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Vanishing current in the ground state of pion-condensed systems

R. F. Sawyer and V. Soni*
Department of Physics, University of California, Santa Barbara, California 93106
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Baym's theorem on the vanishing of the current in the ground state is shown, in the case of a running-wave π^- condensation, to be realized only when a π^+ wave (of smaller amplitude) is added, moving in the *opposite* direction to the π^- wave. Here the division into π^+ and π^- quanta is defined with respect to the standard free-field quantization of the pion field. This ground state was previously noted by Sawyer and Yao. Baym's criticisms of the methods used in previous work are shown to be unjustified.

In a recent publication Baym[1] raised some points, basically concerned with previous work on pion condensation,[2] upon which we would like to comment. The main statements in his letter which bear on the previous work are the following:

(a) that a homogeneous pion-condensed system of nuclear matter will have zero electric current in the ground state,

(b) that in the solution to the pion-condensation problem given in Ref. 2 there is an interaction current term which is in the opposite direction to and of greater magnitude than the free π^--meson current term, and

(c) that the failure of the total current to vanish in this solution is due to an artificial constraint on the nucleon motion.

Baym's observations (a) and (b) are both correct and important; but we disagree completely with his implied assertion (c). The fact is that the method of Ref. 2 gives an answer in which the current is exactly zero, provided that π^+ mesons are included as well as π^- mesons and that certain small nucleon recoil terms discussed in Ref. 2 are neglected. That the energy could be lowered by addition of a π^+ wave was already noted by Sawyer and Yao, and the solution they worked out in Ref. 3 already displays the cancellation of currents between the interaction term and the free-meson terms. The proof of these assertions follows.

(1) In footnote 12 of I it was stated clearly that the imposition of the constraint that the pion momentum be balanced by the proton momentum, in our $\pi^- np$ system, did not lead to significant errors in the calculation of the energy. Since Baym asserts otherwise, we repeat the argument here.

The Hamiltonian of I can be written as

$$H=\frac{1}{2M_p}\int \vec{\nabla}\bar{p}(\vec{x})\cdot\vec{\nabla}p(\vec{x})\,d^3x+H', \tag{1}$$

where H' contains all of the interactions except for the proton kinetic energy. We introduce a new proton field $p'(\vec{x})=p(\vec{x})e^{ikz}$, where k is the momentum of the π^- mode, and we write H as

$$H=\frac{1}{2M_p}\int \vec{\nabla}\bar{p}(x)'\cdot\vec{\nabla}p(x)'\,d^3x+H' +\frac{k^2}{2M_p}\int d^3x\,p'p'+H'' , \tag{2}$$

where

$$H''=-\frac{ik}{2M_p}\int d^3x\left[\left(\frac{\partial}{\partial z}\bar{p}'\right)p'-\bar{p}'\frac{\partial}{\partial z}p'\right]. \tag{3}$$

Now we go through the pion-condensation calculation of Ref. 2. If we omit the term H'' of Eq. (3), the calculation goes through exactly as before even if we omit the nucleon momentum constraint to which Baym objects. That is to say, the Lagrange multiplier λ of Eq. (2.5) of I will now turn out to be numerically zero in the ground state.

Now the previously neglected term, H'', can be treated in perturbation theory; the energy shifts due to H'' are of the order of $k^2XM_p^{-1}$ per nucleon, where X is the fraction of nucleons which are protons. This is a negligible contribution to the condensation energy [Eq. (2.16) of I].

(2) To discuss the cancellation of the current we can simplify the formulas and display the essential point by considering the limit of very large nucleon mass, so that the proton currents can be neglected, whatever momentum the protons carry. This time we refer to Sec. V of Ref. 3, in which a π^+ running wave of momentum $-k\hat{z}$ is added to the π^- running wave of momentum $k\hat{z}$. The energy per baryon in the state of lowest energy with prescribed numbers of π^+ and π^- present is given by Eq. (45) of Ref. 3 (with nucleon kinetic-energy terms dropped):

$$\frac{E}{N}=\omega_k(X+2Y) - 2k\,fM_\pi^{-1}\rho^{1/2}\omega_k^{-1/2}[(X+Y)^{1/2}+Y^{1/2}] \times[X(1-X)]^{1/2}\,, \tag{4}$$

where $\omega_k=(k^2+M_\pi^2)^{1/2}$, f is the pion-nucleon coupling constant, ρ is the number density of baryons, XN is the number of π^- minus the number of π^+, and YN is the number of π^+.

The z component of the current in this system is easily calculated and found to be given by

$$\frac{J_z}{\rho}=2fM_\pi^{-1}\rho^{1/2}\omega_k^{-1/2}[(X+Y)^{1/2}+Y^{1/2}][X(1-X)]^{1/2} -\frac{k}{\omega}(X+2Y)-\frac{2k}{\omega}X^{1/2}(X+Y)^{1/2}\,. \tag{5}$$

The first term here is the interaction current referred to by Baym, the second term is the current of the π^- wave added to that of the π^+ wave, and the third term is an interference term between the π^- wave and the π^+ wave.

In the state of lowest energy we have $dE/dX = dE/dY = dE/dk = 0$. If we combine the equations $dE/dY=0$ and $dE/dk=0$, determined from (4), we obtain directly the equation $J_z=0$.

Finally we remark on the reason, in terms of Baym's proof, that π^+ particles must enter in the above way. Suppose that there exists a homogeneous state with a nonvanishing expectation value of J_z and some expectation value of the Hamiltonian $\langle H\rangle$.

Baym notes that the transformation on all charge-raising fields $\phi(x)$,

$$\phi(\vec{x})\to U^\dagger(q)\,\phi(\vec{x})\,U(q)=\phi(\vec{x})\,e^{iqz}\,, \tag{6}$$

and the corresponding transformation on the charge-lowering fields,

$$\phi(\vec{x})^*\to\phi(\vec{x})^*e^{-iqz}\,,$$

define a transformation on the states of the theory which will invariably lower $\langle H\rangle$, in our current-carrying state, for some positive or negative infinitesimal value of q.

Here we note the effect of the transformation on the states of a free, relativistic, charged-meson field. Let $a_{(-)}(\vec{k})$ be the annihilation operator for a π^- of momentum k and $a^\dagger_{(+)}(-\vec{k})$ be the creation operator for a π^+ of momentum $-\vec{k}$. Defining $\vec{q}=q\hat{z}$ we have

$$U^\dagger(q)\,a_{(-)}(\vec{k})\,U(q)=\frac{1}{2}\left[\left(\frac{\omega_k}{\omega_{k-q}}\right)^{1/2}+\left(\frac{\omega_{k-q}}{\omega_k}\right)^{1/2}\right]\times a_{(-)}(\vec{k}-\vec{q}) +\frac{1}{2}\left[\left(\frac{\omega_k}{\omega_{k-q}}\right)^{1/2}-\left(\frac{\omega_{k-q}}{\omega_k}\right)^{1/2}\right]\times a^\dagger_{(+)}(-\vec{k}+\vec{q})\,. \tag{7}$$

Now suppose we begin with the lowest-energy state of I, which has only a π^- wave, and which has a nonvanishing current. Then we apply the energy-lowering transformation U to this state. Some π^+ wave will be mixed in by the transformation. Minimizing the energy through the correct admixture of π^+'s will automatically eliminate the current.

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