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Electromagnetic mass differences in compensation theory*

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A beginning is made toward developing a model where the electromagnetic mass differences between hadrons are constrained by requiring the finiteness of certain processes to lowest order in all coupling constants. The methods are taken over from a previously worked-out model, called compensation theory, in which electromagnetic masses were neglected. Here it is shown how a certain process, if assumed to be finite, yields the mass relation of Coleman and Glashow. SU(3) symmetry among the hadrons is not even approximately assumed at low energy.

A Lagrangian model ("compensation theory") of the combined strong and weak interactions has been developed over the past few years.[1-3] This model is characterized by an algebra, encompassing SU(3), for the weak currents, and by the absence of SU(3) symmetry for the strong couplings, i.e., for the strong interactions at low energy. The model numerically predicts several strong couplings by requiring the cutoff independence of the individual terms in the strong-interaction Born series. Several mass formulas are also deduced, notably that of Gell-Mann and Okubo, as well as some totally new ones. In the development of the theory to date, electromagnetic and weak mass splittings have been ignored, and the strong couplings were taken to be exactly invariant under isospin rotations.

This note is motivated by the curious fact that, in compensation theory, the Gell-Mann–Okubo formula could be derived without assuming even approximate SU(3) invariance for the strong couplings. This circumstance leads to the speculation that many more results, usually derived from approximate SU(3), may be obtained from compensation theory as well. Here we demonstrate, in particular, that the formula of Coleman and Glashow[4] for the baryonic mass differences,

$$n - p + \Sigma^+ - \Sigma^- + \Xi^- - \Xi^0 = 0, \tag{1}$$

does in fact follow from compensation theory applied to one specific process. Here, however, in contrast to the extensive and apparently complete self-consistency of the previously developed compensation relations, we can think of many processes which no longer seem to compensate as soon as electromagnetic mass differences are turned on. Thus, the calculation presented here only constitutes a beginning toward a complete compensation theory of electromagnetic mass differences: It selects a process whose analysis will have to remain essentially unchanged through future developments of the theory.

In addition, the following points can be made:

(1) The derivation is quite direct and makes almost no assumptions about the detailed nature of the electromagnetic interaction.

(2) The lack of SU(3) invariance, previously predicted for the strong couplings, plays an essential role.

(3) One current used in the calculation is the $V-A$ version of the baryon number current. This appears to provide a hint for a future enlargement of the theory's scope.

The derivation is as follows. We consider the (virtual) decay of a charged pion (e.g., π^+) into two W bosons, one of which is coupled through the $V-A$ baryon number current $\mathcal{B}_{V-A}$, and the other through the $V-A$, SU(3), F-type, isovector octet current $\mathcal{P}^+$. Explicitly, *if electromagnetic interactions are neglected*, the internal quantum numbers are involved as follows[5] in these two currents:

$$\mathcal{B}_{V-A} \propto p^*p + n^*n + \Sigma^{+*}\Sigma^+ + \Sigma^{0*}\Sigma^0 + \Sigma^{-*}\Sigma^- + \Lambda^*\Lambda + \Xi^{-*}\Xi^- + \Xi^{0*}\Xi^0, \tag{2}$$

$$\mathcal{P}^+_{V-A} \propto n^*p + \sqrt{2}\,\Sigma^{-*}\Sigma^0 - \sqrt{2}\,\Sigma^{0*}\Sigma^+ + \Xi^{-*}\Xi^0. \tag{3}$$

The spin structure is $\bar{\psi}\gamma^\mu(1+i\gamma^5)\psi'$ for each term.

The effects of including the electromagnetic coupling are assumed to be as follows, to first order in α:

(i) Some mixing is induced between Σ^0 and Λ:

$$\Sigma^0 \to \Sigma^0 \cos\chi + \Lambda \sin\chi, \tag{4}$$

$$\Lambda \to \Lambda \cos\chi - \Sigma^0 \sin\chi, \tag{5}$$

where

$$\chi = o(\alpha) . \tag{6}$$

In this way one preserves whatever current algebras one had before turning on α. Thus, $\mathcal{B}_{V-A}$ is unaffected, but

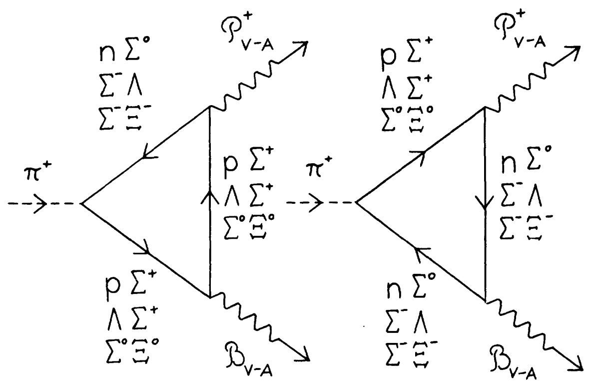

FIG. 1. A π^+ decays virtually into two W bosons. The logarithmically divergent diagrams are shown to lowest order in all couplings, with the possible internal baryons. No Yang-Mills-type interaction between the W bosons is considered in this paper.

$$\mathcal{P}^+_{V-A} \to \mathcal{P}^+_{V-A} + o(\alpha)\times(\Sigma^{-*}\Lambda - \Lambda^*\Sigma^+)_{V-A} + o(\alpha^2). \tag{7}$$

(ii) No degeneracy is left among the baryon octet masses.

(iii) The pion couplings are no longer SU(2)-invariant:

$$|g(p\bar{p}\pi^0)| - |g(n\bar{n}\pi^0)| = o(\alpha), \quad \text{etc.} \tag{8}$$

The process $\pi^+ \to (\mathcal{B}_{V-A})(\mathcal{P}^+_{V-A})$, to first order in the strong (Yukawa) couplings, is illustrated in Fig. 1, with the possible intermediate particles. (The reader will find in Ref. 2 a discussion on the legitimacy of thus isolating the first Born approximation.) No assumptions need be made about the magnitude of the $\mathcal{B}$ and $\mathcal{P}$ vertices, except that they are nonzero. They clearly are not electromagnetic but could be weak or superweak. They must therefore be considered as mere probes of a dynamic situation which they themselves do not cause. Although it might seem mysterious that mass formulas can be derived in this way, it should be remembered that in previous work the weak (and electromagnetic) couplings allowed us to derive mass relations commonly attributed to the medium-strong interactions. *All masses and strong couplings are assumed to possess their true nondegenerate physical value even without the presence of photons.* This is consistent with the "ordinary" form of quantum electrodynamics, which does not try to "explain" electromagnetic mass differences.

If we isolate the (logarithmically) divergent parts of the processes shown in Fig. 1, we discover that the clockwise and counterclockwise loops make identical contributions, which add up to yield the following compensation relation (= requirement for a finite amplitude) between masses and coupling constants:

$$(n-p)g(n\bar{p}\pi^+) + (\Xi^- - \Xi^0)g(\Xi^-\bar{\Xi}^0\pi^+) + (\sqrt{2}\cos\chi)[(\Sigma^- - \Sigma^0)g(\Sigma^-\bar{\Sigma}^0\pi^+) - (\Sigma^0 - \Sigma^+)g(\Sigma^0\bar{\Sigma}^-\pi^+)]$$
$$+ (\sqrt{2}\sin\chi)[(\Sigma^- - \Lambda)g(\Sigma^-\bar{\Lambda}\pi^+) - (\Lambda - \Sigma^+)g(\Lambda\bar{\Sigma}^-\pi^+)] = 0. \tag{9}$$

If this equation is written to order α, one can set

$$g(n\bar{p}\pi^+) = \sqrt{2}\, g_{NN}, \tag{10}$$

$$g(\Xi^-\bar{\Xi}^0\pi^+) = \sqrt{2}\, g_{\Xi\Xi}, \tag{11}$$

$$g(\Sigma^-\bar{\Sigma}^0\pi^+) = -g(\Sigma^0\bar{\Sigma}^-\pi^+) = g_{\Sigma\Sigma}, \tag{12}$$

$$g(\Sigma^-\bar{\Lambda}\pi^+) = g(\Lambda\bar{\Sigma}^-\pi^+) = g_{\Sigma\Lambda}\ . \tag{13}$$

Thus, (9) gives

$$(n-p)g_{NN} + (\Xi^- - \Xi^0)g_{\Xi\Xi} + (\Sigma^- - \Sigma^+)g_{\Sigma\Sigma} + (2\sin\chi)(\Sigma - \Lambda)g_{\Sigma\Lambda} = 0, \tag{14}$$

where we recall from Ref. 2 that

$$g_{NN} : g_{\Xi\Xi} : g_{\Sigma\Sigma} = (N^{-1}) : (\Xi^{-1}) : (-\Sigma^{-1}). \tag{15}$$

In the usual derivation of the Coleman-Glashow formula, one may consider, at first, two symmetry-breaking interactions: a medium-strong SU(3)-breaking parameter κ, and an electromagnetic one, α, which breaks SU(2). The mass differences are then studied when κ and α are simultaneously but independently small.

In our case, there is no parameter κ because SU(3) is not even an approximate symmetry for the strong couplings. However, we can with equivalent effect set the masses $N \approx \Xi \approx \Sigma \approx \Lambda$ equal in the coefficients of Eq. (14). The result, taking (15) into account, is

$$n - p + \Xi^- - \Xi^0 + \Sigma^+ - \Sigma^- = 0,$$

precisely the broken-SU(3) result.

What is the significance of the present approach, and to what extent is it really new? It may at first be thought that, since our assumptions involve SU(3), there is nothing remarkable in the fact that we reproduce the Coleman-Glashow formula. A closer look at the derivation reveals, however, that we make use of an explicit "anti-SU(3)" input, Eq. (15), whose nature is essential to the result. This considerably strengthens the two conjectures that (a) compensation theory will turn out to be equivalent to the standard broken-SU(3) theory in those calculations where the latter is successful, and that (b) the strong-coupling predictions of compensation theory indeed should not follow SU(3).

In conclusion, it should be observed that two possible hints emerge concerning the future evolution of the theory:

(a) In Ref. 2 it was shown that a current $\mathcal{B}_{V-A}$ led to an imperfectly compensated weak process involving the K mesons. But since $\mathcal{B}_{V-A}$ is essential to the present derivation, it appears that some rather minute tampering with the postulates (most likely the inclusion of additional elementary particles) will be needed in the future.

(b) In this paper, we consider the pair of currents $\mathcal{P}^{+}_{V-A}, \mathcal{B}_{V-A}$. Equivalently, we could have considered the $V+A$ versions without affecting the result. Most other possible pairs yield no direct information about mass differences because a knowledge of some coupling-constant differences would also be required. However, some pairs such as ($\mathcal{P}^{+}_{V-A}$, $\mathcal{Y}_{V-A}$) do yield mass relations if one postulates that they, too, correspond to compensated processes. These mass relations are, in general, at variance with the facts and with conventional SU(3) theory. For example, the pair ($\mathcal{P}^{+}, \mathcal{Y}$) would imply $n-p+\Xi^0-\Xi^-=0$ if compensation were postulated. Thus, with the theory in its present form, the class of compensated processes is smaller with electromagnetic perturbations than without. Why does the ($\mathcal{P}^{+}, \mathcal{B}$) pair play a privileged role? It is not unreasonable to speculate that we shall have to invoke the SU(3)-singlet nature of $\mathcal{B}$, or else the related fact that the "complete" $\mathcal{Y}$, $\mathcal{P}^0$, and $\mathcal{I}^0$ (see Ref. 2, Appendix D) should involve mesonic terms as well as baryonic ones, while $\mathcal{B}$ could be entirely restricted to baryonic terms.

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[3]M. Wellner, Ann. Phys. (N.Y.) 76, 549 (1973).

[4]S. Coleman and S. L. Glashow, Phys. Rev. Lett. 6, 423 (1961).

[5]Strictly speaking, a negative-parity, SU(3)-singlet baryon L should be mixed with the Λ (see Ref. 2), but our argument is not modified by leaving it out, as will be done here for simplicity.