer observables D_t, A_t, R_t, A'_t, and R'_t, the projectile nucleon is shown scattering downward in the sketch, unlike the case for all the remaining observables where the projectile is shown scattering upward.

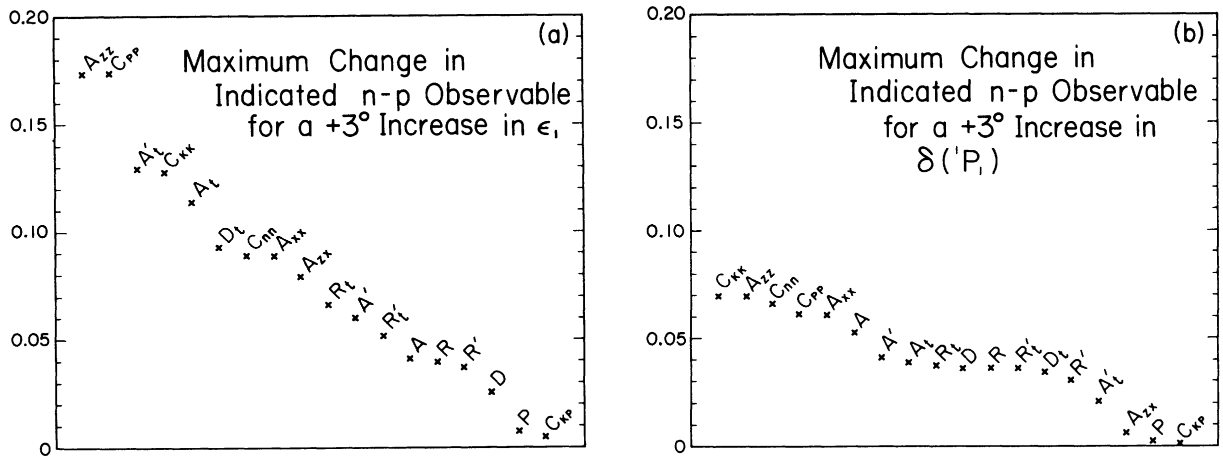

FIG. 4. (a) For each observable along the abscissa, the ordinate is the maximum change in the observable brought about, anywhere in the 0° to 180° c.m. scattering-angle range, by a 3° increase in ϵ_1 from its value in phase-shift set 0, described in the caption of Fig. 1, with all other phase parameters kept at the set-0 values. (b) Same as (a), except that the ordinate is the maximum variation in a given n-p observable due to a 3° increase in $\delta(^1P_1)$ from its value in phase-shift set 0, all other phase parameters being kept at set-0 values. These maximum variations are taken from the sensitivity-study data already graphed in Figs. 1 and 2.

corresponding respectively to $\epsilon_1 = 5° \pm 3°$ and $-4° \pm 4°$. The curve-0 predictions for $D_t(151.5°)$ and $A_{yy}(0°)$ that we used in our parameter studies fall very near these values graphed by Wright *et al.* In fact, we find that despite their predictions, we obtain well-defined minima for χ^2 versus ϵ_1 after inserting the curve-0 fake data points for D_t at 151.5° and A_{yy} at 0°; these minima fall at 0° and 2.5°, respectively, as shown in Figs. 5(a) and 5(b). We are at a loss to explain this disagreement with Wright *et al.*, although we do note that in the case of $D_t(151.5°)$, ϵ_1 at the minimum is shifted slightly toward the B_3 solution.

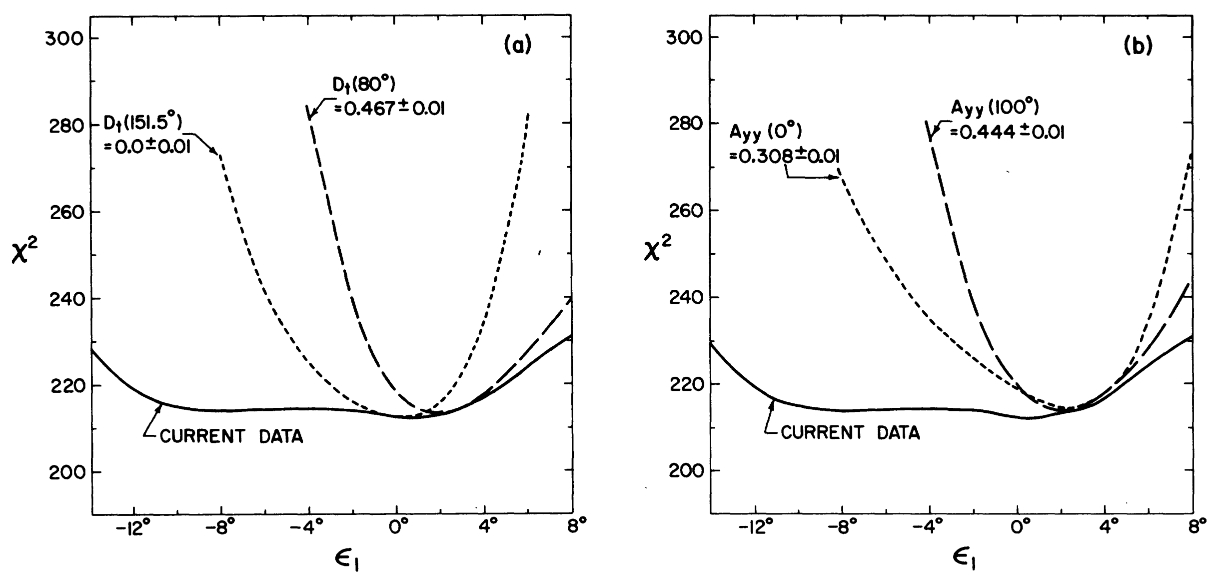

FIG. 5. Parameter studies of χ^2 vs ϵ_1 wherein the data searched against include a fake datum in addition to the experimental 47.5- to 60.9-MeV p-p and n-p data listed in Table I of paper I (Ref. 1). Four separate parameter studies are shown, corresponding to four different fake data points; two D_t studies in Fig. 5(a), two A_{yy} studies in Fig. 5(b). In each case, all phase shifts [except $\delta(^1S_0)_{np}$] below $L = 3$ are allowed to search to minimize χ^2 for the given value of ϵ_1. Note that ϵ_1 is determined to about $\pm 1°$ in each study.

B. Experiments which will best determine $\delta(^1P_1)$

As in the case of ϵ_1, one can scan Figs. 1 and 2 to see which observables are most sensitive to a $+3°$ change in $\delta(^1P_1)$. Figure 4(b) consists of a plot of the maximum variation of each observable (except for σ_{tot} and $d\sigma/d\Omega$) to this $+3°$ change in $\delta(^1P_1)$. One can see that none of these observables is particularly sensitive to $\delta(^1P_1)$. Interestingly enough, the polarization is next to last in sensitivity; this extreme insensitivity shows that better polarization data will be useless in determining $\delta(^1P_1)$. However, $d\sigma/d\Omega$, not indicated in Fig. 4, is quite sensitive to variations in $\delta(^1P_1)$, as can be seen in Fig. 1(b). This oldest of experiments is still the best one to measure to determine $\delta(^1P_1)$. One will observe in Fig. 1(b) that it is in the extreme backward direction that $d\sigma/d\Omega$ is most sensitive to $\delta(^1P_1)$. However, as can be seen in the same figure, $d\sigma/d\Omega$ is also sensitive to the triplet D-waves, for both forward and backward scattering. Thus, as noted in Sec. II, to properly pin down $\delta(^1P_1)$ one must know n-p $d\sigma/d\Omega$ in the extreme forward direction to determine the D-wave phases, and in the extreme backward direction to then determine $\delta(^1P_1)$.

If the data (Harwell[3]) responsible for the current searched value of $\delta(^1P_1)$ are thrown out, the remaining n-p $d\sigma/d\Omega$ data[5] are mostly relative backward data, of unknown normalization. It would seem then that the most urgent need for new n-p $d\sigma/d\Omega$ data is either for more precise absolute $d\sigma/d\Omega$ data at very forward directions or else for good relative $d\sigma/d\Omega$ data spanning the 0–90° range.[7] These new data in conjunction with present σ_{tot} data would then help to fix the normalization of the backward unnormalized $d\sigma/d\Omega$ data. A good relative determination of $[d\sigma/d\Omega(180°)]/[d\sigma/d\Omega(90°)]$ would also be helpful in overriding the Harwell data.[3] For a more detailed discussion of suggested n-p $d\sigma/d\Omega$ experiments, see Sec. VI of paper I.

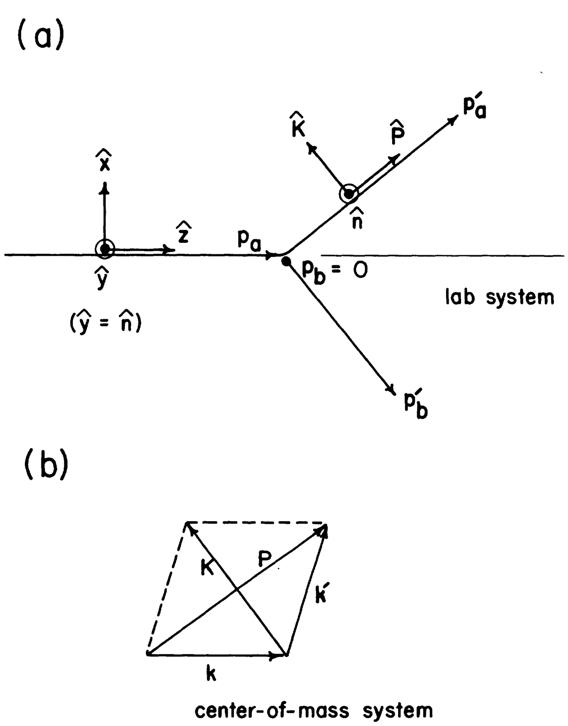

FIG. 6. (a) Sketch of initial nucleon momenta $\vec{p}_a$ and $\vec{p}_b$, and final nucleon momenta $\vec{p}'_a$ and $\vec{p}'_b$, in the laboratory scattering system. Also shown are two Cartesian coordinate systems defined in the text. (b) Sketch of the initial momentum $\vec{k}$ and final momentum $\vec{k}'$ of nucleon a in the center-of-mass system.

ACKNOWLEDGMENTS

We wish to thank Professor Richard Arndt for making the Livermore phase-shift–analysis code available to us, for contributing time and effort in helping to adapt the code to the Cyclotron Institute IBM-7094, and finally for giving helpful advice on the topic of phase-shift analysis. We have also received much help from Professor Lee Northcliffe on the subject of the feasibility of nucleon-nucleon experiments.

APPENDIX: DEFINITION OF SCATTERING OBSERVABLES

Consider an incoming beam of nucleons of type a scattering off a target of nucleons of type b. Let the initial momentum of the beam particles be $\vec{p}_a$ and that of the target particles be $\vec{p}_b$. (The latter momentum will be 0 in the laboratory system.) Let the final momentum of the scattered beam particles be $\vec{p}'_a$ and that of the recoil target particles be $\vec{p}'_b$. Then $\vec{p}_a$, $\vec{p}_b$, $\vec{p}'_a$, and $\vec{p}'_b$ in the laboratory system are as sketched in Fig. 6(a).

The initial wave function for the system may be written

$$\phi = e^{-iEt/\hbar} \exp[i(\vec{p}_a \cdot \vec{r}_a + \vec{p}_b \cdot \vec{r}_b)/\hbar]\, \chi ,$$

where $E = (\vec{p}_a{}^2/2m_a) + (\vec{p}_b{}^2/2m_b)$, $\vec{r}_a$ and $\vec{r}_b$ are the positions of particles a and b, respectively, and χ is a four-component nonrelativistic spinor denoting the mechanical spin of particles a and b.

ϕ can also be written in the form

$$\phi = e^{-iEt/\hbar} \exp[i(\vec{p}_a + \vec{p}_b) \cdot \vec{R}/\hbar]\, e^{i\vec{k}\cdot\vec{r}} ,$$

where $\vec{R} = (m_a \vec{r}_a + m_b \vec{r}_b)/(m_a + m_b)$ is the position of the center of mass, $\vec{r} = \vec{r}_a - \vec{r}_b$ is the relative separation, and $\vec{k} = (m_b \vec{p}_a - m_a \vec{p}_b)/(m_a + m_b)$ is the momentum of particle a in the center-of-mass system [see Fig. 6(b)].

After the scattering when the target and beam particles are separated well beyond the range of nuclear forces, the wave function ψ of the system is

$$\lim_{t\to\infty,\, r\to\infty}\psi = e^{-iEt/\hbar}\exp[i(\vec{p}_a+\vec{p}_b)\cdot\vec{R}/\hbar]$$
$$\times[e^{i\vec{k}\cdot\vec{r}} + r^{-1}e^{ikr}M(\vec{k}',\vec{k}]\chi\ ,$$

where $\vec{k}' = (m_b\vec{p}_a' - m_a\vec{p}_b')/(m_a+m_b)$ is the momentum in the center-of-mass system of the beam particles after the scattering, $k=|\vec{k}|$, and M is the 4×4 scattering matrix.

We list below the formulas for several scattering observables.[8] Two different right-handed coordinate systems are useful in writing these formulas. The first has directions $\vec{P}=\vec{k}'+\vec{k}$, $\vec{K}=\vec{k}'-\vec{k}$, and $\vec{n}=\vec{k}\times\vec{k}'$, where $\vec{K}$ is the momentum transfer and $\vec{n}$ is the normal to the scattering plane. When $m_b=m_a$, then $\theta_{\text{c.m.}}=2\theta_L$ relates the center-of-mass and laboratory scattering angles, $\hat{P}=\hat{p}_a'$, and $\hat{K}=-\hat{p}_b'$, where carets denote unit vectors. We will assume that the masses are indeed equal, in which case $\hat{K}$, $\hat{P}$, and $\hat{n}$ may be drawn as in Fig. 6(a). The circle with the center dot denotes the vector $\hat{n}$ emerging normal to the plane of the paper.

The second coordinate system has directions $\hat{x}$, $\hat{y}$, and $\hat{z}$, where $\hat{x}=\hat{n}\times\hat{p}_a$, $\hat{y}=\hat{n}$, and $\hat{z}=\hat{p}_a$. This system is also sketched in Fig. 6(a).

The formulas for the observables appear below. $\vec{\sigma}^{(a)}$ and $\vec{\sigma}^{(b)}$ are Pauli spin matrices acting on the spinors of nucleons a and b, respectively. Thus $\vec{S}^{(a)}=(\frac{1}{2}\hbar)\vec{\sigma}^{(a)}$ and $\vec{S}^{(b)}=(\frac{1}{2}\hbar)\vec{\sigma}^{(b)}$. I_0 denotes the unpolarized differential cross section $d\sigma/d\Omega$, and $P^{(i)}$ and $\mathfrak{A}^{(i)}$ denote the polarization and asymmetry of particle i, where $i=a$ or b.

$$I_0 = d\sigma/d\Omega = \tfrac{1}{4}\text{Tr}[M^\dagger M]\ ,$$
$$I_0P^{(i)} = \tfrac{1}{4}\text{Tr}[M^\dagger\vec{\sigma}^{(i)}\cdot\hat{n}M]\ ,\quad i=a,b$$
$$I_0\mathfrak{A}^{(i)} = \tfrac{1}{4}\text{Tr}[M^\dagger M\vec{\sigma}^{(i)}\cdot\hat{n}]\ ,\quad i=a,b$$
$$I_0D = \tfrac{1}{4}\text{Tr}[M^\dagger\vec{\sigma}^{(a)}\cdot\hat{n}M\vec{\sigma}^{(a)}\cdot\hat{n}]\ ,$$
$$I_0A = \tfrac{1}{4}\text{Tr}[M^\dagger\vec{\sigma}^{(a)}\cdot\hat{K}M\vec{\sigma}^{(a)}\cdot\hat{z}]\ ,$$
$$I_0R = \tfrac{1}{4}\text{Tr}[M^\dagger\vec{\sigma}^{(a)}\cdot\hat{K}M\vec{\sigma}^{(a)}\cdot\hat{x}]\ ,$$
$$I_0A' = \tfrac{1}{4}\text{Tr}[M^\dagger\vec{\sigma}^{(a)}\cdot\hat{P}M\vec{\sigma}^{(a)}\cdot\hat{z}]\ ,$$
$$I_0R' = \tfrac{1}{4}\text{Tr}[M^\dagger\vec{\sigma}^{(a)}\cdot\hat{P}M\vec{\sigma}^{(a)}\cdot\hat{x}]\ ,$$
$$I_0D_t = \tfrac{1}{4}\text{Tr}[M^\dagger\vec{\sigma}^{(b)}\cdot(-\hat{n})M\vec{\sigma}^{(a)}\cdot(-\hat{n})]\ ,$$
$$I_0A_t = \tfrac{1}{4}\text{Tr}[M^\dagger\vec{\sigma}^{(b)}\cdot(-\hat{P})M\vec{\sigma}^{(a)}\cdot\hat{z}]\ ,$$
$$I_0R_t = \tfrac{1}{4}\text{Tr}[M^\dagger\vec{\sigma}^{(b)}\cdot(-\hat{P})M\vec{\sigma}^{(a)}\cdot(-\hat{x})]\ ,$$
$$I_0A_t' = \tfrac{1}{4}\text{Tr}[M^\dagger\vec{\sigma}^{(b)}\cdot(-\hat{K})M\vec{\sigma}^{(a)}\cdot(\hat{z})]\ ,$$
$$I_0R_t' = \tfrac{1}{4}\text{Tr}[M^\dagger\vec{\sigma}^{(b)}\cdot(-\hat{K})M\vec{\sigma}^{(a)}\cdot(-\hat{x})]\ ,$$
$$I_0C_{PP} = \tfrac{1}{4}\text{Tr}[M^\dagger\vec{\sigma}^{(a)}\cdot\hat{P}\vec{\sigma}^{(b)}\cdot\hat{P}M]\ ,$$
$$I_0C_{KK} = \tfrac{1}{4}\text{Tr}[M^\dagger\vec{\sigma}^{(a)}\cdot(-\hat{K})\vec{\sigma}^{(b)}\cdot(-\hat{K})M]\ ,$$
$$I_0C_{nn} = \tfrac{1}{4}\text{Tr}[M^\dagger\vec{\sigma}^{(a)}\cdot\hat{n}\vec{\sigma}^{(b)}\cdot\hat{n}M]\ ,$$
$$I_0C_{KP} = \tfrac{1}{4}\text{Tr}[M^\dagger\vec{\sigma}^{(a)}\cdot\hat{K}\vec{\sigma}^{(b)}\cdot\hat{P}M]\ ,$$
$$I_0A_{zz} = \tfrac{1}{4}\text{Tr}[M^\dagger M\vec{\sigma}^{(a)}\cdot\hat{z}\vec{\sigma}^{(b)}\cdot\hat{z}]\ ,$$
$$I_0A_{yy} = \tfrac{1}{4}\text{Tr}[M^\dagger M\vec{\sigma}^{(a)}\cdot\hat{y}\vec{\sigma}^{(b)}\cdot\hat{y}]\ ,$$
$$I_0A_{xx} = \tfrac{1}{4}\text{Tr}[M^\dagger M\vec{\sigma}^{(a)}\cdot\hat{x}\vec{\sigma}^{(b)}\cdot\hat{x}]\ ,$$
$$I_0A_{zx} = \tfrac{1}{4}\text{Tr}[M^\dagger M\vec{\sigma}^{(a)}\cdot\hat{z}\vec{\sigma}^{(b)}\cdot\hat{x}]\ .$$

The M matrix which appears in the preceding formulas is assumed to have the most general form consistent with rotation invariance (conservation of total angular momentum). Thus it is a 4×4 matrix of the form

TABLE II. Relationships between polarizations P^a and P^b and asymmetries $\mathfrak{A}^a$ and $\mathfrak{A}^b$ of nucleons a and b, respectively, when isospin I (0 or 1) and parity P are conserved ("good") or not conserved ("bad"); also when the interaction is invariant ("good") or noninvariant ("bad") under time reversal T. Relationships between C_{nn} and A_{yy} are similarly shown.

	I bad P bad T bad	I bad P good T bad	I bad P bad T good	I bad P good T good	I good P bad T bad	I good P good T bad	I good P bad T good	I good P good T good
$P^a \stackrel{?}{=} \mathfrak{A}^a$	No	No	Yes	Yes	No	No	Yes	Yes
$P^b \stackrel{?}{=} \mathfrak{A}^a$	No	No	Yes	Yes	No	No	Yes	Yes
$P^a \stackrel{?}{=} P^b$	No	No	No	No	Yes	Yes	Yes	Yes
$\mathfrak{A}^a \stackrel{?}{=} \mathfrak{A}^b$	No	No	No	No	Yes	Yes	Yes	Yes
$A_{yy} \stackrel{?}{=} C_{nn}$	No	Yes	Yes	Yes	No	Yes	Yes	Yes

$$M=A+B_1(\sigma^{(a)}\cdot\hat{P}+\sigma^{(b)}\cdot\hat{P})+C_1(\sigma^{(a)}\cdot\hat{P}-\sigma^{(b)}\cdot\hat{P})+B_2(\sigma^{(a)}\cdot\hat{K}+\sigma^{(b)}\cdot\hat{K})+C_2(\sigma^{(a)}\cdot\hat{K}-\sigma^{(b)}\cdot\hat{K})$$
$$+B_3(\sigma^{(a)}\cdot\hat{n}+\sigma^{(b)}\cdot\hat{n})+C_3(\sigma^{(a)}\cdot\hat{n}-\sigma^{(b)}\cdot\hat{n})+D_{11}\sigma^{(a)}\cdot\hat{P}\sigma^{(b)}\cdot\hat{P}+D_{12}(\sigma^{(a)}\cdot\hat{P}\sigma^{(b)}\cdot\hat{K}+\sigma^{(a)}\cdot\hat{K}\sigma^{(b)}\cdot\hat{P})$$
$$+E_{12}(\sigma^{(a)}\cdot\hat{P}\sigma^{(b)}\cdot\hat{K}-\sigma^{(a)}\cdot\hat{K}\sigma^{(b)}\cdot\hat{P})+D_{22}\sigma^{(a)}\cdot\hat{K}\sigma^{(b)}\cdot\hat{K}+D_{23}(\sigma^{(a)}\cdot\hat{K}\sigma^{(b)}\cdot\hat{n}+\sigma^{(a)}\cdot\hat{n}\sigma^{(b)}\cdot\hat{K})$$
$$+E_{23}(\sigma^{(a)}\cdot\hat{K}\sigma^{(b)}\cdot\hat{n}-\sigma^{(a)}\cdot\hat{n}\sigma^{(b)}\cdot\hat{K})+D_{33}\sigma^{(a)}\cdot\hat{n}\sigma^{(b)}\cdot\hat{n}+D_{13}(\sigma^{(a)}\cdot\hat{P}\sigma^{(b)}\cdot\hat{n}+\sigma^{(a)}\cdot\hat{n}\sigma^{(b)}\cdot\hat{P})$$
$$+E_{13}(\sigma^{(a)}\cdot\hat{P}\sigma^{(b)}\cdot\hat{n}-\sigma^{(a)}\cdot\hat{n}\sigma^{(b)}\cdot\hat{P}),$$

where A, B_j, C_j, D_{ij}, and E_{ij} $(i,j=1,2,3)$ are functions of $\cos\theta_{\text{c.m.}}$ and the total center-of-mass energy. With this general form, M need not conserve isospin I nor parity P, nor need it be invariant under time-reversal T. When various of these symmetries hold, then various relationships hold between the experimental observables. Some examples are given in Table II. The relationships are obtained according to the following rules: If I is good, then B_1, B_2, C_3, D_{13}, D_{23}, and E_{12} are zero; if T is good, then B_2, C_2, D_{12}, D_{23}, E_{12}, and E_{23} are zero; if P is good, then B_1, C_1, B_2, C_2, D_{13}, E_{13}, D_{23}, and E_{23} are zero. We note that for the nucleon-nucleon system, I is good either when both parity and total mechanical spin $\vec{S}^{(a)}+\vec{S}^{(b)}$ are conserved, or when neither is conserved; I is bad when one but not the other is conserved.

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[1]R. A. Arndt, J. Binstock, and R. A. Bryan, Phys. Rev. D 8, 1397 (1973).

[2]R. A. Bryan and A. Gersten, Phys. Rev. D 6, 341 (1972).

[3]J. P. Scanlon, G. H. Stafford, J. J. Thresher, P. H. Bowen, and A. Langsford, Nucl. Phys. 41, 401 (1963).

[4]The fact that the Harwell n-p differential cross-section data are responsible for the anomalous value of $\delta(^1P_1)$ was observed by one of us (R.A.B.) and R. A. Arndt in 1966 (unpublished). J. K. Perring [Rev. Mod. Phys. 39, 550 (1967)] also observed the non-OPEC behavior of $\delta(^1P_1)$ at 50 MeV and recommended confirmation of the Harwell back-angle $d\sigma/d\Omega$ data. P. S. Signell [in *Proceedings of the Symposium on the Two-Body Force in Nuclei, Gull Lake, Michigan, 1971,* edited by S. M. Austin and G. M. Crawley (Plenum, New York, 1972), p. 9] has similarly remarked on the $\delta(^1P_1)$ anomaly and shown that it is a result of back-angle $d\sigma/d\Omega$ data.

[5]See paper I for a presentation of these data.

[6]R. M. Wright, M. H. MacGregor, and R. A. Arndt, Phys. Rev. 159, 1422 (1967) (paper VI).

[7]The Davis group has recently reported new forward n-p $d\sigma/d\Omega$ data at 50 MeV [T. C. Montgomery, F. P. Brady, B. E. Bonner, W. B. Broste, and M. W. McNaughton, Phys. Rev. Lett. 31, 640 (1973)]. In this letter they also report on the backward n-p $d\sigma/d\Omega$ data which they had kindly permitted us to quote in preliminary form in paper I. These new data are relative data only. We have performed a constrained phase-shift analysis of these new data plus the other world p-p and n-p data in the 47.5 to 60.9 MeV range and find that $\delta(^1P_1)=-4.1°\pm1.0°$. (Our solution is constrained in that ϵ_1 is set equal to 2.78° and not searched, since appropriate data to fix ϵ_1 is lacking, as discussed in Sec. III A.) This value of $\delta(^1P_1)$ is four or five standard deviations away from meson-theoretical predictions, such as those quoted in paper I. If the Harwell n-p $d\sigma/d\Omega$ data are excluded from the search, leaving for n-p $d\sigma/d\Omega$ data only the Davis and Oak Ridge measurements (see the data listing in Table I, paper I), then we find that $\delta(^1P_1)=-7.5°\pm1.8°$. This new value is only a standard deviation or two away from meson theory. Of course it would be nice to reduce the error in the determination of $\delta(^1P_1)$. This will require more accurate data or a more precise determination of the efficiency of the neutron detector used in taking the Davis forward $d\sigma/d\Omega$ measurements. To override the Harwell data, much more accurate data will be required; of course one cannot discount the fact that the Harwell data may be right.

[8]For a more detailed treatment of the formalism of nucleon-nucleon scattering, see N. Hoshizaki, Prog. Theor. Phys. Suppl. 42, 107 (1968). Our conventions are the same as his for all but the definitions of A_t and A_t', where we differ in over-all sign.