Solutions to the "new" equation of motion for classical charged particles*

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The "new" equation of motion for classical charged particles proposed by Mo and Papas to include radiation reaction in a second-order equation is shown to reduce to the usual radiation-neglected equation for charged particles moving in straight lines when acted upon by electric fields only. On the basis of our published calculations, we conclude that the Mo and Papas equation gives what intuitively appears to be unphysical results: The kinetic energy of a pair of like-charged particles can be many times larger *after* collision than before.

Numerical results have recently been presented for the problem of two charged particles of equal mass in straight-line motion using retarded fields without radiation reaction.[1,2] We are now studying this problem with radiation reaction included, using the generally accepted Lorentz-Dirac equation. We have, of course, found runaway solutions and related difficulties for which the third-order Lorentz-Dirac equation is well known. Mo and Papas,[3] in order to avoid such difficulties, proposed a new equation of motion including radiation-reaction effects in a second-order differential equation. They and Shen[4] tested their equation for various cases and found that the results do not differ in any presently physically detectable way from those of the Lorentz-Dirac equation. We therefore decided to apply their expectedly simpler equation to our problem. In the process, we found a curious feature of the Mo-Papas equation: It reduces to the usual *radiation-neglected* equation for the case of charged particles moving in a straight line under the action of electric fields only.

The result can be simply seen as follows. The equation of Mo and Papas, for a particle of charge e and rest mass m, is (in the notation of Ref. 3)

$$m\dot{u}^\mu - (2e^3/3m)F^{\lambda\alpha}\dot{u}_\lambda u_\alpha u^\mu = eF^{\mu\lambda}u_\lambda + (2e^3/3m)F^{\mu\lambda}\dot{u}_\lambda . \qquad (1)$$

Scalar multiplication by $\dot{u}_\mu$ causes the second and fourth terms to vanish, leaving, as Mo and Papas noted,[3]

$$m\dot{u}^\mu\dot{u}_\mu = eF^{\mu\lambda}u_\lambda\dot{u}_\mu . \qquad (2)$$

Combining Eqs. (2) and (1), we obtain

$$m\dot{u}^\mu - eF^{\mu\lambda}u_\lambda = (2e^2/3m)\,(m\dot{u}_\lambda\dot{u}^\lambda u^\mu + eF^{\mu\lambda}\dot{u}_\lambda). \qquad (3)$$

For motion in a straight line we can set $u^\mu = (\gamma, u, 0, 0)$, where $\gamma = (1+u^2)^{1/2}$, and for zero magnetic field we have $F^{\mu\lambda}=0$ unless either $\mu=0$ or $\lambda=0$. Observing that $\dot{u}_\lambda\dot{u}^\lambda = \dot{\gamma}^2 - \dot{u}^2 = -\dot{u}^2/\gamma^2 = -\dot{u}\dot{\gamma}/u\gamma$, we then find that Eq. (3) reduces to

$$m\dot{u} - eF^{10}\gamma = -(2e^2/3m\gamma)\dot{\gamma}(m\dot{u} - eF^{10}\gamma). \qquad (4)$$

The only possible physical solution to the Mo-Papas equation in this case is then just the solution to the radiation-neglected equation

$$m\dot{u} = eF^{10}\gamma . \qquad (5)$$

For the interaction, for example, of two charged particles of equal mass which either (i) have opposite charges and are released from rest at some initial separation d or (ii) have like charges and are thrown at each other with equal initial speeds, only retarded electric fields (no radiation reaction) need be considered in Eq. (5), and the resulting numerical solutions have already been published.[1]

In particular, the results in case (ii) show what intuitively appears to be unphysical behavior: The like-charged particles fly apart with more than their initial energy. The same results are of course solutions of the Mo-Papas equation, but whereas we were able to explain the results in terms of a pulse of negative energy arising from the overlap of radiation fields, in the Mo-Papas formulation the "unphysical" behavior must be related to the extra terms they have added to the energy-momentum tensor.

The "unphysical" nature of the solutions is not insignificant. We found, for example, that for initial kinetic energies equal to $\frac{2}{3}$ of the rest-mass energies, the increase in energy is over $8mc^2$. Comparison with experiment must of course provide the final basis for accepting or rejecting any self-consistent theory, and our calculations were restricted to collisions at zero impact parameter; nevertheless, we feel that our results constitute evidence against the general validity of the equation of motion proposed by Mo and Papas.

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Vanishing current in the ground state of pion-condensed systems

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Baym's theorem on the vanishing of the current in the ground state is shown, in the case of a running-wave π^- condensation, to be realized only when a π^+ wave (of smaller amplitude) is added, moving in the *opposite* direction to the π^- wave. Here the division into π^+ and π^- quanta is defined with respect to the standard free-field quantization of the pion field. This ground state was previously noted by Sawyer and Yao. Baym's criticisms of the methods used in previous work are shown to be unjustified.

In a recent publication Baym[1] raised some points, basically concerned with previous work on pion condensation,[2] upon which we would like to comment. The main statements in his letter which bear on the previous work are the following:

(a) that a homogeneous pion-condensed system of nuclear matter will have zero electric current in the ground state,

(b) that in the solution to the pion-condensation problem given in Ref. 2 there is an interaction current term which is in the opposite direction to and of greater magnitude than the free π^--meson current term, and

(c) that the failure of the total current to vanish in this solution is due to an artificial constraint on the nucleon motion.

Baym's observations (a) and (b) are both correct and important; but we disagree completely with his implied assertion (c). The fact is that the method of Ref. 2 gives an answer in which the current is exactly zero, provided that π^+ mesons are included as well as π^- mesons and that certain small nucleon recoil terms discussed in Ref. 2 are neglected. That the energy could be lowered by addition of a π^+ wave was already noted by Sawyer and Yao, and the solution they worked out in Ref. 3 already displays the cancellation of currents between the interaction term and the free-meson terms. The proof of these assertions follows.

(1) In footnote 12 of I it was stated clearly that the imposition of the constraint that the pion momentum be balanced by the proton momentum, in our $\pi^- np$ system, did not lead to significant errors in the calculation of the energy. Since Baym asserts otherwise, we repeat the argument here.

The Hamiltonian of I can be written as

$$H=\frac{1}{2M_p}\int \vec{\nabla}\bar{p}(\vec{x})\cdot\vec{\nabla}p(\vec{x})\,d^3x+H', \tag{1}$$

where H' contains all of the interactions except for the proton kinetic energy. We introduce a new proton field $p'(\vec{x})=p(\vec{x})\,e^{ikz}$, where k is the momentum of the π^- mode, and we write H as

$$H=\frac{1}{2M_p}\int \vec{\nabla}\bar{p}(x)'\cdot\vec{\nabla}p(x)'\,d^3x+H' +\frac{k^2}{2M_p}\int d^3x\,p'p'+H'' , \tag{2}$$

where

$$H''=-\frac{ik}{2M_p}\int d^3x\left[\left(\frac{\partial}{\partial z}\bar{p}'\right)p'-\bar{p}'\frac{\partial}{\partial z}p'\right]. \tag{3}$$

Now we go through the pion-condensation calculation of Ref. 2. If we omit the term H'' of Eq. (3), the calculation goes through exactly as before even if we omit the nucleon momentum constraint to which Baym objects. That is to say, the Lagrange multiplier λ of Eq. (2.5) of I will now turn out to be numerically zero in the ground state.

Now the previously neglected term, H'', can be treated in perturbation theory; the energy shifts due to H'' are of the order of $k^2XM_p^{-1}$ per nucleon, where X is the fraction of nucleons which are protons. This is a negligible contribution to the con-