Peripherality of tensor and vector exchanges in an approximately unitarized dual $\pi\pi$ model. II*

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We consider a simple $\pi\pi$ model in which we first dynamically generate a low- and intermediate-energy absorptive part A and then calculate effective Regge residues in terms of A via finite-energy sum rules (FESR). In an earlier paper, A was calculated by unitarizing the Lovelace-Veneziano model. In the present paper we follow a somewhat less model-dependent procedure. In states and energy regions where no important narrow resonances are present, we evaluate the (background) A by using approximate unitarity and the duality assumption that amplitudes are well approximated on the average by Regge exchange. Elsewhere the resonances (ρ and f^0) are put in by hand. Pomeranchuk (P) parameters are taken from experiment or calculated from simple models. Exchange degeneracy and two-component duality are not assumed *a priori*, although we find that exchange degeneracy is approximately satisfied by our output Regge residues at $t = 0$. For $t \neq 0$, on the other hand, we find that it is broken. Specifically, ρ exchange is peripheral, as required by the dual absorptive model, whereas f exchange is not; a similar conclusion was reached on purely phenomenological grounds by Barger, Geer, and Halzen for other reactions.

I. INTRODUCTION

In an earlier paper[1] it was found that, if we saturate $\pi\pi$ finite-energy sum rules (FESR) with an approximately unitarized Lovelace-Veneziano model, we obtain a vector exchange which is peripheral, as predicted by the dual absorptive model,[2] and a tensor exchange which is not. This is consistent with recent experimental evidence, which points to a similar conclusion for the s-channel helicity-nonflip amplitudes of other processes.[3] Unfortunately our approach was based on the rather specific Lovelace-Veneziano model, which might be difficult to generalize. In the present paper we shall therefore follow a somewhat more general approach. In states and energy regions where no important narrow resonances are present, the background is generated dynamically from Regge exchange by combining approximate unitarity with simple average duality; although the Lovelace-Veneziano model is still used to calculate certain Regge residues in practice, it is not essential to do so. Elsewhere the resonances are put in by hand, although eventually they could be related to Regge behavior in the crossed channel.

In Sec. II we discuss our model for the background below the $\rho\rho$ threshold. This is then used, along with the ρ and the f^0 resonances, to calculate effective Regge residues via FESR in Sec. III, with the parameters of the Pomeranchukon P taken from experiment. In Appendixes A and B, however, these are calculated theoretically.

II. A SIMPLE MODEL FOR $\pi\pi$ BACKGROUNDS BELOW THE $\rho\rho$ THRESHOLD

If one looks at the empirical situation for $\pi\pi$ scattering below the $\rho\rho$ threshold[4] one finds:

(i) The prominent peaks are the peripheral ρ and f^0 resonances, which dominate in the $0 < s < 2m_\rho{}^2$ and $2m_\rho{}^2 < s < 4m_\rho{}^2$ energy intervals and the $I = 1$, $J = 1$ and $I = 0, J = 2$ partial waves. We shall approximate their contributions to the absorptive part A in the usual way by δ functions, assuming masses $m_\rho = 765$ MeV and $m_f = 1267$ MeV, and widths $\Gamma_\rho = 125$ MeV and $\Gamma_f = 150$ MeV. Any residual backgrounds under these resonances are also assumed to be absorbed by these δ functions. This was checked explicitly for $I = J = 1$ using experimental phase shifts[4] and found to be a good approximation.

(ii) All other partial waves and energy intervals, which then constitute a background, appear to be either smooth or to have, at worst, broad gentle bumps, such as the ϵ. This is in spite of the presence of a second-sheet pole like the S^*, which has very little effect on the $\pi\pi$ cross section, even though it may be prominent in $K\bar{K}$ scattering.[4] In fact we will completely neglect the effect of the $K\bar{K}$ channel and assume that $\pi\pi$ scattering is approximately elastic below the $\rho\rho$ threshold. This may be a poor approximation if we are interested in the fine details of our process, but should be reasonable enough if all we want is the average absorptive part over a broad interval.

In such background situations, where no prominent peaks are present, duality suggests that it

should be reasonable to set $T \approx T_{\text{Regge}}$, at least on the average, even at relatively low energies.[5] Indeed it may even be possible to approximately reproduce broad bumps in this way, at least within a given partial wave.[6] Once we have T, we can generate an absorptive part $A = \text{Im}T$ via the approximate partial-wave unitarity condition[5]

$$A^J \simeq 2(q/\sqrt{s})|T^J_{\text{Regge}}|^2 , \tag{1}$$

where q is the c.m. 3-momentum and $s = (\text{energy})^2$. In this way we obtain a finite absorptive part even in situations where T_{Regge} is purely real, as might happen in an exotic channel.[5]

Since Eq. (1) will be applied at relatively low energies, we will drop the contribution of the P in T_{Regge} and assume that it is given by the Lovelace-Veneziano model.[7] In other words, we are assuming that such a dual exchange-degenerate model is a good zeroth-order approximation to the amplitude and should be adequate for calculating background terms, which are, after all, basically just corrections to the ρ and f^0 resonance contributions. (In a more accurate calculation, of course, $\text{Im}T_{\text{Regge}}$ should be made self-consistent with the output A_{Reg} which we will calculate in Sec. III, while ReT should be calculated from Im T via a dispersion relation.) We then have

$$T^{I=2}_{\text{Regge}} = -\bar{\beta} V(t, u) , \tag{2a}$$

$$T^{I=1}_{\text{Regge}} = -\bar{\beta}[V(s, t) - V(s, u)], \tag{2b}$$

$$T^{I=0}_{\text{Regge}} = -\tfrac{1}{2}\bar{\beta}[3V(s, t) + 3V(s, u) - V(t, u)], \tag{2c}$$

where

$$V(s, t) \simeq \Gamma(1 - \alpha(t))(-s)^{\alpha(t)} , \tag{3}$$

which is just the asymptotic form of

$$V(s, t) = \frac{\Gamma(1 - \alpha(s))\Gamma(1 - \alpha(t))}{\Gamma(1 - \alpha(s) - \alpha(t))} , \tag{4}$$

and where s, t, u are the usual Mandelstam variables, $\alpha(t) = \frac{1}{2}(1 + t/m_\rho^2)$ is the Regge trajectory, and $\bar{\beta}$ is given in terms of the ρ width Γ_ρ. We will take $\bar{\beta} = 0.6$, which corresponds to $\Gamma_\rho = 125$ MeV.

The approximate partial-wave projection of Eqs. (2) is discussed in Appendix C. The resulting T^J_{Regge} were then used to evaluate A^J through Eq. (1) in the $0 < s < 4m_\rho^2$ interval for $I = 0$, $J = 0$ and $I = 2$, $J = 0$, and in the $2m_\rho^2 < s < 4m_\rho^2$ interval for $I = 1$, $J = 1$. The $0 < s < 2m_\rho^2$ interval for $I = 1$, $J = 1$ and the $2m_\rho^2 < s < 4m_\rho^2$ interval for $I = 0$, $J = 2$ are given in terms of the ρ and f^0 resonances, as we have seen already; elsewhere A^J can be neglected since it is a sufficiently high partial wave for $s < 4m_\rho^2$. Combining these background and resonant A^J we can now calculate the full absorptive part A by using the partial-wave expansion:

$$A = \sum_J (2J + 1)A^J P_J(1 + t/2q^2) . \tag{5}$$

In calculating the background A^J we have assumed that Regge behavior has some kind of validity even at rather low energies. This may seem somewhat dubious for $s \lesssim 2m_\rho^2$. Nevertheless, it turns out that the resulting average A^J for $0 < s \lesssim 2m_\rho^2$ is within roughly 10% of the value obtained from the experimental phase shifts for the $I = 0$, $J = 0$ state.[4] The situation is somewhat harder to check directly for the $I = 2$, $J = 0$ state, where the phase shift is very poorly known.[8] However, in this case Eq. (4) is already a smooth function and so we can calculate A_J directly without using the asymptotic form (3) (see Appendix C). Moreover, it can be explicitly shown that for other exotic channels, such as K^+p and pp scattering, the absorptive part is given to within roughly 30% by P exchange.[9] If we assume the same thing for $I = 2$ $\pi\pi$ scattering, with the P residue given by factorization in terms of the πp and pp cross sections, we find that the resulting absorptive part agrees quite well with our calculated value. Actually it turns out that even if we reduce the contribution of the $I = 2$, $J = 0$ state for $s < 2m_\rho^2$ by a factor of 2, none of our qualitative conclusions are changed thereby, and even our quantitative results are not changed by more than about 10%.

III. EFFECTIVE ρ AND f REGGE RESIDUES

As in Ref. 1, we will now make the Harari assumption that the high-energy absorptive part (but not necessarily ReT) can be approximated by the effective Regge behavior

$$A_{\text{Reg}} = \sum_i b_i(t) s^{\alpha_i(t)} , \tag{6}$$

with $\alpha_i(t) = \alpha_i(0) + \alpha_i' t$. This assumption is approximately consistent with experiment and leads to the FESR

$$\int_0^N ds(A - A_{\text{Reg}}) = 0 , \tag{7}$$

where we take A from Eq. (5). Equations (6) and (7) now enable us to calculate the $b_i(t)$.

Although our numerical values are somewhat different, our qualitative conclusions are identical with those obtained in Sec. III of Ref. 1 [but with the central T_J "backgrounds" now given by Eq. (1)]. If we also take the same N, α_i, a $[= b_P'/b_P]$, and R $[= b_P(0)/b_f(0)]$,[10] we obtain $b_\rho(0) = 0.523$ and a zero of $b_\rho(t)$ at $t \simeq -0.225$ GeV^{-2}; this corresponds to a zero of $J_0(r\sqrt{-t})$ with $r \simeq 1$ fermi, in agreement with the dual absorptive model (DAM). For $I_t = 0$ we obtain $b_P(0) = 0.844$, which gives an asymptotic cross section $\sigma_{\text{tot}} = 11.2$ mb and $b_f(0)/$

$b_\rho(0)=1.61$. Experimentally $\sigma_{\text{tot}} \approx 15$ mb, while exchange degeneracy gives an f-ρ ratio equal to 1.5. On the other hand, with $\alpha_P' = 0.5\alpha_\rho'$, the first zero of $b_f(t)$ appears at $t \simeq 0.57$ GeV^{-2}, in complete disagreement with DAM. To get agreement, one needs a much larger α_P'. For example, $\alpha_P' = \alpha_\rho'$ gives $b_f(0)=0$ at $t \simeq -0.24$ GeV^2. However, such an α_P' seems to be difficult to reconcile with experiment.[1,3,11]

It should again be emphasized that our conclusions were based mainly on certain general properties of the resonance and background terms and are not sensitive to the details of our model. This can be checked, for example, by repeating the calculation with a difference of two FESR, instead of using a single FESR as in Eq. (7). Specifically we took

$$\int_{2m_\rho{}^2}^{4m_\rho{}^2} ds(A - A_{\text{Reg}}) = 0 \ . \tag{8}$$

This involves a rather small energy interval but has the advantage that we do not have to assume any kind of Regge behavior at very low energies. None of our qualitative conclusions were affected. The results for $b_\rho(0)$, $b_f(0)$, $b_P(0)$ were at worst within about 10% of the ones using Eq. (7). The zero of b_ρ moved to $t=-0.16$ GeV^2, while that of b_f disappeared completely for $\alpha_P' \lesssim 0.5\alpha_\rho'$, so that ρ exchange is somewhat more peripheral and f exchange is even less peripheral than before. With $\alpha_P' \simeq \alpha_\rho'$, the zero of b_f again occurred at about the value predicted by DAM, although this slope is much larger than the experimental pp value, as we have seen.

IV. CONCLUSION

In both Ref. 1 and the present paper we used FESR to calculate effective Regge residues in terms of low-energy absorptive parts. In Ref. 1 the latter was taken from an approximately unitarized Lovelace-Veneziano model. In Sec. II, on the other hand, it was assumed to be given by a sum of peripheral resonances plus a background generated by combining unitarity with the assumption that, in nonresonant situations, $T \approx T_{\text{Regge}}$, at least on the average, even at relatively low energies. Otherwise the arguments and conclusions are very similar, however, with the background of Sec. II playing the same sort of role that the central T_J do in Ref. 1. In both cases we are led naturally to a peripheral ρ and nonperipheral f exchange.

In Sec. II the Lovelace-Veneziano model was still used for the ρ and f Regge residues in the background equation (1). In a more complete calculation, however, these residues should instead be made self-consistent with the output ρ and f residues which come out of the sum rule (7). Instead of using a simple Regge-pole form for T_{Regge} we should probably also use Regge behavior only for ImT, where it seems to work better,[2] particularly at lower energies. Then ReT could be calculated from ImT via a dispersion relation. In effect this is what we already did for $s < 2m_\rho{}^2$ in the $I_s=2$ state. Finally, we should add P exchange to the ρ and f in Eq. (1). Of course this kind of calculation would probably be much more involved than the one reported above.

So far, we have only applied our approach for $t \leq 0$. By extending it to $t>0$ we could also calculate output ρ and f^0 resonance widths from the $b_\rho(m_\rho{}^2)$ and $b_f(m_f{}^2)$ which come out of Eqs. (6) and (7). We could then require these to be self-consistent with the input resonant A^J, which we inserted into Eq. (5). This would lead to nonlinear relations. In contrast with certain earlier dual "bootstrap" programs it should therefore be capable of fixing the absolute values as well as the ratios of these widths.

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APPENDIX A: EXTENSION OF OUR METHOD ABOVE THE $\rho\rho$ THRESHOLD

In this appendix the parameters $b_P(0)$ and a will be calculated by extending our method above the $\rho\rho$ threshold. As long as we are still in the quasi-two-body region we shall assume that the dominant intermediate states are $\pi\pi$ and RR, where R is the low-energy 2π system. Something like πA_2 is actually dual to RR so that we would be double counting if we included both at the same time, at least in any given region of phase space. On the other hand, the known resonances in this region, such as the g (Ref. 12) and the recently discovered $\rho'(1500)$ tend to decay primarily into RR.[8] This choice is also the intermediate state most consistent with the Amati-Bertocchi-Fubini-Stanghellini-Tonin model.[13]

Above the $\rho\rho$ threshold the only prominent peaks, such as the g, appear to be in the $I_s=1$ state. None of the narrow high-mass objects reported previously have been confirmed,[8] and even if they were there, their contribution to FESR should be small relative to the background. The broad gentle bumps which are in fact observed[8] should be well approximated on the average by a background-generating expression such as Eq. (1), provided we also include the RR intermediate state. Diagrammatically

this is equivalent to calculating the $I_s=0, 2$ absorptive parts from Fig. 1. We will denote the contributions of Figs. 1(a) and 1(b) as $A^{\pi\pi}$ and A^{RR}, respectively.

Production experiments suggest that the horizontal Regge lines of Fig. 1(b) are well approximated by elementary one-pion exchange, provided we put in an off-shell correction. For the latter we will use the Lovelace prescription[7] of merely putting in an over-all function of the virtual pion mass. In addition we will assume that the low-energy absorptive part A of Fig. 2, out of which Fig. 1(b) can be constructed,[13] can be approximated by a single δ function:[14]

$$A_{I_t}(s,t) \simeq C_{I_t}(t)\delta(s-m_R{}^2)\,, \tag{A1}$$

with

$$C_{I_t}(t) \simeq \int_0^{4m_\rho{}^2} ds\, A_{I_t}(s,t)\,, \tag{A2}$$

where A is given by Eq. (5) and $m_R{}^2 \simeq 2m_\rho{}^2$, which is the midpoint of the elastic region $0<s<4m_\rho{}^2$ in our model. Figure 1(b) then gives

$$A^{RR}_{I_t}(s,t) = |C_{I_t}(t)|^2 w(s,t)\theta(s-4m_R{}^2)\,, \tag{A3}$$

where θ is the usual step function and $w(s,t)$ is a function which contains all the off-shell corrections but is independent of I_t.

In some average sense our quasi-two-body approximation should now be reasonable up to $s \approx 9m_R{}^2$, at which point the RRR intermediate state can be expected to become important. We will therefore take $N=9m_R{}^2$ with $A=A^{\pi\pi}+A^{RR}$ in Eq. (7) and subtract from this the sum rule (7) with $N=4m_\rho{}^2$, which was already enforced separately in Sec. III. Using Eq. (6), we find that the resulting $I_s=0$ and $I_s=2$ sum rules now become

$$\int_{4m_\rho{}^2}^{9m_R{}^2} ds[(\tfrac{1}{3}b_f+b_\rho)s^{\alpha_\rho}+\tfrac{1}{3}b_P s^{\alpha_P}-A^{\pi\pi}_{I_s=0} - (\tfrac{1}{3}|C_0|^2+|C_1|^2+\tfrac{5}{3}|C_2|^2)w]=0\,, \tag{A4}$$

$$\int_{4m_\rho{}^2}^{9m_R{}^2} ds[(\tfrac{1}{3}b_f-\tfrac{1}{2}b_\rho)s^{\alpha_\rho}+\tfrac{1}{3}b_P s^{\alpha_P}-A^{\pi\pi}_{I_s=2} - (\tfrac{1}{3}|C_0|^2-\tfrac{1}{2}|C_1|^2+\tfrac{1}{6}|C_2|^2)w]=0\,. \tag{A5}$$

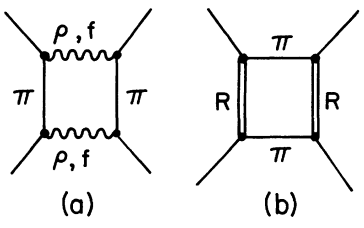

FIG. 1. Unitarity diagrams giving (a) $A^{\pi\pi}$ and (b) A^{RR} for $s>4m_\rho{}^2$. All our external lines are pions.

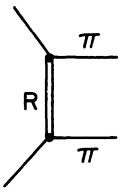

FIG. 2. Low-energy off-shell A which can be used to construct Fig. 1(b).

We have neglected any kind of low-lying $I_t=2$ Regge exchange, which should be quite reasonable for $s>4m_\rho{}^2$.

To evaluate $A_{\pi\pi}$ we could again use Eq. (1), as in Sec. II. Since we are assuming that this is entirely given by background, however, Eqs. (2) may be too crude in this case. Instead we shall take advantage of the fact that $A^{\pi\pi}=f(t)s^{2\alpha_R(t/4)-1}$ for large s, at least to within logarithms and P exchanges, which we shall ignore. This in turn implies that $A^{\pi\pi}$ itself satisfies FESR[9,15]:

$$\int_0^N ds\, A^{\pi\pi} = f(t)N^{2\alpha_R(t/4)}/2\alpha_R(t/4)\,. \tag{A6}$$

To determine $f(t)$ we took $N=4m_\rho{}^2$ and used the expressions for $A=A^{\pi\pi}$ as calculated in Sec. II for $s<4m_\rho{}^2$. The contributions of the $A^{\pi\pi}$ to Eqs. (A4) and (A5) can then be calculated by taking $N=9m_R{}^2$ in Eq. (A6) and subtracting from it the same FESR with $N=4m_\rho{}^2$. If we now eliminate $\int w\,ds$ between Eqs. (A4) and (A5), we obtain a single relation between the Regge parameters at a given value of t. For simplicity we only considered this relation and its t derivative at $t=0$ (the differentiation was carried out numerically).

If we now combine these two constraints with those obtained in Sec. III we obtain $b_f(0)=1.074$, $b_P(0)=0.643$, and $a=4.35-3.20\alpha_P'$. Our asymptotic cross section is thus $\sigma_{\rm tot}\simeq 8.6$ mb, which is smaller than the experimental value of 15 mb. Our ratio $R=0.60$ is also somewhat smaller than the value $R\simeq 1$ suggested by πp and pp experiments, although it must be remembered that this ratio is difficult to extract experimentally. On the other hand, our ratio $b_f(0)/b_\rho(0)=2.05$ is somewhat larger than the exchange-degeneracy prediction of 1.5. However, this ratio is a fairly sensitive one in this calculation and there is in any case no particular reason to expect exchange degeneracy to be very accurate in view of its failure for $t\neq 0$.

If we insert our calculated value of a into Eq. (6), we obtain the diffraction-peak width parameter $w=8.70+9.61\alpha_P'$ at $s=3000$ GeV^2. With $\alpha_P'=0.5\ \alpha_\rho'=0.425$ GeV^{-2} this gives a value $w\simeq 12.8$ GeV^{-2}, which is about the value observed at ISR for pp scattering, while $\alpha_P'=\alpha_\rho'$ gives a value $w=16.9$ GeV^{-2}, which is larger than the experimental pp

value. This would be an added reason for not taking such a large value of α_P'. On the other hand, one needs a large value of α_P' in order to get the sort of zero of $b_f(t)$ predicted by DAM, as we have seen.

APPENDIX B: MULTIPERIPHERAL MODEL CONSTRAINTS

Up to now, we used an $\alpha_P(t)$ inferred from experiment. In the Appendix of Ref. 1, however, we saw that an additional relation between the f and P parameters at any t can be obtained by assuming a simple J-plane structure consistent with a broad class of multiperipheral models.[16,17] By combining this relation and its t derivative at $t=0$ with the constraints of Sec. III, we obtain approximately the same results for $b_P(0)$ and $b_f(0)$ as were quoted in Sec. III. This time, however, we also calculate α_P at the same time, obtaining $\alpha_P(0)=0.986$ and $\alpha_P'=0.218\ \mathrm{GeV}^{-2}$. This sort of α_P', as we saw in Sec. III, leads to a nonperipheral f exchange and is consistent with experiment.

APPENDIX C: CALCULATION OF BACKGROUNDS

Since only the small-t region is important asymptotically, we make the exponential approximation

$$\Gamma(1-\alpha(t))\simeq\Gamma(1-\alpha_0)e^{-t\alpha'\psi(1-\alpha_0)} \tag{C1}$$

in Eqs. (2)–(4), where $\psi=\Gamma'/\Gamma$ and $\alpha_0=\alpha(0)$. We also set $(-s)^\alpha=e^{-i\pi\alpha}s^\alpha$. It is now trivial to make the partial-wave projection

$$T^J_{\mathrm{Regge}}=\int_{-2q^2}^{0}\frac{dt}{q^2}\,T_{\mathrm{Regge}}P_J\left(1+\frac{t}{2q^2}\right), \tag{C2}$$

where we have put in an extra factor of 2 to take into account the contribution of the backward direction and put in the lower limit to correspond to $\theta=90°$, so as to avoid double counting. For $s>2m_\rho^2$ it was also a reasonable approximation to take the lower limit at $t=-\infty$. This gave

$$(T^0_{\mathrm{Regge}})_{I=2}=\frac{2\bar\beta\Gamma(1-\alpha_0)(\alpha's)^{\alpha_0-1}}{\psi_0-\ln\alpha's}, \tag{C3}$$

$$(T^0_{\mathrm{Regge}})_{I=0}=(-\tfrac{1}{2}T^0_{\mathrm{Regge}})_{I=2}\left(1+3i\,\frac{\psi_0-\ln\alpha's}{\psi_0-\ln\alpha's+i\pi}\right), \tag{C4}$$

$$(T^1_{\mathrm{Regge}})_{I=1}=-i\,\frac{2\bar\beta\Gamma(1-\alpha_0)(\alpha's)^{\alpha_0-1}}{\psi_0-\ln\alpha's+i\pi}\times\left(1+\frac{2}{\alpha's}\,\frac{1}{\psi_0-\ln\alpha's+i\pi}\right), \tag{C5}$$

where $\psi_0=\psi(1-\alpha_0)$. These expressions were then inserted into Eq. (1).

In carrying out the s integrals of Eq. (7) for $2m_\rho^2<s<4m_\rho^2$, it was assumed that the variation of the $\ln\alpha's$ terms in Eqs. (C3)–(C5) could be neglected. All these terms were therefore evaluated at $s=1.5/\alpha'=3m_\rho^2$. The integrals $\int_0^{2m_\rho^2}\times A^I(s)ds$ were evaluated by using a simple Simpson three-point rule. In evaluating A^I through Eq. (1) it was important to keep the correct lower limit in Eq. (C2) for $s<2m_\rho^2$, however. For $I=2$, we also followed the more accurate procedure of using Eq. (4) instead of Eq. (3) in evaluating Eq. (2a) for $s<2m_\rho^2$, even though the final results are not too different from each other.

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